When Geography Matters for Growth: Market Inefficiencies and Public Policy Implications

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Abstract

We propose a unique market and social planner solution for a generalized New Economic Geography and Growth model to highlight the importance of taking account of the existence of agglomeration externalities in an analysis of market inefficiencies, which allows us to provide some important implications for public policy. This framework among other things, allows us to disentangle an insufficient growth condition from an under-investment in R&D condition which in turn allows us to explain various market steady state situations. For instance, it provides an explanation for situations where the market economy grows too slowly and over-invest in R&D (as opposed to an a-spatial model). By evaluating the effects of two strategic policies, namely innovation policy and industrial policy, on market inefficiencies, we show that (1) the efficiency of a policy evolves strongly with the market economy situation and no policy is the most efficient in all situations, (2) the geography of economic activities and the question of over or under-agglomeration of the market economy plays a central role on the relative efficiency of policies and (3) industrial and innovation policies are only partially complementary but policy-mixes can be justified if some market gaps are more important than others.

Keywords: agglomeration, growth, spatial income inequality, innovation and industrial policies.
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1 Introduction

The relation between agglomeration and growth has attracted a great deal of interest from economists and generated numerous debates. This is probably because although growth is seen always to be desirable, this does not apply to agglomeration. From a theoretical point of view, several approaches have been applied to try to explain why economic activities are agglomerated, and more importantly, how such agglomeration benefits economic growth. Most of this work relies on the existence of localized external effects such as Marshall-Arrow-Romer spillovers (related to specialization), Jabobs spillovers (related to diversity) or Porter spillovers (related to local competition). Although these different externalities may be at the heart of the growth-enhancing effect of agglomeration, several economists have pointed to the potential negative effects of spatial concentration such as congestion and pollution. Very few empirical studies investigate the relation between growth and agglomeration and they do not identify a clear relationship.

For instance, Crozet and Koenig (2007) exploit data for the EU regions for the 1980-2000 period and find a strong growth-promoting effect of agglomeration, especially for Northern regions. The more recent study by Brülhart & Sbergami (2009) uses a larger panel of 102 countries over the 1960-2000 period and finds evidence of an inverted-U shaped relationship between agglomeration and growth according to the level of development of the country. More precisely, their results suggest that agglomeration fosters growth in the early stages of development then becomes an obstacle in more developed countries. Henderson (2003) using panel data for up to 70 countries over the period 1960-1990 finds no-promoting (or reducing) effect of urbanization on growth. Overall, the existing empirical work does not reject the existence of a growth promoting-effect of agglomeration especially in relation to European countries [see Geppert et al. (2008) and Gardiner et al. (2010)].

Another aspect of the spatial agglomeration[1] of economic activities is spatial income inequalities. Indeed, agglomeration is not neutral for local wages, employment, profitability, consumption and housing prices, etc. This is one of the main features of the New Economic Geography [Krugman (1991), Fujita and Krugman (2004)] literature which links the level of spatial agglomeration to the level of (spatial) income inequality. In those models, the spatial concentration of firms influences both nominal and real income levels in each location. On the one hand, the spatial concentration of firms agglomerates the demand for labor in that location which can lead to higher wages and nominal income for workers. On the other hand, the geography of economic activities influences the price index in each location as soon as traded products are subject to transport costs.[2] These two effects lead to most NEG models suggesting that agglomeration tends to spur spatial income inequalities.

If an agglomeration-growth link exists, agglomeration will influence both the level and "the inclusive" part of economic growth.[3] The issue that arises from this observation is whether there are potential trade-offs between spatial dispersion, spatial income equality, and economic growth. Few models link these three endogenous market economy indicators because this would require a complex multi-country framework of trade and growth. However, the New Economic Geography and Growth (NEGG) framework includes endogenous growth in a New Economic Geography (NEG) framework. This explains endogenously why economic activities (and especially innovative activities) are spatially agglomerated, why capital accumulation dynamics are spurred by the spatial concentration of firms, and how agglomeration can increase spatial income inequality. There are three key elements at the heart of these economic relations: the presence of increasing returns, the presence of transport costs for goods, and the presence of localized knowledge spillovers. The last

[1] Especially in a context where agglomeration spurs economic growth.

[2] A more concentrated economic geography will imply lower price index for the inhabitants of the country where the firms are agglomerated.

[3] Assuming that the "inclusive-growth" effect of agglomeration is well-described by the effect of agglomeration on the level of spatial income inequalities.

two elements are particularly interesting because they reflect trade and technological integration measures which are 1) at the core of the globalization process, and (2) have opposing influences on the benefits of agglomeration.

The main objective in this paper, is to show the importance of taking account of the existence of a growth-agglomeration-inequality link in analyses of market inefficiencies and discussion of the capacity of different policies to bring the economy closer to its optimum.

The first contribution of this paper is that it develops a generalized NEGG model à la Martin and Ottaviano (1999) which combines an endogenous growth model à la Grossman and Helpman (1991) with the Footloose Capital model of Martin and Rogers (1995). Therefore, we are working with an asymmetric two country model where trade is subject to transport costs, production is subject to increasing returns to scale, trade in capital is costless, and labor is immobile. The only source of nominal income inequality is related to a different initial capital endowment. The presence of localized knowledge spillovers from production to R&D (Jacobs spillovers) implies that the agglomeration of industrial and innovative firms spurs economic growth by increasing inventors' access to knowledge. The model proposed in this paper preserves this general framework, and introduces a Benassy (1998) Constant Elasticity of Substitution (CES) function and eliminates the presence of scale effects in the growth rate. The use of this CES function allows us to disentangle the taste of variety (which corresponds to the "returns to specialization/innovation") from the monopoly markup and the share of demand for differentiated products. Consequently, a situation of over-investment in R&D can occur at the market equilibrium, something that is impossible in NEGG models (which endogenously assume under-investment in R&D). Concerning the scale effects on the growth rate, we eliminate them by assuming the presence of duplication in R&D activities via scale effects on the R&D costs.

The second contribution of this paper is that it provides a social planner version of the model. To the best of our knowledge, the literature so far does not propose a social planner solution for a NEGG model. We investigate fundamentals issues such as the optimal level of agglomeration, growth, and spatial income inequality. Concerning the geography of economic activities, three effects of agglomeration that the market does not internalize create a (new) gap between the market and optimal levels of agglomeration. Our proposed model highlights two key parameters which influence the importance of these three external effects - the level of trade integration and the level of technological integration. We show that the social planner has a greater incentive to choose a more dispersed spatial equilibrium if technological integration is high, and a higher incentive to choose a more concentrated spatial equilibrium if trade integration is high. If there is perfect technological integration between locations, the social planner equilibrium tends towards a dispersed geography of economic activities (which is not possible in a market economy.)

The third contribution of this paper is that it provides a detailed analysis of the outcome differentials (market gaps) between the market and social planner solutions. We use Grossman and Helpman (1991) method to distinguish analytically the different sources of market inefficiencies, and link them to key gaps in the model.[4] This highlights the importance of each sources of market inefficiency, and especially those related to agglomeration externalities. Also, the analysis shows the strength of the proposed model. We model six potential market economy situations (e.g., combinations of key gaps) compared to the two enabled by NEGG models (Martin 1999, Baldwin and Forslid 2000, Baldwin, Martin and Ottaviano 2001) and one in first generation of endogenous growth models (Romer 1990, Grossman and Helpman 1991, Aghion and Howitt 1992). The gaps identified are due to two main elements. The first is related to the fact that in the model, the geography of economic activities has a positive influence on both economic growth and R&D productivity. The under-growth condition (optimal growth is higher than market growth) becomes different

[4] The key gaps between social and market outcomes include a growth gap, an agglomeration gap, an income inequality gap and an R&D investment gap.

from the under-investment condition (optimal R&D investment is higher than market R&D investment) which explains situations where the market leads the economy to grow too slowly but to over-invest in R&D (which is not possible in a-spatial growth model). The second element is the introduction of the Benassy CES function which allows potential over-investment in R&D.

The final contribution of this paper is that it provides some implications for public policy. The analysis focuses on the capacity of two strategic policies - innovation and industrial - to reduce the gaps between the market and the optimal outcomes. The choice of these two public policies is motivated by debate in the literature on the need for, the use, and the effect of these policies. To examine the capacity of a policy to reduce market gaps, requires an evaluation of its effects on the market equilibrium at all possible (initial) steady states. Among the six potential states of the economy, there are only two cases where one of these strategic policies is able to reduce the key gaps[5]. These necessarily imply that the market economy is growing too slowly and there is under-investment in R&D. In the other four cases, the policies implemented increase at least one of the gaps. It seems also that the implementation of innovation and industrial policies lead the market economy far away of its optimal steady state if the market economy grows too quickly and over-invest in R&D. Overall, our results show that in a situation of over-agglomeration in the market economy, innovation policy tends to be superior to industrial policy while in a situation of lack of spatial agglomeration of firms, industrial policy tends to be superior. Thus, one explanation for the relative efficiency of public policies is related to the geography of economic activities. From a policy mix perspective, our results point to a lack of complementarity between innovation and industrial policies. However, in some cases, this policy-mix is justified, e.g. if government's aim is to target specific gaps, or attach different weights to each gaps.

The remainder of the article is organized as follows. Section 2 presents our framework and the market economy solution to the model. Section 3 develops a social planner model and highlights the agglomeration externalities induced by the introduction of an agglomeration-growth-inequality link. Section 4 proposes an analysis of the differential outcomes between the market and social planner solutions by relating the various market inefficiencies to different market gaps. Section 5 provides some public policy implications by analyzing the effects of industrial and innovation policies. Section 6 concludes and suggests some directions for further research.

2 The model

The model developed in this paper is a generalization of Martin and Ottaviano (1999). We consider a world composed of two open countries denoted a and b which differ only in relation to their initial capital endowment $K_a(0) > K_b(0)$. Their technology and labor forces are identical, their labor is immobile geographically but not sectorally, and capital mobility is perfect between countries.

2.1 Demand side

The utility function is assumed to be the same in both locations. We consider the following utility function for a representative consumer in country a:

$$U^a = \int_0^\infty \log \left[C_z^a(t)^{1-\alpha} C_m^a(t)^\alpha \right] e^{-\rho t} dt \tag{2.1}$$

[5] Note that the policies studied are not able to achieve a first best situation since they do not correct for the markup power of firms.

where C_z represents the consumption of a homogeneous good, and C_m represents the consumption of a composite good made up of a large number of differentiated products according to a Benassy-type (1998) CES function:

$$C_m^a(t) = \left[\int_{i=0}^{N_a(t)} c_i^a(t)^{\frac{\sigma-1}{\sigma}} di + \int_{j=0}^{N_b(t)} c_j^a(t)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}} N(t)^{\frac{\mu}{\alpha}-\frac{1}{\sigma-1}} \tag{2.2}$$

where $N = N_a+N_b$ is the number of varieties available to produce the composite good, and c_i^a and c_j^a represent the demand for the variety produced in countries a and b related to the consumption of the composite good. Using this modified Dixit-Stiglitz (1977) CES function, we disentangle the taste for variety ($\mu \in [0,1[$) from the elasticity of substitution between varieties ($\sigma > 1$). In what follows we make the dependence of variables on time implicit apart from the variables with the subscript 0. Saving occurs in the form of a riskless asset that pays an interest rate r, or in the form of investment in the shares of M firms on a world stock market. As each consumer offers inelastically one unit of labor, we can write the intertemporal budget constraint as:

$$\dot{S}_a = w + rS_a - E_a \tag{2.3}$$

where S represents the consumer's saving, w is the wage, r is the interest rate, and E represents expenditures on consumption:

$$E_a \equiv C_z^a + \int_{i=0}^{N_a} p_i^a c_i^a di + \int_{j=0}^{N_b} \tau p_j^a c_j^a dj \tag{2.4}$$

where p_i^a and p_j^b are the producer prices in countries a and b respectively. Following the NEG models, we introduced transaction costs $\tau > 1$ on the differentiated goods, in the form of iceberg costs, while trade in the homogeneous good is costless. We define the price of the composite good denoted P as an index of the prices of all varieties:

$$P_a \equiv N^{\frac{1}{(\sigma-1)}-\frac{\mu}{\alpha}} \left(\int_{i=0}^{N_a} (p_i^a)^{1-\sigma} di + \int_{j=0}^{N_b} (\tau p_j^a)^{1-\sigma} dj\right)^{1/(1-\sigma)} \tag{2.5}$$

Utility-maximization gives rise to a two-stage maximization problem. First, consumers determine their demand for the homogeneous and differentiated goods using utility functions (2.1) and (2.2) subject to the budget constraint (2.3). For a representative consumer living in country a, we obtain the usual consumer demands:

$$\begin{aligned} C_z^a = (1-\alpha)E^a \qquad & C_m^a = \frac{\alpha E^a}{P_a} \\ c_i^a = & \left(\frac{p_i^a}{P_a}\right)^{-\sigma} N^{\left(\frac{\mu(\sigma-1)}{\alpha}-1\right)} C_m^a \\ c_j^a = & \left(\frac{\tau p_j^a}{P_a}\right)^{-\sigma} N^{\left(\frac{\mu(\sigma-1)}{\alpha}-1\right)} C_m^a \end{aligned} \tag{2.6}$$

Second, consumers trade off current consumption against future consumption, taking as given the relative price of consumption over time. This leads to the classical Euler equation:

$$\frac{\dot{E}_a}{E_a} = r - \rho \tag{2.7}$$

Note that the demands of a representative consumer living in country b are symmetric to those of a consumer living in country b. Thus, the demands of a representative consumer in country b are given simply by changing the subscripts a by b in expression (2.6), adding the transport cost to p_i, and removing the transport cost to p_j. The Euler equation is the same for both countries due to free capital movements so that $\dot{E}_a/E_a = \dot{E}_b/E_b = r - \rho$.

2.2 Production side

Both countries use similar technologies. There are three production sectors (Z,M,I), and two inputs - capital and labor. Capital is mobile geographically, and labor is mobile sectorally.

The T sector produces a homogeneous good in a competitive environment with a constant returns to scale technology using labor:

$$Y_Z = L_Z \tag{2.8}$$

where L_Z is the labor used in the Z sector. Using this good as the numéraire, profit maximization gives $p_z = w_z = 1$ at all period of time where w_z represents the wage rate in the Z sector. We assume also that the production of the homogeneous good takes place in both countries so that the wage rate in the three sectors and in both countries is given by $w = 1$.

The M sector produces differentiated goods in a monopolistic environment with increasing returns to scale technology using labor as variable input, and one unit of capital as the fixed cost:

$$Y_M = \frac{L_M}{\beta} \tag{2.9}$$

where L_M is the labor used in the M sector. As $\sigma > 1$ is the elasticity of substitution between varieties, the profit maximization rule in a monopolistic market gives $p_i = p_j = p = \beta\sigma/(\sigma - 1)$ so that the producer prices of all varieties in both countries are the same. M firms finance the acquisition of capital by issuing shares (which are bought by consumers with their savings) on a world stock market. As one unit of capital is required to start the production of each variety, the total number of M firms (N) is fixed by the number of world patents (K):

$$N_a + N_b = K_a + K_b = K$$

The I sector produces capital which takes the form of an infinitely lasting patent for the design of a new variety in a competitive environment with constant returns to scale technology, and using labor and localized knowledge spillovers:

$$Y_I = \dot{K} = L_I \frac{KW}{\eta L} \tag{2.10}$$

where L_I is the labor used in sector I. Profit maximization implies that the price of one unit of capital is equal to the marginal cost of the patent: $p_K = \eta L/KW$. The localized spillovers are measured by $KW = K \max\{W_a; W_b\}$ where $W_a \equiv s_n + \lambda(1 - s_n)$ and $W_b \equiv \lambda s_n + (1 - s_n)$ measures the diffusion of the total spillovers in each country. Note that we assume $K_a(0) > K_b(0)$ with free capital movement, equilibrium location of M-firms $s_n \in [1/2; 1]$ so that $W_a \geq W_b$. This implies in our case that $W = W_a \equiv s_n + \lambda(1 - s_n)$.

When $W_a > W_b$, the marginal cost of the patent is lower in country a so that all R&D activities are located in country a and $L_I = L_I^a$ as $L_I^b = 0$. When $W_a = W_b$, then both locations offers same marginal cost, so that R&D firms are indifferent between the two locations. In this latter case, the location of the R&D firms does not matter for the economic outcome. Thus, we know that a solution with $L_I^a \geq 0$ and $L_I^b = 0$ (driven by the market but also by a social planner) will always be a possible equilibrium (and exactly the equilibrium for $s_n > 1/2$). This justifies the focus in the rest of the paper on solutions where the I-sector is active only in the country a.

2.3 Equilibrium relations

Consumer demands for differentiated goods, and the location of M-firms

Inserting the producer prices into the consumer demands for differentiated goods given in (2.6), we obtain:

$$c_i^a = \frac{\sigma - 1}{\beta\sigma} \frac{\alpha E_a}{N_a + \phi N_b} \qquad\qquad c_j^a = \frac{\sigma - 1}{\beta\sigma} \frac{\alpha E_a \tau^{-\sigma}}{N_a + \phi N_b} \tag{2.11}$$

and symmetrically, we have the following demands for country b:

$$c_i^b = \frac{\sigma - 1}{\beta\sigma} \frac{\alpha E_b}{\phi N_a + N_b} \qquad\qquad c_j^b = \frac{\sigma - 1}{\beta\sigma} \frac{\alpha E_b \tau^{-\sigma}}{\phi N_a + N_b} \tag{2.12}$$

Given this result, market effects lead to the following equilibrium relationships between the consumption levels in the two countries:

$$c_i^a \phi = c_j^a \tau \qquad\qquad c_j^b \phi = c_i^b \tau \qquad\qquad c_j^b = \frac{E_b}{E_a} \left(\frac{s_n + \phi(1 - s_n)}{\phi s_n + (1 - s_n)} \right) c_i^a \tag{2.13}$$

where $s_n \equiv N_a/N$ is the share of M-firms located in country a. The location of firms is free and we assume no relocation costs, so that if a firm owned by an agent in country a locates in country b, then the operating profits of this firm are repatriated to country b. At equilibrium, production equals consumption so that the production of M-firms is given by $y_i^a = L(c_i^a + \tau c_i^b)$ in country a, and by $y_j^b = L(c_j^b + \tau c_j^a)$ in country b. Since capital flows are unrestricted, location equilibrium implies that no M-firms have an incentive to delocate, i.e., both locations offer the same operating profits ($\pi_a = \pi_b$). This condition holds if $y_i^a = y_j^b = y$. We thus obtain the following equilibrium location condition:

$$s_n \equiv N_a/N = \frac{1}{2} + \frac{1 + \phi}{1 - \phi} \left(s_e - \frac{1}{2} \right) \tag{2.14}$$

and the following level of production for M firms

$$y = \alpha L \frac{\sigma - 1}{\beta\sigma} \frac{E}{N} \tag{2.15}$$

Equilibrium condition (2.15) says that the location with the largest market size or the highest expenditure level will attract the most firms. Due to transaction costs and increasing returns, firms want to be located near to the largest markets. This result is the famous "home market effect" highlighted in the NEG literature (Krugman, 1991). When transaction costs are low, i.e., ϕ is large, the sensitivity of the location decision to market size differentials increases because it makes it easier for firms to locate in the largest market and export to the other location.

Labor market

Global labor supply is given by $2L$. Labor demand from the Z sector is given by $L_Z = L(1 - \alpha)E$. Labor demand from the M sector is given by $L_M = \beta N y = \alpha L(\sigma - 1)E/\sigma$. Labor demand from the I sector is given by $L_I = \eta L\dot{K}/KW$ so that equilibrium in the labor market is:

$$2L = g\left(\frac{\eta L}{W}\right) + \left(\frac{\sigma - \alpha}{\sigma}\right) LE \tag{2.16}$$

where $g = \dot{K}/K$ is the patent growth rate. From this equation we can see immediately that a balanced growth path is possible only if world expenditures are constant over time. From (2.7), it follows that $r = \rho$.

Financial market

Recall that saving takes place in the form of a riskless asset which pays an interest rate r, or in the form of investment in the shares of M-firms in the world stock market. As an infinite lasting lived patent is required to start production of a new variety, the firm that buys the patent has perpetual monopoly over particular good. The value of an M firm on the stock market is the present discounted value of all future operating profits. These operating profits will be the same in both locations so long as there are no capital movement restrictions. This implies also that the value of any firm in the world is:

$$v(t) = \int_t^{\infty} e^{-[R(s)-R(t)]} \frac{\beta y(s)}{\sigma - 1} ds \tag{2.17}$$

where R(t) represents the cumulative discount factor applicable to the profits earned at time t. Differentiating with respect to time, we get the arbitrage condition on capital markets:

$$\frac{\beta y}{\sigma - 1} + \dot{v} = rv \tag{2.18}$$

which says that the returns on the different riskless assets must be equalized. On an investment of size v in a firm, the return is equal to the operating profits (or the dividends paid to the shareholders) plus the change in the value of the firm (the capital gains or losses). Due to monopolistic competition in the M sector, the equilibrium value of an M firm should be equal to the value of its asset, i.e, the value of a patent. Therefore, we have the following equality: $v = p_K = \eta L/KW$ and $\dot{v}/v = -g$ implying that the value of a firm decreases at the same rate as the rate of entry of new competitors in the market.

The growth rate of capital and living standards

Inserting (2.14) and (2.15) in (2.18), and using the fact that at equilibrium we have $r = \rho$, $v = \eta L/KW, \dot{v}/v = -g$, we can express the agregate growth rate of capital as:

$$g = \max \left\{ 0, \frac{2\alpha W}{\sigma \eta} - \rho \left(\frac{\sigma - \alpha}{\sigma} \right) \right\} \tag{2.19}$$

This expression shows that agglomeration spurs economic growth. We see also that the market economy growth rate is completely independent of the returns to innovation measured by μ, but is a function of the share of differentiated goods (α), the elasticity of substitution (σ), and the interest rate (ρ). Note that if expenditures are constant in both countries at the equilibrium, the consumption of the composite good C_m evolves over time. Inserting equilibrium levels of consumption (2.11) in (2.2), we can express the rate of growth of consumption of the composite good (which is the same in both countries) as:

$$\frac{\dot{C}_m}{C_m} = \frac{\mu}{\alpha} g$$

so that consumption of the composite good increases over time.

In the remainder of the paper, we measure the standard of living of consumers based on their real income. The real income of a consumer is equal to the nominal income divided by the price index. Given our CES function, and the prices charges by M firms, the price indices of composite goods are given by:

$$\begin{aligned} P_a &= N^{\frac{1}{(\sigma-1)} - \frac{\mu}{\alpha}} \left(\frac{\beta\sigma}{\sigma - 1} \right) \left(\int_{i=0}^{N_a} di + \int_{j=0}^{N_b} \phi dj \right)^{1/(1-\sigma)} = N^{-\frac{\mu}{\alpha}} \left(\frac{\beta\sigma}{\sigma - 1} \right) [s_n + \phi(1 - s_n)]^{1/(1-\sigma)} \\ P_b &= N^{\frac{1}{(\sigma-1)} - \frac{\mu}{\alpha}} \left(\frac{\beta\sigma}{\sigma - 1} \right) \left(\int_{i=0}^{N_a} \tau di + \int_{j=0}^{N_b} di \right)^{1/(1-\sigma)} = N^{-\frac{\mu}{\alpha}} \left(\frac{\beta\sigma}{\sigma - 1} \right) [\phi s_n + (1 - s_n)]^{1/(1-\sigma)} \end{aligned} \tag{2.20}$$

Given consumers' preferences given by (2.1) and (2.2) and the normalization of the homogeneous good price, the global price indices at equilibrium are given by $P_a^* = P_a^\alpha$ and $P_b^* = P_b^\alpha$. Consequently, the real income in both countries denoted by E_a^* and E_b^* respectively, evolves according to:

$$\frac{\dot{E}_a^*}{E_a^*} = \frac{\dot{E}_b^*}{E_b^*} = \mu g_K \tag{2.21}$$

Thus, at equilibrium, the standard of living of consumers evolves at the same rate in both countries. The improvement in living conditions in both countries depends on the capital (or patent) growth rate but also on the taste for variety. Consequently, the strength of capital growth is not sufficient to ensure an important improvement in living conditions over time.

Inequality of revenues

Using (2.16), (2.19) and the fact that at equilibrium $r = \rho, \dot{v}/v = -g$, per capita expenditure at the world level is given by:

$$E = E_a + E_b = 2 + \rho\frac{\eta}{W} \tag{2.22}$$

Per capita world income is equal to the world labor per capita income (twice the wage rate given by 1) plus the income from investments which is the value of the firms owned in each location ($rS^a = \rho v K_a/L$ in country a and $rS^b = \rho v K^b/L$). Given that $v = \eta L/KW$, it is straightforward that per capita income equals per capita expenditure at both the national and world levels. Thus, we can write:

$$E_a = 1 + \rho\frac{\eta s_k}{W} \qquad E_b = 1 + \rho\frac{\eta(1-s_k)}{W}$$

where $s_k = K_a/K$ is the share of capital owned in each location. Therefore, the nominal income inequality is given by:

$$s_e = E_a/E = \frac{1}{2} + \frac{\alpha\rho(2s_k - 1)}{2\sigma(g+\rho)} \tag{2.23}$$

Expression (2.23) shows that a higher growth rate decreases the level of nominal income inequality. This is because more growth implies more competition in the market and less profit for firms. The value of capital decreases which in turn reduces the nominal income inequality because capital endowment is the only pure source of income inequality in the market. Consequently our model predicts a negative relationship between nominal income inequality, and the level of growth. Since agglomeration spurs growth, increased agglomeration decreases the nominal income inequality. Nevertheless, unambiguously, the negative relationship disappears if we consider the effect of agglomeration on the real income inequality. More agglomeration implies a decrease in the price index for the inhabitants of country a, whereas it increases the price index for inhabitants of country b (see 2.20).

2.4 The steady state

The model steady sate is defined by the triplet (s_n, s_e, g) which satisfies equilibrium conditions (2.14), (2.19) and (2.23). Inserting (2.19) and (2.23) in (2.14), we can express the equilibrium level of agglomeration (s_n) as:

$$s_n = \min\left\{1, \frac{1}{2} + \frac{1}{4}\frac{\sqrt{(1+\lambda+\rho\eta)^2(1-\phi)^2 + 4\eta\rho(1-\phi^2)(1-\lambda)(2s_k-1)} - (1+\lambda+\rho\eta)(1-\phi)}{(1-\phi)(1-\lambda)}\right\} \tag{2.24}$$

This expression highlights the forces at play for and against spatial concentration in a market economy. The first is the initial capital endowment inequality (s_k) which constitutes a centripetal force. Indeed, the higher the capital inequality the higher the spatial concentration in the core country (see 2.14). The second is the level of trade integration (ϕ) which is also a centripetal force as $\partial s_n/\partial\phi > 0$. As trade integration increases, firms have an interest in locating in the core country in order to benefit from scale economies, and to export to the periphery country (which is less costly due to trade cost reductions). Note that complete agglomeration ($s_n = 1$) occurs as soon as $\phi > \tilde{\phi} = [1+\rho\eta(1-s_k)]/[1+\rho\eta s_k]$. The third force is the level of technological integration (λ) which is a centrifugal force as $\partial s_n/\partial\lambda \leq 0$ if $\phi \in [0, \tilde{\phi}]$. This relation is intuitive because all else being equal, an increase in λ, implies better transmission of knowledge between countries, higher growth (and stronger competition) and less income inequality. This reduces the incentives to locate in the core country. If technology integration is a centrifugal force, this is not sufficient to generate a dispersed economic geography. As soon as capital inequality exists ($s_k > 1/2$), the dispersion case is never a spatial equilibrium of a market economy whatever the level of technological integration (including the perfect integration case where $\lambda = 1$). To summarize, the centripetal force of trade integration is able to generate a core-periphery structure whereas the centrifugal force of technology integration is not able to generate a dispersed structure.

These elements together lead to the following proposition:

Proposition 1: *In a market economy composed of two asymmetric countries with capital mobility and labor immobility, (1) a core-periphery structure occurs only if trade integration is sufficiently high, (2) a dispersed structure is never a spatial equilibrium, and (3) the returns to specialization do not influence the steady state.*

Inserting expression (2.24) into (2.19) and (2.23), we obtain the equilibrium level of growth, and nominal income inequality. To fully describe the market steady state, we can express the labor employed in each sector as:

$$L_I = \frac{\eta L g}{W} \qquad L_M = \alpha L \left(\frac{\sigma-1}{\sigma}\right)\left(2+\frac{\eta\rho}{W}\right) \qquad L_Z = (1-\alpha)L\left(2+\frac{\eta\rho}{W}\right) \tag{2.25}$$

Expressions (2.25) shows that a higher spatial concentration is associated with a higher level of employment in the R&D sector, and a lower level of employment in the two good producer sectors. The ratio of labor devoted to differentiated products, and to the homogeneous product (L_M/L_Z) is given by the marginal rate of substitution between the composite good and the homogeneous good ($\alpha/(1-\alpha)$) multiplied by the inverse of the markup charged by M firms ($(\sigma-1)/\sigma$). Using (2.25), we can express each country's demand for labor:

$$\begin{aligned} L_I^a &= \frac{\eta L g}{W} & L_I^b &= 0 \\ L_M^a &= s_n L_M & L_M^b &= (1-s_n)L_M \\ L_Z^a &= L - s_n L_M - L_I^a & L_Z^b &= L - (1-s_n)L_M \end{aligned}$$

3 The social planner model

In this section, we propose the social planner version of the model presented in section 2. We assume that the homogeneous good is produced in both countries. Then the social planner problem is given by:

$$\underset{\{C_i,s_n\}}{\text{maximize}} \int_0^\infty \left(L\log\left[C_z^a(t)^{1-\alpha}C_m^a(t)^\alpha\right] + L\log\left[C_z^b(t)^{1-\alpha}C_m^b(t)^\alpha\right]\right)e^{-\rho t}dt$$

$s.t.$

$(A.1)\quad \dot{K} = L_I\dfrac{KW}{\eta L}, \qquad (A.2)\quad L_M = \beta Y_M, \qquad (A.3)\quad L_Z = Y_Z$

$(B)\quad 2L = L_I + L_M + L_Z$

$(C.1)\quad Y_Z^a + Y_Z^b = L(C_z^a + C_z^b), \qquad (C.2)\quad Y_M^a + Y_M^b = LN[s_n(c_i^a + \tau c_i^b) + (1-s_n)(\tau c_j^a + c_j^b)]$

$(D.1)\quad L_I^a \geq 0 \wedge L_I^b = 0, \qquad (D.2)\quad L(C_z^a + C_z^b) > L$

$(E.1)\quad s_n \in [0,1], \qquad (E.2)\quad \forall i, C_i \in \mathbb{R}_+$

$(F)\quad K(0) = K^a(0) + K^b(0) > 0$

where $C_i = \{C_z^a, C_z^b, c_i^a, c_i^b, c_b^j, c_b^j\}$ represents the set of consumption variables.

In the maximization program, (A.1)-(A.3) represents technology constraints including the capital accumulation dynamics, (B) is the world labor market clearing condition, (C.1) and (C.2) are world product market clearing conditions for the homogeneous and the differentiated goods, (D.1) and (D.2) are specific constraints imposed on the model. (E.1) and (E.2) represent constraints on the controls, and (F) is our initial condition on world capital stock. Note that as we do not consider capital stock depreciation, we have $K(t) > 0, \forall t$ and we have not pure state constraints in our problem. Inserting constraints (A.), (B) and (C.) into the utility function, and exploiting the fact that $N = K$, we obtain the following present value Hamiltonian:

$$\begin{aligned} H =& L\log\left[(C_z^a)^{1-\alpha}K^{\mu+\alpha}\left(s_n(c_i^a)^{\frac{\sigma-1}{\sigma}} + (1-s_n)(c_j^a)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\alpha\sigma}{\sigma-1}}\right] \\ &+L\log\left[(C_z^b)^{1-\alpha}K^{\mu+\alpha}\left(s_n(c_i^b)^{\frac{\sigma-1}{\sigma}} + (1-s_n)(c_j^b)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\alpha\sigma}{\sigma-1}}\right] \\ &+\psi\left[K\frac{W}{\eta}\Big(2 - \beta K s_n[c_i^a + \tau c_i^b] - \beta K(1-s_n)[c_j^b + \tau c_j^a] - C_z^a - C_z^b\Big)\right] \end{aligned} \tag{3.1}$$

The related Lagrangian - which includes constraints (D.1) and (D.2) is given by:

$$\mathcal{L} = H(t,K,C_i,s_n,\psi) + \theta_1[2 - \beta K s_n[c_i^a + \tau c_i^b] - \beta K(1-s_n)[c_j^b + \tau c_j^a] - C_z^a - C_z^b] + \theta_2[L(C_z^a + C_z^b) - L] + \theta_3 s_n + \theta_4[1-s_n]$$

3.1 Necessary and sufficient conditions

Due to the log-utility function, we ensure that H is concave with respect to the controls (C_i, s_n) and the state variable (K). The social planner solution has to satisfy the following necessary (and sufficient) conditions:

$$
\begin{array}{llll}
(a) & \dfrac{\partial \mathcal{L}}{\partial C_i} \leq 0, & C_i \geq 0, & C_i \dfrac{\partial \mathcal{L}}{\partial C_i} = 0 \\
(b) & \dfrac{\partial \mathcal{L}}{\partial s_n} = 0 & & \\
(c) & \dfrac{\partial \mathcal{L}}{\partial \theta_i} \geq 0, & \theta_i \geq 0, & \theta_i(t) \dfrac{\partial \mathcal{L}}{\partial \theta_i} = 0 \\
(d) & \dfrac{\partial \mathcal{L}}{\partial \psi} = \dot{K} & & \\
(e) & \dfrac{\partial \mathcal{L}}{\partial K} = -[\dot{\psi} - \rho\psi] & & \\
(f) & \lim_{t\to\infty} e^{-\rho t}\psi(t) \geq 0, & & \lim_{t\to\infty} e^{-\rho t}\psi(t)K(t) = 0
\end{array}
$$

In the following, we focus on an interior solution to this problem, i.e., we look for an optimal social path implying $L_I^i > 0$. Using the necessary condition (c) implies $\theta_1 = \theta_2 = 0$.

3.2 Consumption levels

Using condition (a), we obtain the following core relationships between consumption values:

$$
\begin{aligned}
C_z^a &= C_z^b = (1-\alpha)\frac{\eta L}{\psi K W} \\
c_i^a &= \frac{\tau}{\phi} c_j^a = \alpha \frac{\eta L}{\psi K^2 \beta W [s_n + \phi(1-s_n)]} \\
c_j^b &= \frac{\tau}{\phi} c_i^b = \left(\frac{s_n + \phi(1-s)}{\phi s_n + (1-s)}\right) c_i^a
\end{aligned}
\tag{3.2}
$$

Using these preliminary results and bearing in mind that the social planner problem assigns equal weights to the consumer's utility whatever the consumer's location, it is straightforward to see that the social planner chooses consumption levels such that the total consumption cost is the same in both countries. In other words, (spatial) income inequality disappears in the social planner's choice, and we have $E_a = E_b$ where $E_a = C_z^a + N_a c_i^a + N_b \tau c_j^a$ and $E_b = C_z^b + N_a \tau c_i^b + N_b c_j^b$. Consequently, the optimal level of income inequality is given by $s_e^* = 1/2$.

3.3 Shadow price of capital, and the social planner's growth rate

Using equilibrium consumption relationships (3.2) with conditions (d) and (e), we obtain the following expression for the dynamics of capital accumulation:

$$
\dot{K} = 2\frac{KW\psi - L\eta}{\eta\psi} \tag{3.3}
$$

and the following dynamics for the shadow price of capital:

$$
\dot{\psi} = \rho\psi + 2\frac{L\eta(1-\mu) - KW\psi}{K\eta} \tag{3.4}
$$

Based on the fact that at the steady state the consumption of the homogeneous good is stable over time $\dot{C}_z^a = 0$, it follows that the dynamics of the shadow price of capital is given by $\dot{\psi} = -\psi\dot{K}/K$. Combining this last result with (3.3) and (3.4), we can express the shadow price of capital as:

$$\psi = \frac{2\mu L}{\rho K} \tag{3.5}$$

The shadow price of capital at time t is not directly influenced by the geography of economic activities. In fact, the influence is indirect (via the growth rate) since the dynamics of the shadow price of capital is given by $\dot{\psi}/\psi = -g$. We can now determine the aggregate growth rate of capital by inserting (3.5) in (3.3):

$$g^* = \max\left\{0, \frac{2W^*}{\eta} - \frac{\rho}{\mu}\right\} \tag{3.6}$$

This expression highlights some core properties of the first-best growth rate. The social growth rate is linearly (and positively) related to the level of agglomeration ($W^* = s_n^* + \lambda(1 - s_n^*)$) but negatively related to the technology parameter η and the time preference parameter ρ. In contrast to the rate of growth of the market economy, the social growth rate does not depend on the elasticity of substitution between varieties (σ), or the share of the differentiated good (α). Instead, it is linearly dependent on the returns to innovation (the taste for variety parameter) μ, while this parameter is absent from the market solution. It is clear that this is due to the fact that the market reacts only to profit-incentives, whereas the social planner takes account of the consumer-surplus generated by innovation.

3.4 The steady state of the social planner problem

The optimal level of agglomeration (s_n^*) has to respect condition (b):

$$\frac{\partial \mathcal{L}}{\partial s_n} = \frac{\partial H}{\partial s_n} + \theta_3 - \theta_4 = 0 \tag{3.7}$$

Using the previous intermediate results, we can express the partial derivative of the Hamiltonian with respect to the level of spatial concentration (s_n) as:

$$\frac{\partial H}{\partial s_n} = -\frac{\alpha L}{(\sigma - 1)} \frac{(1-\phi)^2(2s_n - 1)}{[s_n + \phi(1 - s_n)[\phi s_n + (1 - s_n)]} - \frac{2L(1-\lambda)}{s_n + \lambda(1 - s_n)} + \frac{4\mu L(1-\lambda)}{\rho\eta} \tag{3.8}$$

This above expression highlights the three effects that are not internalized by investors when they choose their location. The first (transport cost effect) is related to the effect of the location' choice on the transport costs induced by international trade. This effect is negative since the global transport cost increases with spatial concentration. The second (nominal consumption effect) is related to effect of the location' choice on the nominal consumption level. This second effect is also negative because more spatial concentration increases the labor devoted to the R&D sector, and thus decreases the labor force available to produce consumption goods. The third (growth effect) is related to effect of the location' choice on the aggregate growth rate. Since spatial concentration is pro-growth, this last effect is positive. Consequently, whereas the equilibrium market location choice is driven only by relative location profitability, the social planner takes account of these three effects.

Expression (3.8) shows that, in contrast to the market solution, the social planner level of agglomeration integrates the level of taste for variety (μ), the elasticity of substitution between varieties (σ) but also the share of differentiated goods in total consumption (α). As μ only influences the positive growth effect, a higher taste for variety gives the social planner a greater incentive to concentrate economic activities. σ and α influence only the transport cost burden effect but in opposite ways. Indeed, an increase in σ will decrease the negative transport cost burden effect, whereas an increase in α increases the importance of this negative effect. In other words, a higher σ gives the social planner a greater incentive to concentrate economic activities, whereas an increase in α gives the social planner a greater incentive to disperse economic activities.

If $\partial H/\partial s_n > 0$, then condition (b) is satisfied if $\theta_3 < \theta_4$. As $\theta_3 \geq 0$ (see condition (c)), thus we have $\theta_4 > 0$ which implies that $s_n^* = 1$. If $\partial H/\partial s_n < 0$, then $\theta_3 > \theta_4$ which implies that $s_n^* = 1/2$. The last case implies that $\partial H/\partial s_n = 0$ which corresponds to the case where the three effects in (3.7) compensate for each other. This last constraint corresponds to the roots of a third degree polynomial equation in s_n. This is not surprising given the presence of the three effects. It is due also to the fact that if the growth effect and the nominal consumption effect influence both locations uniformly, the trade cost effect is not the same in both locations. Indeed this effect is positive for consumers located in country a but negative for those located in country b. Since there always exists at least a real solution to a third degree polynomial equation, we can express the steady state level of spatial concentration as:

$$s_n^* = \min\{1, \max\{1/2, s^*\}\} \tag{3.9}$$

where s^* is the real solution to $\partial H/\partial s_n = 0$. The trigonometric expressions for s^* are reported in appendix A. There are two important remarks related to the optimal level of agglomeration obtained by analyzing the sign of $\partial H/\partial s_n$. The first point concerns the relative importance of the consumption and growth effect. Indeed, we can show easily that the positive growth effect is higher than the negative nominal consumption effect. The second point concerns the sensitivity of the three effects to the level of trade integration (ϕ) and the level of technological integration (λ). Expression (3.8) shows that the level of trade integration only influences the transport cost effect. More precisely, the (negative) transport cost effect is decreasing and convex with ϕ. This implies that in a context of increasing trade integration, the negative (transport cost) effect of more agglomeration decreases increasingly. Consequently, when trade integration tends to be perfect then only nominal consumption and growth effects are at play. Since the growth effect is stronger than the nominal consumption effect, the optimal level of spatial concentration is the maximum feasible, i.e., a complete agglomeration ($s_n^* = 1$).

Expression (3.8) shows also that the level of technological integration influences only the (negative) nominal consumption effect and the (positive) growth effect. The (negative) nominal consumption effect is decreasing and convex with λ. This means that in a context of increasing technological integration, the negative (consumption) effect of more agglomeration decreases at an increasing rate. The (positive) growth effect decreases linearly with the level of technological integration. This implies that in a context of increasing technological integration, the positive (growth) effect of agglomeration decreases. When technological integration tends to be perfect (global spillovers case), then only the (negative) transport cost effect matters. In that case, agglomeration has a negative influence on welfare so that the optimal level of spatial agglomeration is the minimum feasible, i.e., complete dispersion ($s_n^* = 1/2$).

To summarize, an increase in trade integration provides the social planner with a greater incentive to agglomerate economic activities, while an increase in technological integration gives the social planner a greater incentive to disperse economic activities. This last result has important implications in a context of increasing integration: (1) the optimal level of agglomeration depends fundamentally on the gap between trade and technological integration and (2) heterogeneity in the dynamics of trade integration and technological integration imply strong changes to the optimal level of spatial concentration over time.

The previous results lead to the following proposition

Proposition 2: *In a social economy with capital mobility and labor immobility, (1) a core-periphery structure will arise if trade integration is sufficiently high (or if the technology integration is sufficiently* *low compared to trade integration), (2) a dispersed geography is never a spatial steady state if technological integration is not perfect but (3) it becomes the spatial steady state if technological integration is perfect.*

To obtain the optimal growth rate, it is necessary to insert expression (3.9) into (3.6). To fully describe the social planner steady state, we can express the labor employed in each sector as:

$$L_I^* = \frac{\eta L g^*}{W^*} \qquad L_M^* = \alpha \frac{\eta \rho L}{\mu W^*} \qquad L_Z^* = (1-\alpha)\frac{\eta \rho L}{\mu W^*} \tag{3.10}$$

There are two main points related to the allocation of labor between sectors. First, it is easy to see that spatial concentration increases the labor devoted to the R&D sector (as $\partial L_I / \partial s_n > 0$) and decreases the labor devoted to the two consumption-good producing sectors. The ratio of labor devoted to the homogeneous product to the labor devoted to the differentiated products (L_Z/L_M) is given by the marginal rate of substitution between the homogeneous and the composite good ($\alpha/(1-\alpha)$). Using (3.10), we can express labor demand in each country as:

$$\begin{aligned} L_I^{a*} &= \frac{\eta L g^*}{W^*} & L_I^{b^*} &= 0 \\ L_M^{a*} &= s_n^* L_M^* & L_M^{b*} &= (1-s_n^*)L_M^* \\ L_Z^{a*} &= L - s_n^* L_M^* - L_I^a & L_Z^{b*} &= L - (1-s_n^*)L_M^* \end{aligned}$$

4 Analysis of the social planner and market outcome differential

In this section, we analyze the differences between the social planner and the market outcome and highlight the different market inefficiencies. Our objective is to compare the allocation of labor between sectors given by the market (2.25) and by the social planner (3.10).

A first difference concerns the relative allocation of labor between the good-producing sectors. Indeed, we have:

$$\frac{L_M}{L_Z} = \frac{\alpha}{1-\alpha}\left(\frac{\sigma-1}{\sigma}\right) < \frac{\alpha}{1-\alpha} = \frac{L_M^*}{L_Z^*} \tag{4.1}$$

so that the social planner allocates relatively more labor to the M sector than the market. The difference equals the markup charged by M firms in the market outcome. The non-competitive price in the market economy leads to too low a level of demand for differentiated goods such that an insufficient level of resources is devoted to the industrial sector. This is the first market inefficiency in our model. Sorensen (2006) suggests two instruments to correct this market failure: (1) a subsidy that covers part of the purchasing costs on the differentiated goods (demand-oriented), or (2) a subsidy proportional to the production of differentiated goods (supply-oriented). According to Sorensen, both instruments give firms incentives to increase their production, and hence to increase the labor employed in the differentiated-good producing sector. In the next section we analyze the effect of different public policies and show that in a trade and growth model such subsidies do not always increase production.

Another difference is related to the allocation of labor to the R&D sector. Traditionally, (especially) the first generation of endogenous growth models (Romer 1990, Grossman and Helpman 1991, Aghion Howitt 1992) at this step, conclude that the market economy allocates less input to the R&D sector than the social planner would allocate. Using steady state levels of labor in R&D given by (2.25) and (3.10), we obtain the following under-investment in R&D condition:

$$L_I^* > L_I \Leftrightarrow \frac{g^*-g}{g} > \frac{(s_n^*-s_n)(1-\lambda)}{W} \tag{4.2}$$

Inequality (4.2) shows that in our model the under-investment condition ($L_I^* > L_I$) is not equivalent to the under-growth condition ($g^* > g$). This is an important difference with traditional endogenous growth models where these two conditions match. This break is related directly to the introduction of a geography-growth link via localized intertemporal knowledge spillovers. It allows the possibility of market equilibria where under-growth is associated with over-investment in R&D, and vice-versa. Note that assuming global knowledge spillovers ($\lambda = 1$), brings us back to the classical growth result. According to (4.2), the under-investment condition is stronger than the under-growth condition when under-agglomeration occurs ($s_n^* - s_n > 0$), whereas the under-investment condition becomes less strong than the under-growth condition when over-agglomeration occurs ($s_n^* - s_n < 0$). In other words, when under-agglomeration occurs in the market economy, under-investment in R&D is always associated with under-growth, whereas over-investment in R&D can occur in both conditions of under-growth and over-growth. When over-agglomeration occurs in the market economy, over-investment in R&D is always associated with over-growth whereas under-investment in R&D can occur in the condition of under-growth and over-growth.

Using expression for growth rates in the market economy (2.19) and in social economy (3.6), we can write the under-growth and under-investment in R&D conditions as:

$$\begin{aligned} g^* > g &\Leftrightarrow \frac{W^*}{W}\left[\frac{\sigma}{\alpha}g + \rho\left(\frac{\sigma-\alpha}{\alpha}\right)\right] - \frac{\rho}{\mu} - g > 0 \\ L_I^* > L_I &\Leftrightarrow \frac{W^*}{W}\left[\left(\frac{\sigma-\alpha}{\alpha}\right)(g+\rho)\right] - \frac{\rho}{\mu} > 0 \end{aligned} \tag{4.3}$$

These expressions show more clearly that the under-investment condition becomes stronger than the under-growth condition in the case of under-agglomeration and vice-versa. Using growth rates expressions (2.19) and (3.6), we can verify that the minimum (positive) value of the social planner growth rate for complete agglomeration is always higher than the maximum value of the market growth rate (whatever the level of the market spatial equilibrium including complete agglomeration). Consequently, if the optimal geography is a core-periphery equilibrium which happens when trade integration becomes sufficiently high, or when technological integration is sufficiently low, then the market leads the economy to grow too slowly and to under-invest in R&D.

To complete our analysis and highlight the role of agglomeration externalities, we relate the under-investment and under-growth conditions (4.3) to the market inefficiencies present in our model. We use Grossman and Helpman's (1991, p.82-83) method to measure the external benefits and costs created by the introduction of marginally greater variety at time t assuming that the externally provided product does not affect the geography of activities s_n (and hence the growth rate and income inequality). By so doing, we exclude the role of agglomeration externalities and highlight only the other sources of inefficiencies. Now, imagine that an external agent provides an additional product at all times after time t, and repatriates all profits that accrue to the extra product. We consider the effects of this perturbation on the welfare of agents in the market economy. Inserting market prices and consumption values into utility function (2.1), we can express the utility of an agent living in country a at time t[6] as:

$$\begin{aligned} U^a &= \int_t^{+\infty} e^{-\rho(\theta-t)} \log(\chi N(\theta)^{\mu} E_a(\theta))d\theta \\ \chi &= \alpha^{\alpha}(1-\alpha)^{1-\alpha}\left(\frac{\sigma-1}{\beta\sigma}\right)[s_n + \phi(1-s_n)]^{\alpha/(\sigma-1)} \end{aligned} \tag{4.4}$$

[6]Note that for our purpose the symmetric result would be obtain for consumer living in country b.

Using the Leibniz rule, the effect of the extra variety provided at all moments after t on the welfare of agents living in country a (other than the external one) is given by:

$$\frac{dU^a}{dN} = \int_t^{+\infty} e^{-\rho(\theta-t)} \frac{\mu}{N(\theta)} d\theta + \int_t^{+\infty} e^{-\rho(\theta-t)} \frac{1}{E_a(\theta)} \frac{dE_a(\theta)}{dN(\theta)} d\theta \tag{4.5}$$

because we assume that the extra variety does not influence the level of spatial concentration, and it also does not influence the equilibrium level of income inequality. Thus, we can replace E_a by E in expression (4.5). As shown in Appendix B, we can rewrite (4.5) as:

$$\frac{dU^a}{dN} = \int_t^{+\infty} e^{-\rho(\theta-t)} \frac{\mu}{N(\theta)} d\theta - \int_t^{+\infty} e^{-\rho(\theta-t)} \frac{\alpha}{(\sigma-\alpha)N(\theta)} d\theta + \int_t^{+\infty} e^{-\rho(\theta-t)} \frac{\alpha g}{(\sigma-\alpha)(g+\rho)N(\theta)} d\theta \tag{4.6}$$

The above expression highlights the three external effects that are related to innovation (sources of inefficiencies) without considering the role of the economic geography. The first term in (4.6) represents the "consumer surplus effect", i.e., the marginal benefit to consumers at initial prices, from the extra diversity in consumption. The second term in (4.6) represents the "profit destruction effect", i.e., the old producers' marginal loss plus the incumbent's marginal gain. The third term in (4.6) represents the "knowledge externalities effect", i.e., the marginal (cost) gain for new inventors due to the availability of new knowledge. We can easily show that the profit-destruction effect is higher than the knowledge externalities effect. The sum of these three external effects is positive if:

$$\frac{dU^a}{dN} > 0 \Leftrightarrow \left[\frac{\sigma-\alpha}{\alpha} g + \rho \left(\frac{\sigma-\alpha)}{\alpha}\right)\right] - \frac{\rho}{\mu} > 0 \tag{4.7}$$

It is straightforward to see that condition (4.7) corresponds to the under-growth and under-investment in R&D conditions when optimal and market agglomeration match. Consequently, the differences between inequalities (4.3) and (4.7) allow us to determine the influence of the introduction of a geography-growth link on the gap between optimal and market R&D investment. Simple comparison of the expressions shows immediately that under-agglomeration increases the potential for both under-growth and under-investment in R&D, while the reverse applies in a situation of over-agglomeration. Finally, we can now distinguish the impact of agglomeration externalities on the growth and under-investment gaps:

$$\begin{aligned} \Phi_g &= (g^* - g) - (g^* - g)_{|s_n^* = s_n} = 2\left(\frac{W^* - W}{\eta}\right) \\ \Phi_{L_I} &= (L_I^* - L_I) - (L_I^* - L_I)_{|s_n^* = s_n} = \eta L \frac{\rho}{\mu}\left[\frac{1}{W} - \frac{1}{W^*}\right] \end{aligned} \tag{4.8}$$

The first expression in (4.8) indicates that the agglomeration externalities increase (decrease) the gap between optimal and market growth when $s_n^* > s_n$ ($s_n^* < s_n$). We obtain the same result from the second expression in (4.8). Thus, it is clear that in situations where the market economy over-agglomerates economic activities the need for public policies is reduced while in a situation of under-agglomeration of economic activities, the need for public policies increases.

In relation to the income inequality gap ($s_e^* - s_e$) and the agglomeration gap ($s_n^* - s_n$), it is important to remember that the social planner chooses $s_e^* = 1/2$ which implies a systematic too high level of inequality in the market economy. Finally, even if the trigonometric expression for the optimal level of agglomeration does not allow us to provide an analytical condition for under-agglomeration, the analyses in sections 2 and 3 show that spatial under-agglomeration is more likely when the following conditions are (simultaneously) verified:

- initial capital inequality is low
- trade integration is high because the negative transport cost burden effect decreases with ϕ
- technology integration is low(er than) trade integration because the positive growth effect decreases with λ
- preference for variety (returns to specialization) is high (relative to private returns/interest rate)

Obviously, the opposite conditions are likely to lead the market to over-agglomerate economic activities.

The previous results lead to the following proposition:

Proposition 3:*In a two-country trade and growth model with capital mobility and labor immobility, (1) the under-investment condition is not equivalent to the under-growth condition, (2) over-investment in R&D potentially could be associated with under-growth, and vice-versa, and (3) under(over)-agglomeration increases (decreases) the potential for situations of under-growth and under-investment in R&D.*

5 Public policy implications

In section 4, we investigated the differences between the model's social and market outcomes, and highlighted the various sources of the identified gap. The natural next step to complete the present study is to draw some implications for public policy. In the following, we will distinguish and discuss the effects of two main strategic policies. The first is innovation policy which includes subsidies and taxes that influence R&D costs. The second we call "industrial" policy which includes all subsidies and taxes that might influence directly the location of industrial activities between countries.

5.1 Innovation policy

In this paper, we assume a supranational government which implements a non place-based R&D policy (government subsidizes all R&D activities similarly, regardless of their location, see Montmartin (2013) for a formal analysis). For simplicity, we assume that central government implements a subsidy proportional to total R&D costs. In the market economy developed in section 1, the cost of patent production is modified and given by:

$$F_I = \frac{L\eta(1-S)}{KW} \tag{5.1}$$

where S denotes the R&D subsidy rate. Inserting (5.1) in the world labor market condition (2.16) gives:

$$2L = g_K \frac{L\eta(1-S)}{W} + \left(\frac{\sigma-\alpha}{\sigma}\right) LE \tag{5.2}$$

Using arbitrage condition (2.18), (5.1) and (5.2), we can express the rate of growth of our economy as:

$$g = \frac{2\alpha W}{\eta\sigma(1-S)} - \rho\left(\frac{\sigma-\alpha}{\sigma}\right) \tag{5.3}$$

Using the definition of world income in the market economy given by (2.22) and the fact that $v = F$ at equilibrium, we can express the income in both countries as:

$$E_a = 1 + \rho\frac{\eta s_k(1-S)}{W} \qquad E_b = 1 + \rho\frac{\eta(1-s_k)(1-S)}{W} \tag{5.4}$$

However, the expression of income inequality (s_e) is not influenced by the introduction of an R&D subsidy policy. Indeed, using (5.4) we obtain the same expression as given by (2.23). Consequently, the policy will influence income inequality only through its growth effects. Using (2.14), (2.23) and (5.3), we can determine the steady state level of agglomeration under an innovation policy:

$$s_n = \frac{1}{2} + \frac{\sqrt{[(1-\phi)(1+\lambda+\rho\eta(1-S)]^2 + 4\rho\eta(1-\phi^2)(1-\lambda)(2s_k-1)(1-S)} - (1-\phi)[1+\lambda+\rho\eta(1-S)]}{4(1-\phi)(1-\lambda)} \tag{5.5}$$

In Appendix C, we evaluate the market economy's response to a change in the level of R&D subsidy. The following highlights the policy's core effects:

$$\frac{ds_n}{dS} < 0, \qquad \frac{ds_e}{dS} < 0, \qquad \frac{dg}{dS} > 0, \qquad \frac{L_I}{dS} > 0$$

Note that, in the case that central government chooses $S = 1$ (so that all R&D costs are publicly financed), the spatial equilibrium is dispersed ($s_n = 1/2$). The following economic mechanisms imply this result. Subsidies decrease the R&D costs which lead to more inventors undertaking R&D activities as the sector is in perfect competition. The level of investment in R&D increases which also increases the aggregate growth rate. As the growth rate increases, the value of capital decreases which reduces spatial income inequality. Since the location of firms in the market economy depends only on the income differential, it follows that spatial concentration in the core country decreases. This last effect reduces the increased R&D investment and growth but also decreases income inequality. Overall, an R&D subsidy policy increases growth and investment in R&D but decreases the spatial concentration of economic activities and income inequality. This outcome of innovation policy has been well documented in the NEGG literature (see Martin (1999) and Montmartin (2013)). According to Martin (1999), it is the only policy that allows the trade-off between equity and efficiency to be overcome. Our analysis suggests a less favorable view of a centralized innovation policy. Although it might help to bring growth, income inequality, and R&D investment levels[7] closer to their optimal level, it may also cause the level of spatial agglomeration to deviate far more from its optimum in the case of under-agglomeration. In some senses, it would seem that innovation policy focuses only on the number of researchers in trying to bring the economy closer to its optimum, and ignores agglomeration as a driver of R&D productivity and growth.

5.2 Industrial policy

In this paper we assume a supranational government that implements a place-based industrial policy (government subsidizes the industrial activities differently depending on their location). In what follows, we consider two opposite cases: (1) subsidies for location in the core country, and (2) subsidies for location in the periphery country.

Industrial policies favoring the core country

For simplicity, we assume that central government implements a fiscal subsidy on the operating profit in the core country. Again, we do not consider the cost of such a policy. This policy modifies the equilibrium location condition which becomes $\pi_i(1+B) = \pi_j$. We then obtain the following location equilibrium condition:

$$s_n = \frac{1}{2} + \frac{(1+\phi)}{2(1-\phi)} \left(\frac{[(2+B)s_e - 1](1-\phi) + \phi B}{(1-\phi) + B[s_e - \phi + \phi s_e]} \right) \tag{5.6}$$

[7] In the case where the market economy grows too slowly and under-invests in R&D

Due to the fiscal subsidies, the equilibrium level of production will differ between locations at the equilibrium:

$$
\begin{aligned}
y_a &= \alpha LE(\sigma-1)(1-\phi)\frac{(1-\phi)+B[s_e-\phi+\phi s_e]}{N\sigma\beta[1+B-\phi][1-\phi(1+B)]} \\
y_b &= (1+B)y_a
\end{aligned}
\tag{5.7}
$$

We can see that the production of a typical M firm in country b will be higher than the production of a typical M firm in country a. The location condition requires firms to achieve the same level of profit in both locations. As the subsidy increases the profit on each unit produced in country a, country b's producers have to produce more units than country a's' in order to obtain the same level of profit. In contrast to innovation policy which influences R&D costs directly, and thus the labor market equilibrium condition (2.16), this industrial policy does not generate similar modifications. The R&D cost remains equal to $F_I = L\eta/KW$), and the labor market equilibrium condition is not modified since the total labor in the M sector is still given by $L_M = \beta Y_M = \beta[N_i x_i + N_j x_j] = \alpha LE(\sigma-1)/\sigma$. Using the labor market condition (2.16), the arbitrage condition (2.18), (5.6) and (5.7), we can express the equilibrium growth rate of the market economy as:

$$
g = \frac{2\alpha W(1+B) - \eta\rho(\sigma-\alpha)[1+B(1-s)]}{\eta[\sigma(1+B[1-s]) + Bs\alpha]} \tag{5.8}
$$

Note that unlike innovation policy, the industrial policy does not influence nominal income because it does not influence the equilibrium value of capital (which is equal to the marginal cost of the patent). Nevertheless, its direct effect on profitability in the location means that this industrial policy modifies the (spatial) income inequality which becomes:

$$
s_e = \frac{1}{2} + \frac{\alpha\rho(2s_k-1)(1+B)}{2(g+\rho)[\sigma(1+B[1-s]) + Bs\alpha]} \tag{5.9}
$$

Finally, using the three equilibrium relations (5.6),(5.8) and (5.9), we can express the steady state level of agglomeration with fiscal subsidies to the industrial sector (in the core) as:

$$
\begin{aligned}
s_n &= \frac{1}{2} + \frac{\Lambda + \sqrt{\Lambda + (1+\phi)(1-\phi)^2(1-\lambda)(B+2)[B(1+\phi)(1+\lambda) + 2\rho\eta([(2+B)s_k-1](1-\phi) + B\phi]}}{2(1-\phi)^2(1-\lambda)(B+2)} \\
\Lambda &\equiv B[2\phi - \lambda - \lambda\phi^2 - \rho\eta(1-\phi)(k-\phi+k\phi)] - (1-\phi)^2(1+\lambda+\rho\eta)
\end{aligned}
\tag{5.10}
$$

Inserting (5.10) in (5.8) and (5.9), we obtain the economic growth and income inequality steady state values. Similar to how we proceeded for innovation policy, we evaluate the market economy's response to a change in the level of fiscal subsidies to the industrial sector. Appendix D1 provides the proofs of the following results:

$$
\frac{ds_n}{dB} > 0, \qquad \frac{ds_e}{dB} < 0, \qquad \frac{dg}{dB} > 0, \qquad \frac{L_I}{dB} > 0
$$

In other words, the industrial policy is able to increase spatial agglomeration, economic growth, and R&D investment while reducing spatial income inequality. The mechanisms underlying these results are the following. Subsidizing the core market increases the profitability of that location which leads some M firms to relocate from country b to country a. More generally, this policy increases the profitability of industrial activities which in turn, leads to increase new firms' entry to the market, and increase the demand for knowledge capital. This leads to an increase in R&D investment and growth. Since this industrial policy increases growth and agglomeration, the value of capital decreases more quickly, implying a decrease in nominal income inequality.

Industrial policies aimed at the periphery market

In the spirit of the previous subsection, we assume that central government implements fiscal subsidies proportional to the operating profit in the periphery market. We do not consider the cost of such a policy. Clearly, this policy modifies the location condition which now is given by $\pi_i = \pi_j(1 + B)$. We obtain the following location equilibrium condition:

$$s_n = \frac{1}{2} + \frac{(1+\phi)}{2(1-\phi)} \left(\frac{(2s_e - 1)(1-\phi) - B(1 - s_e + s_e\phi)}{(1-\phi) + B[1 - s_e - \phi s_e]} \right) \tag{5.11}$$

Due to the fiscal subsidies, the equilibrium level of production in countries a and b differs:

$$\begin{aligned} y_a &= \alpha LE(\sigma - 1)(1-\phi)(1+B)\frac{(1-\phi) + B[1 - s_e - \phi s_e]}{N\sigma\beta[1 + B - \phi][1 - \phi(1+B)]} \\ y_b &= \frac{y_a}{1+B} \end{aligned} \tag{5.12}$$

Here we obtain the opposite result to that obtained in the previous subsection. With this policy, the production of a typical M firm in country b will be lower than the production of a typical M firm in country a. Since the fiscal subsidy increases the profit on each unit produced in country b, country a's producers must produce more units than country b's producers in order to achieve the same level of profit. For the same reason as in the previous case, this policy does not modify the R&D cost or the labor market equilibrium condition.

Using the labor market condition (2.16), and the arbitrage condition (2.18), (5.11) and (5.12), we can express the equilibrium growth rate of the model as:

$$g = \frac{2\alpha W(1+B) - \eta\rho(\sigma - \alpha)[1 + Bs]}{\eta[\sigma(1 + Bs_n) + B\alpha(1 - s_n)]} \tag{5.13}$$

Due to its direct effect on the profitability of the location, the industrial policy also modifies the income inequality expression, which becomes:

$$s_e = \frac{1}{2} + \frac{\alpha\rho(2s_k - 1)(1+B)}{2(g+\rho)[\sigma(1+Bs) + B\alpha(1-s)]} \tag{5.14}$$

Finally, using the three equilibrium relations (5.11),(5.13) and (5.14), we can express the steady state level of agglomeration as:

$$\begin{aligned} s_n &= \frac{1}{2} + \frac{\Lambda + \sqrt{\Lambda + (1+\phi)(1-\phi)^2(1-\lambda)(B+2)[2\rho\eta(1-\phi)(2s_k-1) - 2B\rho\eta(1 - s_k + s_k\phi) - B(1+\phi)(1+\lambda)}}{2(1-\phi)^2(1-\lambda)(B+2)} \\ \Lambda &\equiv B[2\lambda\phi + \rho\eta(1-\phi)(k + k\phi - 1) - (1+\phi^2)] - (1-\phi)^2(1 + \lambda + \rho\eta) \end{aligned} \tag{5.15}$$

Inserting (5.15) in (5.13) and (5.14), we obtain the economic growth and income inequality steady state values. As before, we evaluate the market economy's response to a change in the level of fiscal subsidies to the industrial sector. Appendix D2 provides the proofs of the following results:

$$\frac{ds_n}{dB} < 0, \qquad \frac{ds_e}{dB} > 0, \qquad \frac{dg}{dB} < 0, \qquad \frac{L_I}{dB} < 0$$

In other words, the industrial policy decreases spatial agglomeration, economic growth, and R&D investment, and increases income inequality. The mechanisms underlying these results are the following. The fiscal subsidies towards the periphery increases profitability in that location which lead some M firms to relocate from country a to country b. More generally, this policy increases the profitability of industrial activities which generates the entry of new firms to the market. This direct effect tends to increase the demand for knowledge capital, R&D investment, and growth. However, this direct effect is counterbalanced by an indirect effect related to the reduced spatial agglomeration which in turn reduces R&D productivity and growth. Appendix D2 shows that the latter effect dominates the former which leads to a policy effect of decreased economic growth and reduced R&D investment. This reduction in the economic growth rate increases nominal income inequality.

5.3 Relative efficiency and complementarity of the two strategic policies

Table 1 summarizes the effects of the three policies studied on four key gaps in the model: (1) the agglomeration gap denoted $\Omega_{s_n} =\mid s_n^* - s_n \mid$, (2) the growth gap denoted $\Omega_g =\mid g^* - g \mid$, (3) the R&D investment level gap denoted $\Omega_{L_I} =\mid L_I^* - L_I \mid$ and (4) the income inequality gap denoted $\Omega_{s_e} =\mid s_e^* - s_e \mid$. More precisely, one of the main advantage of our model compared to the classical NEGG model is that it greatly increases the number of potential market outcome in terms of gaps. Table 1 presents the six potential gap situations in the market economy. Note that the previous NEGG models consider only the last two cases because they always involve under-investment in R&D, and under-growth.

Table 1: Effect of policies on market inefficiencies

initial equilibrium	$\Omega_{s_n} < 0$	$\Omega_{s_n} > 0$
Innovation policy (RDP)		
$\Omega_g < 0 \wedge \Omega_{L_I} < 0$	$\downarrow \Omega_{s_n}, \uparrow \Omega_g, \uparrow \Omega_{L_I}, \downarrow \Omega_{s_e}$	$\uparrow \Omega_{s_n}, \uparrow \Omega_g, \uparrow \Omega_{L_I}, \downarrow \Omega_{s_e}$
$\Omega_g < 0 \wedge \Omega_{L_I} > 0$	$\downarrow \Omega_{s_n}, \uparrow \Omega_g, \downarrow \Omega_{L_I}, \downarrow \Omega_{s_e}$	N/A
$\Omega_g > 0 \wedge \Omega_{L_I} < 0$	N/A	$\uparrow \Omega_{s_n}, \downarrow \Omega_g, \uparrow \Omega_{L_I}, \downarrow \Omega_{s_e}$
$\Omega_g > 0 \wedge \Omega_{L_I} > 0$	all gaps decrease	$\uparrow \Omega_{s_n}, \downarrow \Omega_g, \downarrow \Omega_{L_I}, \downarrow \Omega_{s_e}$
Place-based industrial policy 1 (IP1)		
$\Omega_g < 0 \wedge \Omega_{L_I} < 0$	$\uparrow \Omega_{s_n}, \uparrow \Omega_g, \uparrow \Omega_{L_I}, \downarrow \Omega_{s_e}$	$\downarrow \Omega_{s_n}, \uparrow \Omega_g, \uparrow \Omega_{L_I}, \downarrow \Omega_{s_e}$
$\Omega_g < 0 \wedge \Omega_{L_I} > 0$	$\uparrow \Omega_{s_n}, \uparrow \Omega_g, \downarrow \Omega_{L_I}, \downarrow \Omega_{s_e}$	N/A
$\Omega_g > 0 \wedge \Omega_{L_I} < 0$	N/A	$\downarrow \Omega_{s_n}, \downarrow \Omega_g, \uparrow \Omega_{L_I}, \downarrow \Omega_{s_e}$
$\Omega_g > 0 \wedge \Omega_{L_I} > 0$	$\uparrow \Omega_{s_n}, \downarrow \Omega_g, \downarrow \Omega_{L_I}, \downarrow \Omega_{s_e}$	all gaps decrease
Place-based industrial policy 2 (IP2)		
$\Omega_g < 0 \wedge \Omega_{L_I} < 0$	$\downarrow \Omega_{s_n}, \downarrow \Omega_g, \downarrow \Omega_{L_I}, \uparrow \Omega_{s_e}$	$\uparrow \Omega_{s_n}, \downarrow \Omega_g, \downarrow \Omega_{L_I}, \uparrow \Omega_{s_e}$
$\Omega_g < 0 \wedge \Omega_{L_I} > 0$	$\downarrow \Omega_{s_n}, \downarrow \Omega_g, \uparrow \Omega_{L_I}, \uparrow \Omega_{s_e}$	N/A
$\Omega_g > 0 \wedge \Omega_{L_I} < 0$	N/A	$\uparrow \Omega_{s_n}, \uparrow \Omega_g, \downarrow \Omega_{L_I}, \uparrow \Omega_{s_e}$
$\Omega_g > 0 \wedge \Omega_{L_I} > 0$	$\downarrow \Omega_{s_n}, \uparrow \Omega_g, \uparrow \Omega_{L_I}, \uparrow \Omega_{s_e}$	all gaps increase

Note: N/A indicates that this initial equilibrium does not exist.

Before beginning the analysis, we need to specify that our analysis excludes one of the gaps present in the model, i.e, the misallocation of labor between the two good producing sectors. This misallocation is not treated by the policies studied because it would imply some modification of the relative demand between the homogeneous and differentiated goods. To achieve this would require demand-oriented policy. This implies also that none of the supply-oriented policies studied in this paper could achieve a first best. Therefore, we propose a model where a mix of demand and supply policies is a necessary condition to achieve a first best.

Table 1 shows that both innovation policy and the industrial policy aimed at the core (IP1) are able to reduce the four gaps. However, this positive result is achievable only in two different potential equilibria.

Specifically, the innovation policy reduces the four gaps if the market economy initially grows too slowly, under-invests in R&D, and has an overly concentrated geography of economic activities. The industrial policy achieves this result if the market economy initially grows too slowly, under-invests in R&D, and has an overly dispersed geography of economic activities. In the other four potential equilibria, all the policies increase at least one gap. This means that in these four scenarios, industrial and innovation policies potentially could lead the market economy away from its optimal steady state. This applies especially to industrial policy towards the periphery (IP2) if implemented when the market economy initially is growing too slowly, under-investing in R&D, and has a too dispersed geography of economic activities. In this last case, IP2 increases all four gaps which applies also to the case of IP1 and RDP policies if the market economy is growing too quickly and there is over-investment in R&D. In both these cases, the two policies increase at least two gaps.

Table 2 present a simpler view of the effect of the different policies by comparing the capacity of each to decrease the gaps. Note that the ranking in Table 2 values each gap equally, i.e., two policies are considered to be approximatively equivalent if they decrease the same number of gaps.

Table 2: Relative efficiency of industrial and innovation policies

initial equilibrium	$\Omega_{s_n} < 0$	$\Omega_{s_n} > 0$
$\Omega_g < 0 \wedge \Omega_{L_I} < 0$	$IP2 > RDP > IP1$	$IP1 \approx IP2 > RDP$
$\Omega_g < 0 \wedge \Omega_{L_I} > 0$	$RDP > IP1 \approx IP2$	N/A
$\Omega_g > 0 \wedge \Omega_{L_I} < 0$	N/A	$IP1 > RDP > IP2$
$\Omega_g > 0 \wedge \Omega_{L_I} > 0$	$RDP > IP1 > IP2$	$IP1 > RDP > IP2$

It can be seen from Table 2 that the capacity of the policies studied to reduce the gaps clearly evolves with the initial equilibrium situation. It can be seen also that in the case of the market economy leading to a too low level of spatial agglomeration, the industrial policy towards the core (IP1) is superior (at least slightly) to other policies. In the case where the market economy leads to a too high level of spatial agglomeration, the situation changes completely. If the economy grows too quickly, and there is over-investment in R&D, the most efficient policy is the industrial policy towards the periphery (IP2), while the industrial policy towards the core (IP1) is the worst policy. If the economy under-invests in R&D, the innovation policy becomes the most efficient policy while the industrial policy towards the periphery (IP2) becomes the worst policy. Thus, contrary to the widespread belief among economists and politicians, innovation policy is not always desirable, and in most cases, is not the most appropriate tool to bring a market economy closer to its optimum.

The last issue on which the model throws some light is that of the complementarity/substitutability of innovation and industrial policies. Recall that only the innovation policy (RDP) and the industrial policy 1 (IP1) are able to decrease all four gaps. However, this is achieved in two different initial steady states. In both cases, innovation policy and industrial policy 1 are only partially complementary because in each case, the "non-best" policy decreases some gaps but also increases one gap. More generally, in our model, a policy-mix will always imply some kind of trade-off in the sense that if two different policies are able to decrease certain gaps, this necessarily implies forgetting the objective of reducing all gaps and accepting the possibility that some gaps might increase. Generally, a combination of innovation and industrial policies produced mixed results from a welfare point of view, so these policies are only partially complementary. Policy-mixes clearly are justified if the gaps vary in their importance, and if government's objective is to reduce the most important gaps.

The above results lead to the following proposition:

Proposition 4:*In a two-country trade and growth model with capital mobility and labor immobility, (1) the efficiency of innovation and industrial policies evolves strongly with the market economy situation, (2) no policy is the most effective in all situations, (3) the geography of economic activities is at the heart of the* *relative efficiency of policies, and (4) industrial and innovation policies are only partially complementary but policy-mixes can be justified.*

6 Conclusion

The main objective of this paper was to show the importance of linking the processes of agglomeration and growth to analyze market inefficiencies, and their public policy implications. To achieve this, we have made four contributions to the literature. First, we developed a New Economic Geography and Growth (NEGG) model which is a generalization of Martin and Ottaviano's (1999) model. The model disentangles the restrictive relations between key parameters, and allows potential situations of over-investment in R&D. Second, we provided the social planner version of the model, and thus highlight first best characteristics. To our knowledge, this is the first demonstration of the first best solution in a NEGG model. Third, we analyzed market economy inefficiencies which we link directly to the gap between market and social outcomes. The growth-agglomeration link appears to have some important effects. For instance, it disentangles the under-growth and under-investment in R&D conditions. In other words, the market can induce a situation where the economy grows too slowly but over-invests in R&D. The fourth contribution is that it offers some implications for public policy. Specifically, we evaluated the effect of two strategic policies (innovation and industrial policy) on the model's key endogenous variables[8] and discussed the rationale for their implementation according to different (potential) steady states. The first main result is that none of these policies is able by itself to achieve a first best. The second is that there are only two situations where innovation policy or industrial policy is able to reduce all gaps.[9]. In other situations, these policies increase at least one gap. In some cases, innovation and industrial policies can the market economy away to its optimal steady state. The third and last result of our public policy analysis concerns the relative efficiency of the policies under studied. We found that industrial policy is clearly superior to innovation policy if the initial level of agglomeration is below its optimal level. However, when there is over-agglomeration at the initial steady state, then innovation policy is superior to industrial policy except in the case that the market economy over-invests in R&D and grows too quickly. Our results imply that (1) policy efficiency evolves strongly with the market economy situation's and no one policy is the most efficient in all situations, (2) the geography of economic activities and the question of over- or under-agglomeration play a central role in the relative efficiency of industrial and innovation policies, and (3) industrial and innovation policies are only partially complementary but policy mixes can be justified especially if some gaps are more important than others.

All of these results highlight the importance of considering the links between agglomeration, economic growth, and spatial income inequality in economic analyze. Numerous of the elements discussed in this paper highlight the pertinence of the NEGG framework for the study of market inefficiencies and public policies. This largely under-developed literature has the potential to provide promising results in the future. More generally, more theoretical and empirical works is needed, focused on the analysis of market inefficiencies in a trade and growth framework.

Theoretically, several developments of the present model can be considered. For instance, we could choose to relax the unambiguously positive relation between growth and agglomeration in our model. Empirical studies suggests the development of framework that allows both negative and positive relations to result from the market economy. This could be achieved in our framework by considering a more general form of intertemporal knowledge spillovers. More precisely, if we were to assume that each region had specific capabilities to access and use its own knowledge, and to access and use foreign knowledge, then under certain conditions imposed on these parameters, a negative relation between agglomeration and growth might

[8] I.e., the level of agglomeration, the economic growth rate, the level of investment in R&D, and the level of income inequality.
[9] Except the one related to the market power of industrial firms

emerge. One way to achieve such result would be to replace the localized knowledge spillovers denoted $W = \min\{s_n + \lambda(1 - s_n); \lambda s_n + (1 - s_n)\}$ with a more generalized one $W = \min\{\lambda_1 s_n + \lambda_2(1 - s_n); \lambda_3 s_n + \lambda_4(1 - s_n)\}$. Another interesting development that would provide new policy implications would be to modify the link between the levels of competition and growth. In the model developed in this paper, there is a negative relationship between the Lerner index and economic growth. Aghion et al. (2005) found evidence of a inverted-U relation between the level of competition and the level of growth. This inverted-U shape can be achieved by linking the elasticity of substitution with the market share of differentiated goods (see Bucci, 2005). However, probably, the most obvious development of this paper from a public policy perspective would be to determine the optimal policy mix to achieve a first best.

From an empirical perspective, it would be necessary to evaluate the importance of the different inefficiencies highlighted by the comparison of market and optimal outcomes. Empirical work evaluating the social returns to R&D does not distinguish and evaluate the different inefficiencies (see Montmartin and Massard (2015) for a discussion). This is a major problem because different policy tools are needed to correct different inefficiencies. In our view, a reduced form of the Welfare function in our model (see 4.4) could be estimated which would allow empiricists to distinguish the importance (on the gap between market and optimal solution) of four sources of inefficiencies: agglomeration externalities, knowledge externalities, rent transfers, and consumer surplus. This would help our understanding of which policy would be the most effective for bringing the economy to its optimal steady state, and would enable a better appreciation of the policy mixes used in each country.

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Appendix

Appendix A

In the social planner problem, the optimal level of agglomeration is achieved when the partial derivative of the Hamiltonian with respect to the level of agglomeration (see 3.9) is equal to zero. Solving this constraint is equivalent to find the roots of a third degree polynomial equation in the level of agglomeration $f(s_n) = as_n^3 + bs_n^2 + cs_n + d = 0$ where

$$\begin{aligned}
a &= -4\mu(1-\lambda)^2(1-\phi)^2(\sigma-1) \\
b &= -2(1-\phi)^2(1-\lambda)[2\mu(2\lambda-1)(\sigma-1)+\eta\rho(\alpha+1-\sigma)] \\
c &= 2(1-\lambda)(\sigma-1)[(1-\phi)^2(2\lambda\mu-\eta\rho)+2\mu\phi(1-\lambda)] - \alpha\eta\rho(1-\phi)^2(3\lambda-1) \\
d &= 2\phi(1-\lambda)(2\lambda\mu-\eta\rho)(\sigma-1)+\alpha\lambda\eta\rho(1-\phi)^2
\end{aligned}$$

Using Cardano's and trigonometric method, we can express the optimal level of agglomeration as:

$$s_n^* = \min\{1, \max\{1/2, s^*\}\}$$

$$\begin{aligned}
s^* &= 2\sqrt{\frac{-p}{3}}cos\left[\frac{1}{3}arccos\left(\frac{3q}{2p}\sqrt{\frac{-3}{p}}\right)\right] - \frac{b}{3a} && \text{if } 4p^3+27q^2 \leq 0 \text{ and } p<0 \\
s^* &= -2\frac{|q|}{q}\sqrt{\frac{-p}{3}}\cosh\left[\frac{1}{3}arcosh\left(\frac{-3|q|}{2p}\sqrt{\frac{-3}{p}}\right)\right] - \frac{b}{3a} && \text{if } 4p^3+27q^2 > 0 \text{ and } p<0 \\
s^* &= -2\sqrt{\frac{p}{3}}sinh\left[\frac{1}{3}arsinh\left(\frac{3q}{2p}\sqrt{\frac{3}{p}}\right)\right] - \frac{b}{3a} && \text{if } p>0
\end{aligned}$$

with

$$p = \frac{3ac-b^2}{3a^2} \qquad q = \frac{2b^3-9abc+27a^2d}{27a^3}$$

Note that the optimal level of agglomeration is easily found when we use specific links between the level of trade integration and the level of technology integration. More precisely, if we assume $\lambda = \phi$ or $\lambda = 1/\phi$, then the problem is reduced to finding the roots of a second degree polynomial equation. The optimal level of agglomeration when $\lambda = \phi$ is given by

$$s_n^* = \min\left\{1, \frac{1}{2} + \frac{\sqrt{\rho\eta(\sigma-1-\alpha)^2+4\mu(\sigma-1)^2(1+\phi)(\mu+\mu\phi-\rho\eta)}+\rho\eta(\sigma-1-\alpha)}{4\mu(1-\phi)(\sigma-1)}\right\}$$

and by

$$s_n^* = \min\left\{1, \frac{1}{2} + \frac{\sqrt{\phi\rho\eta(\sigma-1-\alpha)^2+4\mu(\sigma-1)^2(1+\phi)(\mu+\mu\phi-\phi\rho\eta)}+\phi\rho\eta(\sigma-1-\alpha)}{4\mu(1-\phi)(\sigma-1)}\right\}$$

when $\lambda = 1/\phi$. We can see that, in these two specific cases, the optimal level of agglomeration is always higher than 1/2. It shows that the impact of trade integration is stronger than the impact of technological integration.

Appendix B

We depart from (4.5) which evaluates the effect of the extra variety provided at anytime after t on the welfare of agents living in country a (other than the external one):

$$\frac{dU^a}{dN} = \int_t^{+\infty} e^{-\rho(\theta-t)}\frac{\mu}{N(\theta)}d\theta + \int_t^{+\infty} e^{-\rho(\theta-t)}\frac{1}{E_a(\theta)}\frac{dE_a(\theta)}{dN(\theta)}d\theta$$

Using (2.3) and equilibrium wage ($w = 1$), the world expenditure level is given by $E = 2 + (\Pi - v\dot{K})/L$, where $\Pi = N\pi$, where $\pi = \alpha LE/\sigma N$ is the profit/dividend of a representative firm and $v = F$ is the market value of capital/the cost of patents. The variation of profit as a result of the marginal addition in variety is given by:

$$\frac{d\pi}{dN} = -\frac{\alpha LE}{\sigma N^2} + \frac{\alpha L}{\sigma N}\frac{dE}{dN}$$

The change in profits of the N producers other than the external one is given by:

$$\frac{d\Pi}{dN} = N\frac{d\pi}{dN} = -\frac{\alpha LE}{\sigma N} + \frac{\alpha L}{\sigma}\frac{dE}{dN}$$

The change in total expenditure of the N agents other than the external one is given by:

$$\frac{dE}{dN} = \frac{1}{L}\left(\frac{d\Pi}{dN} - \dot{N}\frac{dF}{dN}\right)$$

Using the expressions of changes in profit and in total expenditure, we can express the variation of expenditure (E) as a result of the marginal addition in variety:

$$\frac{dE}{dN} = -\frac{\alpha E}{(\sigma - \alpha)N} + \frac{\sigma}{(\sigma - \alpha)}\frac{\eta g}{NW}$$

Inserting this last expression into (4.5), we can write:

$$\frac{dU^a}{dN} = \int_t^{+\infty} e^{-\rho(\theta - t)}\frac{\mu}{N(\theta)}d\theta - \int_t^{+\infty} e^{-\rho(\theta - t)}\frac{\alpha}{(\sigma - \alpha)N(\theta)}d\theta + \int_t^{+\infty} e^{-\rho(\theta - t)}\frac{\alpha g}{(\sigma - \alpha)(g + \rho)N(\theta)}d\theta$$

Appendix C: Effect of innovation policy (RDP)

Using equilibrium relations (5.3), (2.23) and (2.14), we can write the following equalities:

$$\frac{ds_n}{dS} = \frac{ds_n}{ds_e}\frac{ds_e}{dS} = \frac{1 + \phi}{1 - \phi}\frac{ds_e}{dS}$$

$$\frac{ds_e}{dS} = \frac{ds_e}{dg}\frac{dg}{dS} = -\frac{\alpha\rho(2s_k - 1)}{\sigma(g + \rho)^2}\frac{dg}{dS}$$

$$\frac{dg}{dS} = \frac{\partial g}{\partial S} + \frac{\partial g}{\partial s_n}\frac{ds_n}{dS}$$

$$\frac{\partial g}{\partial S} = \frac{2\alpha W}{\sigma\eta(1 - S)^2} > 0$$

$$\frac{\partial g}{\partial s_n} = \frac{2\alpha(1 - \lambda)}{\sigma\eta(1 - S)} > 0$$

Then, we depart from the following equality:

$$\frac{ds_n}{dS} = \frac{ds_n}{ds_e}\frac{ds_e}{dg}\frac{dg}{dS}$$

Using previous equalities, we can rewrite this last expression as:

$$\frac{ds_n}{dS} = \frac{ds_n}{ds_e}\frac{ds_e}{dg}\left[\frac{\partial g}{\partial S} + \frac{\partial g}{\partial s_n}\frac{ds_n}{dS}\right]$$

which is equivalent to:

$$\frac{ds_n}{dS} = \frac{\frac{ds_n}{ds_e}\frac{ds_e}{dg}\frac{\partial g}{\partial S}}{1 - \frac{ds_n}{ds_e}\frac{ds_e}{dg}\frac{\partial g}{\partial s_n}}$$

As $ds_n/ds_e > 0$, $ds_e/dg < 0$, $\partial g/\partial S > 0$, the numerator of this last expression is negative. As $\partial g/\partial s_n > 0$, the denominator is positive. Thus, this last expression is unambiguously negative implying that an increase in R&D subsidies decreases the spatial agglomeration in the core. Using the fact that,

$$\frac{ds_n}{dS} = \frac{ds_n}{ds_e}\frac{ds_e}{dS} = \frac{1+\phi}{1-\phi}\frac{ds_e}{dS}$$

we know that $sign(ds_n/dS) = sign(ds_e/dS)$. This implies that an increase of R&D subsidies decreases the spatial income inequality. Since

$$\frac{ds_e}{dS} = \frac{ds_e}{dg}\frac{dg}{dS} = -\frac{\alpha\rho(2s_k - 1)}{\sigma(g+\rho)^2}\frac{dg}{dS}$$

we know that $sign(ds_e/dS) = -sign(dsg/dS)$. This implies that an increase in R&D subsidies increases the economic growth. The last element to analyze is the effect of the innovation policy on the level of R&D investment. Remember that the level of investment in R&D L_I is given by (see 5.2):

$$L_I = g\frac{L\eta(1-S)}{s_n + \lambda(1-s_n)}$$

Inserting the expression of growth given by 5.3, we obtain:

$$L_I = 2\frac{\alpha L}{\sigma} - \rho\frac{(\sigma-\alpha)L\eta(1-S)}{\sigma[s_n + \lambda(1-s_n)]}$$

We can thus write:

$$\frac{dL_I}{dS} = \frac{\partial L_I}{\partial S}\frac{dS}{dS} + \frac{\partial L_I}{\partial S} = L\eta\rho\frac{(\sigma-\alpha)}{\sigma[s_n+\lambda(1-s_n)]}\left[1 + \frac{(1-\lambda)(1-S)}{[s_n+\lambda(1-s_n)]}\frac{ds_n}{dS}\right]$$

Now, we evaluate the differential of s_n with respect to S:

$$\frac{ds_n}{dS} = \frac{\eta\rho}{4}\left[\frac{\Lambda + (1-4s_k(1-\lambda)(1+\phi) - 3(\lambda-\phi) - \lambda\phi - \eta\rho(1-\phi)(1-S)}{(1-\lambda)\Lambda}\right]$$

$$\Lambda \equiv \sqrt{[(1-\phi)(1+\lambda+\eta\rho(1-S)]^2 + 4\eta\rho(1-\phi^2)(1-\lambda)(2s_k-1)(1-S)}$$

Using (5.5), we can write:

$$\Lambda = 4\left(s_n - \frac{1}{2}\right)(1-\lambda)(1-\phi) + (1-\phi)[1+\lambda+\rho\eta(1-S)]$$

Inserting this last expression into the expression of ds_n/dS, we obtain:

$$\frac{ds_n}{dS} = -\frac{\eta\rho[s_k(1+\phi) - s_n(1-\phi) - \phi]}{4\left(s_n - \frac{1}{2}\right)(1-\lambda)(1-\phi) + (1-\phi)[1+\lambda+\rho\eta(1-S)]}$$

Now using the expression of dL_I/dS, we write the condition under which R&D subsidy increases the level of investment in R&D as:

$$4W\left[s_n - \frac{1}{2}\right](1-\phi) + W\frac{(1-\phi)}{(1-\lambda)}[1+\lambda+\rho\eta(1-S)] > (1-S)\rho\eta[s_k(1+\phi) - s_n(1-\phi) - \phi]$$

We factor this expression by s_n. We obtain a second degree polynomial equation in s_n, where the coefficient of the second degree is positive. Then, we calculate the two real roots:

$$s_1 = \frac{1}{2} - \frac{\chi + \sqrt{\chi^2 + 8(1-\phi)[2\rho\eta(s_k(1+\phi)(1-\lambda) + \lambda\phi - 1)(1-S) - (1+\lambda)^2(1-\phi)]}}{8(1-\phi)(1-\lambda)}$$

$$s_2 = \frac{1}{2} + \frac{-\chi + \sqrt{\chi^2 + 8(1-\phi)[2\rho\eta(s_k(1+\phi)(1-\lambda) + \lambda\phi - 1)(1-S) - (1+\lambda)^2(1-\phi)]}}{8(1-\phi)(1-\lambda)}$$

$$\chi \equiv 2\rho\eta(1-\phi)(1-S) + 3(1-\phi)(1+\lambda)$$

To evaluate the sign of dL_I/dS, we compare these two solutions with the market level of spatial agglomeration given by (5.5). We know that if $s_n \in]s_1; s_2[$, then $dL_I/dS < 0$ whereas in the other cases $dL_I/dS \geq 0$. The first solution s_1 is strictly lower than 1/2 and thus lower than the optimal level of spatial agglomeration given by (5.5). We can also verify that s_n given by (5.5) is higher than s_2. Consequently, we know that, given the market level of agglomeration given by (5.5), an increase in R&D subsidy rises the level of R&D investment ($dL_I/dS > 0$).

Appendix D

D.1. The effects of industrial policies towards the core country (IP1)

Using equilibrium relations (5.6), (5.8) and (5.9), we have the following equalities:

$$\frac{\partial s_n}{\partial B} = \frac{s_e(1-s_e)(1+\phi)^2}{[(1-\phi) + B(s_e - \phi + \phi s_e)]^2} > 0$$

$$\frac{\partial s_n}{\partial s_e} = \frac{(1+\phi)(1+B-\phi)(1-B-B\phi)}{(1-\phi)\left[(1-\phi) + B(s_e - \phi + \phi s_e)\right]^2} > 0 \text{ if } B < \frac{1-\phi}{\phi}$$

$$\frac{\partial g}{\partial B} = \frac{\alpha(2W + \rho\eta)(\sigma - \alpha)s_n}{\eta[\sigma(1 + B[1-s]) + B\alpha s]^2} > 0$$

$$\frac{\partial g}{\partial s_n} = \alpha B \frac{(2W + \rho\eta)(\sigma - \alpha)(1+B)}{\eta[\sigma(1 + B[1-s]) + B\alpha s]^2} > 0$$

$$\frac{\partial s_e}{\partial B} = \frac{\alpha\rho(2s_k - 1)(\sigma - \alpha)s}{2(g+\rho)[\sigma(1 + B[1-s]) + B\alpha s]^2} > 0$$

$$\frac{\partial s_e}{\partial g} = -\frac{\alpha\rho(2s_k - 1)(1+B)}{2(g+\rho)^2[\sigma(1+Bs) + B\alpha(1-s)]} < 0$$

$$\frac{\partial s_e}{\partial s_n} = \frac{\alpha\rho B(2s_k - 1)(1+B)(\sigma - \alpha)}{2(g+\rho)[\sigma(1+Bs) + B\alpha(1-s)]^2} > 0$$

Now using the expressions for the level of income in each country given by $E_a = 1 + \rho\eta s_k/W$ and $E_a = 1 + \rho\eta(1-s_k)/W$, we can write the income inequality as $s_e = [W + \rho\eta s_k]/[2W + \rho\eta s_k]$. Using this last expression, we can write:

$$\frac{ds_e}{dB} = -(1-\lambda)\rho\eta\frac{2s_k - 1}{(2W + \rho\eta)^2}\frac{ds_n}{dB} = \frac{ds_e}{ds_n}\frac{ds_n}{dB}$$

Now using (5.6), we can write the differential of s_n with respect to B as:

$$\frac{ds_n}{dB} = \frac{\partial s_n}{\partial B} + \frac{\partial s_n}{\partial s_e}\frac{ds_e}{ds_n}$$

Remplacing ds_e/dB into this last expression allow us to rewrite the differential of s_n with respect to B as:

$$\frac{ds_n}{dB} = \frac{\frac{\partial s_n}{\partial B}}{1 - \frac{\partial s_n}{\partial s_e}\frac{ds_e}{ds_n}}$$

Assuming $B < (1-\phi)/\phi$, we have $\partial s_n/\partial B > 0$, $\partial s_n/\partial s_e > 0$ and $ds_e/ds_n < 0$, we immediately see that $ds_n/dB > 0$. Given that $ds_e/ds_n = -\text{sign}(ds_n/dB)$, we have $ds_e/dB < 0$. Now, using expression (5.8), we can write:

$$\frac{dg}{dB} = \frac{\partial g}{\partial B} + \frac{\partial g}{\partial s_n}\frac{ds_n}{dB}$$

As $\partial g/\partial B > 0$, we have $\partial g/\partial s_n > 0$ and $ds/dB > 0$, it becomes evident that the policy increase the aggregate growth rate ($dg/dB > 0$). Now to close our analysis, we evaluate the effect of the policy upon the private R&D investment ($L_I = gL\eta/[s_n + \lambda(1-s_n])$. We thus have:

$$\frac{dL_I}{dB} = \frac{\partial L_I}{\partial g}\frac{dg}{dB} + \frac{\partial L_I}{\partial s_n}\frac{ds_n}{dB}$$

The first term of this differential is positive but the second term is negative as an increase in agglomeration increases the R&D productivity. The question is to know which of these two effects is the most important. For that, we will first evaluate the effect of the policy on the labor employed in the industrial and traditional sector. We know that the labor in the M and Z sectors are given by $L_M = \alpha LE(\sigma-1)/\sigma$ and $L_Z = (1-\alpha)LE$ respectively. Using the fact that at equilibrium, the wage rate is equal to 1 and that the value of capital is equal to the marginal cost to produce this capital, we have $F = L\eta/KW$ and $E = 2 + \rho\eta/W$. Thus, we can write the following equality

$$\frac{dL_M}{dB} = \frac{dL_M}{dE}\frac{dE}{ds_n}\frac{ds_n}{dB} < 0 \quad \frac{dL_Z}{dB} = \frac{\partial L_Z}{\partial E}\frac{dE}{ds_n}\frac{ds_n}{dB} < 0$$

Given that the labor force is fixed over time, we know that if the policy decreases the labor devoted to good-producing sector, it will increases the labor devoted to the innovative sector. In other words, we have shown that $dL_I/dB > 0$.

D.2. The effects of industrial policies towards the periphery country (IP2)

Using equilibrium relations (5.11), (5.13) and (5.14), we can write the following equalities:

$$\frac{\partial s_n}{\partial B} = -\frac{s_e(1-s_e)(1+\phi)^2}{[(1-\phi)+B(1-s_e-\phi s_e)]^2} < 0$$

$$\frac{\partial s_n}{\partial s_e} = \frac{(1+\phi)(1+B-\phi)(1-B-B\phi)}{(1-\phi)\left((1-\phi)+B[1-s_e-\phi s_e]\right)^2} > 0 \text{ if } B < \frac{1-\phi}{\phi}$$

$$\frac{\partial g}{\partial B} = \frac{\alpha(2W+\rho\eta)(\sigma-\alpha)(1-s_n)}{\eta[\sigma(1+Bs_n)+B\alpha(1-s_n)]^2} > 0$$

$$\frac{\partial g}{\partial s_n} = \alpha(1+B)\frac{2\sigma(1-\lambda)+B[2\alpha-2\sigma\lambda-\rho\eta(\sigma-\alpha)]}{\eta[\sigma(1+Bs)+B\alpha(1-s)]^2} > 0 \text{ if } B < \frac{2\sigma(1-\lambda)}{2\sigma\lambda+\rho\eta(\sigma-\alpha)-2\alpha}$$

$$\frac{\partial s_e}{\partial B} = \frac{\alpha\rho(2s_k-1)(\sigma-\alpha)(1-s_n))}{2(g+\rho)[\sigma(1+Bs_n)+B\alpha(1-s_n)]^2} > 0$$

$$\frac{\partial s_e}{\partial g} = -\frac{\alpha\rho(2s_k-1)(1+B)}{2(g+\rho)^2[\sigma(1+Bs_n)+B\alpha(1-s_n)]} < 0$$

$$\frac{\partial s_e}{\partial s_n} = -\frac{\alpha\rho B(2s_k-1)(1+B)(\sigma-\alpha))}{2(g+\rho)[\sigma(1+Bs)+B\alpha(1-s)]^2} < 0$$

As we have made in appendix D.1, we use the expression for the income level in each country to express the nominal income inequality as $s_e = [W + \rho\eta s_k]/[2W + \rho\eta s_k]$. This last expression implies that:

$$\frac{ds_e}{dB} = -(1-\lambda)\rho\eta\frac{2s_k - 1}{2W + \rho\eta)^2}\frac{ds_n}{dB} = \frac{ds_e}{ds_n}\frac{ds_n}{dB}$$

Combining this last expression with the expression of the differential of s_n with respect to B from (5.11) (see Appendix D.1), we can write:

$$\frac{ds_n}{dB} = \frac{\frac{\partial s_n}{\partial B}}{1 - \frac{\partial s_n}{\partial s_e}\frac{ds_e}{ds_n}}$$

Assuming $B < (1-\phi)/\phi$, $\partial s_n/\partial B < 0$, $\partial s_n/\partial s_e > 0$ and $ds_e/ds_n < 0$, we immediately see that $ds_n/dB < 0$. Given that $ds_e/ds_n = -\text{sign}(ds_n/dB)$, we thus have $ds_e/dB > 0$, i.e., the policy increases the nominal income inequality.

Now, using expression (5.13), we have:

$$\frac{dg}{dB} = \frac{\partial g}{\partial B} + \frac{\partial g}{\partial s_n}\frac{ds_n}{dB}$$

Given that $\partial g/\partial B > 0$, $\partial g/\partial s_n > 0$ and $ds_n/dB < 0$, the direct effect of the policy upon the growth rate is positive whereas its indirect effect is negative (due to the decrease of spatial concentration). Obviously, the question is which of these two effects is the most important. To answer this question, we first evaluate the effect of the policy on the labor employed in the industrial and traditional sectors. We know that the labor in the M and Z sectors are given by $L_M = \alpha LE(\sigma-1)/\sigma$ and $L_Z = (1-\alpha)LE$ respectively. At the steady state, the wage rate is equal to 1 and that the value of capital is equal to the marginal cost of capital, we have $F = L\eta/KW$ and $E = 2 + \rho\eta/W$. Thus, we can write the following equalities

$$\frac{dL_M}{dB} = \frac{dL_M}{dE}\frac{dE}{ds_n}\frac{ds_n}{dB} > 0 \qquad \frac{dL_Z}{dB} = \frac{dL_Z}{dE}\frac{dE}{ds_n}\frac{ds_n}{dB} > 0$$

Given that the labor force is fixed over time, we know that if the policy increases the labor devoted to good-producing sector, it decreases the labor devoted to the innovative sector. In other words, we have $dL_I/dB < 0$ and this result will give us the sign of dg/dB. Indeed, we have $L_I = gL\eta/[s_n + \lambda(1-s_n)]$ and we can write:

$$\frac{dL_I}{dB} = \frac{\partial L_I}{\partial g}\frac{dg}{dB} + \frac{\partial L_I}{\partial s_n}\frac{ds_n}{dB}$$

As $dL_I/dB < 0$, $\partial L_I/\partial g > 0$, $\partial L_I/\partial s_n < 0$ and $ds_n/dB < 0$, it becomes clear that $dg/dB < 0$. Thus the policy decreases the number of researchers, the productivity of R&D activities (due to a less concentrated economic geography) and finally the economic growth. Contrary to the innovation policy, this industrial policy is not able to increase economic growth because the negative indirect effect is lower than its positive direct effect.

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