

Dynamical Interaction Between Financial and Business Cycles*

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Abstract

We adopt the Dynamical Influence model from computer science and transform it to study the interaction between business and financial cycles. For this purpose, we merge it with Markov-Switching Dynamic Factor Model (MS-DFM) which is frequently used in economic cycle analysis. The model suggested in this paper, the Dynamical Influence Markov-Switching Dynamic Factor Model (DI-MS-DFM), allows to reveal the pattern of interaction between business and financial cycles in addition to their individual characteristics. More specifically, with the help of this model we are able to identify and describe quantitatively the existing regimes of interaction in a given economy, and we allow them to switch over time. We are also able to determine the direction of causality between the two cycles for each of the regimes. The model estimated on the US data demonstrates reasonable results, identifying the periods of higher interaction between the cycles in the beginning of 1980s and during the Great Recession, while in-between the cycles evolve almost independently. The output of the model can be useful for policymakers since it provides a timely estimate of the current interaction regime, which allows to adjust the timing and the composition of the policy mix.

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1 Introduction

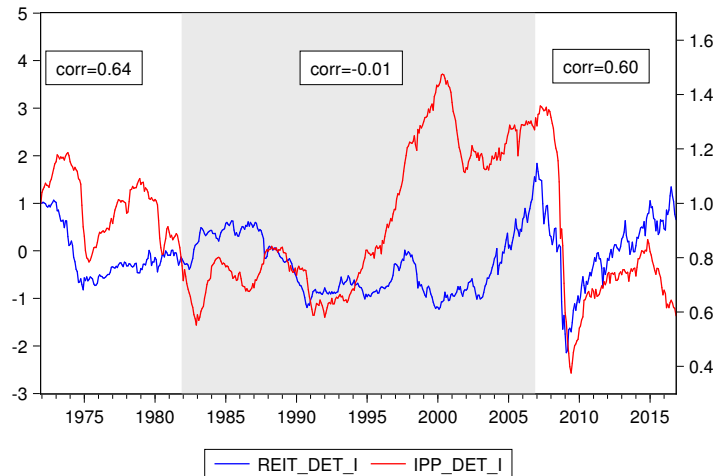
Throughout the history, the financial sector has been given an increasing role with respect to the business cycle: from neutral intermediary in the theory of Modigliani-Miller to the early-warning indicator revealing the expectations of the economic agents about the business cycle in the framework of the efficient market hypothesis, then further to financial accelerator exacerbating the shocks in the real economy in models with financial frictions, and finally, to the independent source of shocks, on a par with technology and preference shocks in the New Keynesian DSGE models. Given the fast development and the increasing importance of the financial sector, the understanding of the interaction between the financial sector and the business cycle has become crucial for coordination of fiscal, monetary and macroprudential policies. For this purpose, the quantitative estimates of the role of the financial sector are essential.

The study of the financial sector and financial crises in particular gave rise to the notion of the financial cycle. For the moment, there is no single definition of the financial cycle. Instead, in most applied papers researchers refer to the fluctuations of credit, equity and house prices. In spite of the fact that these represent different parts of the financial sector, they possess similar cyclical features, and are therefore considered as the features of the financial cycle. Hubrich et al. (2013), Borio (2014) find that the financial cycles are longer than the real business cycles and last about 12-15 years in US, France and Italy. Drehmann et al. (2012), Ciccarelli et al. (2016), Canova and Ciccarelli (2009), Canova and Ciccarelli (2012) find that the amplitude and duration of the financial cycle evolve. Borio (2007) states that they actually depend on financial regime (liberalized market, controlled market), monetary policy (high and variable inflation causes financial instability) and the state of the business cycle (recession or expansion). In the same time, most of the studies agree that the business cycle, in turn, depends on the financial cycle, with the real shocks being more significant during the episodes of financial instability (see, for example, Bernanke and Gertler (1999), Kiyotaki and Moore (1997), Borio (2014), Hubrich et al. (2013), Claessens et al. (2012)).

The particularly interesting feature of interdependence - the causality direction between the cycles - has been studied in many papers. In the same time, there is no consensus on whether the financial cycle leads the real cycle (Borio (2014)) or lags behind it (Runstler and Vlekke (2015)). This, however, is consistent with the fact that the financial cycles evolve over time and are longer than business cycles.

Given the changing character of the cycles, it is natural to expect that the interaction between the two is also evolving. Indeed, a brief look on the dynamics of the business and financial cycle in the US (approximated by the index of industrial production and the index of house prices) shows that the two cycles have different degree of synchronization in different periods of time (Figure 1). Thus, the cycles are much more correlated in 1970s-beginning of 1980s and after the Global Financial Crisis (with correlation about 0.60), and much less in-between (the correlation is zero).

Figure 1: US industrial production index and US index of house prices



Note: US index of industrial production (red line, right axis, source: Federal Reserve Bank of St. Louis), US index of house prices (blue line, left axis, source: FTSE NAREIT US Real Estate Composite Index). Both series are detrended and seasonally-adjusted.

Taking into consideration all above, an econometric model that is used to study the joint dynamics of business and financial cycles should allow for the dynamical feedback between them. This idea was implemented in several different approaches. Among the popular parametric models are the time-varying VAR (in Hubrich et al. (2013)), Markov-Switching VAR model with time-varying transition probabilities (as, for example, in Anas et al. (2007)), versions of multivariate structural time series models (STSMs) (see Runstler and Vlekke (2015)), and time-varying Panel Bayesian VAR used by Ciccarelli et al. (2016) for the analysis of the macro-financial linkages between countries.

In this paper we attempt to reconcile previous findings on the evolving character of the interaction between business and financial cycles. For this purpose we suggest an alternative model, the Dynamical Influence Markov-Switching Dynamic Factor Model (DI-MS-DFM). We assume that each of the cycles can be in several states (expansion and recession in case of the business cycle, boom and disruption in case of the financial cycle), and that there are several regimes of interaction, which differ in degree of interdependence and leading/lagging behavior of the two cycles. This type of interaction is realized with the help of a specific hierarchical structure, where an exogenous Markov chain governs the size of the impact one cycle has on the other, which is the critical feature of this framework. Due to the specific parametrization of the interaction, the model can be transformed into a classical Markov-Switching model and so allows to obtain very rich statistical inference. Besides average duration, qualitative characteristics, and filtered and smoothed probabilities of each state for each of the cycles, we get the same inference for the existing influence regimes. Also, we are able to detect the channels of transmission of states, identify the direction of causality between cycles and evaluate the relative impact of one cycle on the other, which is the main contribution of the paper. These estimates allow us, first, to obtain the dating of recessions for each cycle separately, second, to determine the leading-lagging relationship between business cycle and financial cycle during different periods of history, third, to identify and characterize the existing influence regimes as well as to make probabilistic

inference on the current interaction regime and finally, provide forecasts of future states of each chain given the current influence regime. All these estimates obtained in real time are particularly important for the policymakers since they can be used in the design and adjustment of the policy mix. Moreover, if we consider the notion of systemic risk in a broader sense, i.e. as a risk of a joint recession (both in the financial and the business cycle simultaneously), the smoothed probability of the influence regime corresponding to high interaction (as influence regime 2 in our empirical exercise below) can serve as an early-warning indicator of systemic risk.

The paper is organized as follows. In Section 2 we introduce the model, describe the underlying interaction mechanism and define Granger causality. We discuss the Maximum Likelihood estimation and derive the indicators of in-sample inference in Section 3. The h -step ahead forecasts are presented in Section 4. Section 5 demonstrates the quality of in-sample and out-of-sample performance of the model. Section 6 describes the data used in the empirical exercise, whereas its results are discussed in Section 7. Section 8 concludes.

2 The model

We adopt the Dynamical Influence model from computer science by Pan et al. (2012) and transform it to study the interaction between business and financial cycles. For this purpose, we merge it with Markov-Switching Dynamic Factor Model (MS-DFM) which is frequently used in economic cycle analysis. The resulting model, the Dynamical Influence Markov-Switching Dynamic Factor Model (DI-MS-DFM), is presented below.

At date t , $t = 1, \dots, T$, economic agents observe (or infer) the business cycle RF_t and the financial cycle FF_t which have the following dynamics

$$RF_t = \mu(S_t^1) + \varphi(L)RF_t + \sigma(S_t^1)\varepsilon_t, \quad (1)$$

$$FF_t = \beta(S_t^2) + \psi(L)FF_t + \theta(S_t^2)\xi_t, \quad (2)$$

where S_t^1 and S_t^2 are unobservable discrete processes which are associated with a finite number of states and which govern the dynamics of the business cycle and the financial cycle, correspondingly, $\varphi(L) = \varphi_1L + \dots + \varphi_{p_1}L^{p_1}$ and $\psi(L) = \psi_1L + \dots + \psi_{p_2}L^{p_2}$ are lag polynomials of finite order p_1 and p_2 correspondingly, $\{\varepsilon_t\}$ and $\{\xi_t\}$ are independent standard Gaussian white noises. The functions $\mu(\cdot), \sigma(\cdot), \beta(\cdot), \theta(\cdot)$ are known functions of the specified arguments with unknown parameters.

We assume that the interaction between the cycles happens at the level of unobservable processes S_t^1 and S_t^2 , but not observations, which means that (1)-(2) is a restricted VAR.¹

The current values of S_t^1 and S_t^2 are each dependent on the past of both processes and a variable r_t governing the interaction between S_t^1 and S_t^2 , which is the crucial feature of the model:

$$P(S_t^1 | S_{t-1}^1, S_{t-1}^2, r_t) = A(S_{t-1}^1, S_{t-1}^2, r_t), \quad (3)$$

¹When the interaction on the level of observation is also allowed for, the identification of each channel can be an issue.

$$P(S_t^2|S_{t-1}^1, S_{t-1}^2, r_t) = B(S_{t-1}^1, S_{t-1}^2, r_t), \quad (4)$$

$$P(r_t|r_{t-1}) = Q, \quad (5)$$

where r_t is a Markov chain of first order² with a finite number of regimes and the transition probability matrix Q , $A(\cdot)$, $B(\cdot)$ are known functions with unknown parameters. We assume that the initial $r_0, S_0^1, S_0^2, RF_0, FF_0$ are not random.

For the sake of simplicity, we suppose here that the Markov processes r_t , S_t^1 and S_t^2 are of order 1 with two states each ($S_t^1 = 1$ in case of expansion and $S_t^1 = 2$ in case of recession; $S_t^2 = 1$ in case of financial boom and $S_t^2 = 2$ in case of financial disruption; the interpretation of the states of $r_t = \{1, 2\}$ is determined by the degree of mutual influence between the two chains in each regime estimated within the model³). Nevertheless, the analysis can be easily extended to incorporate chains of a higher (and different) order and with more states. Similarly, it is also feasible to allow the past of RF_t , FF_t or some observable covariate cause S_t^1 and S_t^2 .

Unlike classic Markov-switching models used in business cycle analysis, strictly speaking, the processes S_t^1 and S_t^2 are not Markov chains since the current state of each of them depends on the past of the other chain, too. Moreover, the process (S_t^1, S_t^2) is not Markov either as it depends on all its lags. Nevertheless, for the ease of exposition, we address to S_t^1 and S_t^2 as “chains”.

To understand the dynamics of the model, we present the conditional distributions of \underline{RF}_t , \underline{FF}_t , \underline{S}_t^1 , \underline{S}_t^2 , \underline{r}_t using a generic notation $\underline{x}_t = (x_t, x_{t-1}, \dots, x_0)$:

$$\mathfrak{L}(r_t|\underline{RF}_{t-1}, \underline{FF}_{t-1}, \underline{S}_{t-1}^1, \underline{S}_{t-1}^2, \underline{r}_{t-1}) = \mathfrak{L}(r_t|r_{t-1}), \quad (6)$$

$$\mathfrak{L}(S_t^1|\underline{RF}_{t-1}, \underline{FF}_{t-1}, \underline{S}_{t-1}^1, \underline{S}_{t-1}^2, \underline{r}_t) = \mathfrak{L}(S_t^1|S_{t-1}^1, S_{t-1}^2, r_t), \quad (7)$$

$$\mathfrak{L}(S_t^2|\underline{RF}_{t-1}, \underline{FF}_{t-1}, \underline{S}_{t-1}^1, \underline{S}_{t-1}^2, \underline{r}_t) = \mathfrak{L}(S_t^2|S_{t-1}^1, S_{t-1}^2, r_t), \quad (8)$$

$$\mathfrak{L}(RF_t|\underline{RF}_{t-1}, \underline{FF}_{t-1}, \underline{S}_t^1, \underline{S}_t^2, r_t) = N(\mu(S_t^1) + \varphi(L)RF_t, \sigma^2(S_t^1)), \quad (9)$$

$$\mathfrak{L}(FF_t|\underline{FF}_{t-1}, \underline{RF}_t, \underline{S}_t^1, \underline{S}_t^2, r_t) = N(\beta(S_t^2) + \psi(L)FF_t, \theta^2(S_t^2)). \quad (10)$$

The fundamental assumptions of the model are:

1. r_t is autonomous, S_t^1 and S_t^2 do not cause r_t in the Granger sense since $\underline{S}_{t-1}^1, \underline{S}_{t-1}^2$ do not appear in its conditional distribution.
2. RF_t and FF_t do not Granger cause S_t^1, S_t^2 and r_t .
3. S_t^1 and S_t^2 are conditionally independent given $\underline{RF}_{t-1}, \underline{FF}_{t-1}, \underline{S}_{t-1}^1, \underline{S}_{t-1}^2, \underline{r}_t$.
4. The process (S_t^1, S_t^2, r_t) is an autonomous Markov chain.
5. RF_t and FF_t are conditionally independent given r_t, S_t^1 and S_t^2 .

²We assume that.

³For simplicity we assume that the number of states is equal, although this assumption is not restrictive.

To summarize, the dynamics of the model can be represented in the following way (with $\omega_t = (r_t, S_t^1, S_t^2, RF_t, FF_t)$):

$$r_t | \omega_{t-1} = r_t | r_{t-1}, \quad (11)$$

$$S_t^1 | r_t, \omega_{t-1} = S_t^1 | r_t, S_{t-1}^1, S_{t-1}^2, \quad (12)$$

$$S_t^2 | r_t, S_t^1, \omega_{t-1} = S_t^2 | r_t, S_{t-1}^1, S_{t-1}^2, \quad (13)$$

$$RF_t | r_t, S_t^1, S_t^2, \omega_{t-1} = RF_t | S_t^1, \quad (14)$$

$$FF_t | RF_t, r_t, S_t^1, S_t^2, \omega_{t-1} = FF_t | S_t^2. \quad (15)$$

2.1 The interaction mechanism

The exogenous influence regime process r_t is an ergodic first-order Markov Chain with 2 states, i.e.

$$P(r_t = j | r_{t-1} = i, r_{t-2} = k, \dots) = P(r_t = j | r_{t-1} = i) = q_{ij}, \quad j, i, k = \{1, 2\},$$

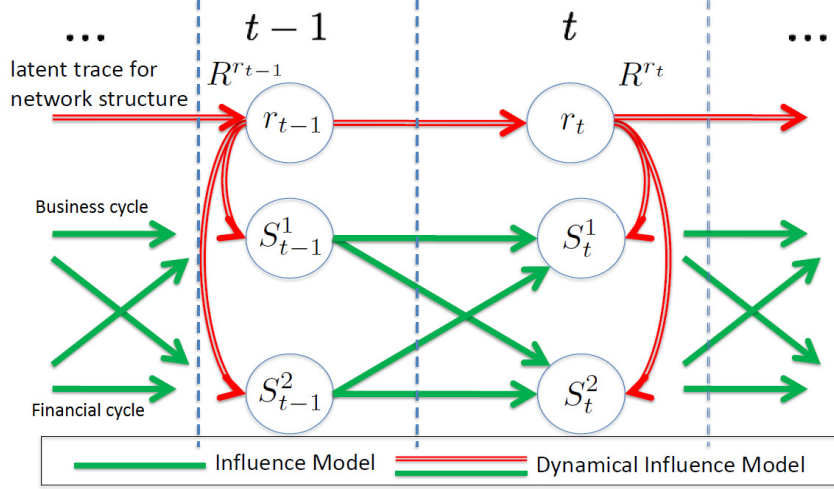
So r_t switches states according to the transition probabilities matrix $Q = \begin{bmatrix} q_{11} & 1 - q_{11} \\ 1 - q_{22} & q_{22} \end{bmatrix}$

At period t the state of the business cycle S_t^1 depends on its own past S_{t-1}^1 and also on the previous state of the financial cycle S_{t-1}^2 . Similar logic applies to the financial cycle. The relative importance of each chain is determined by a matrix R^{r_t} , where $r_t = \{1, 2\}$, which assigns weights to S_{t-1}^1 and S_{t-1}^2 , thus determining their self-impact and the influence of the other chain given the current influence regime r_t . Therefore, the probability that the business cycle is in state S_t^1 , given the states S_{t-1}^1, S_{t-1}^2 and r_t , is a weighted average of probabilities to switch from $S_{t-1}^1 = a$ to $S_t^1 = b$ and from $S_{t-1}^2 = c$ to $S_t^1 = b$, where $a, b, c, = \{1, 2\}$, with weights determined by r_t :

$$P(S_t^i = a | S_{t-1}^i = b, S_{t-1}^k = c, r_t) = R_{i,i}^{r_t} \times P_{ba}^i + R_{k,i}^{r_t} \times C_{ca}^{k,i}, \quad (16)$$

with $i, k = \{1, 2\}$, $i \neq k$, $a, b, c = \{1, 2\}$, and where $X_{i,j}$ denotes the element of the i -th row and j -th column of the matrix X . Here $P^i, i = 1, 2$, is a 2×2 matrix of parameters capturing the self-state transition, so that the element $P_{1,1}^1$ shows the probability that the first chain stays in the regime 1 “expansion”. Similarly, the matrix $C^{k,i}$, $i, k = \{1, 2\}$, $i \neq k$, is a 2×2 matrix of parameters that capture cross-chain transition, so that, for example, the element $C_{1,1}^{1,2}$ shows the probability that expansion in the first chain induces expansion in the second chain. Importantly, self-state transitions and cross-chain transitions do not depend on r_t . The value $R_{k,i}^{r_t}$ shows the relative importance (the weight) of the past of chain k on the present of the chain i given the current influence regime $r_t = \{1, 2\}$. Therefore, the larger are the diagonal elements of this matrix, the higher is the self-impact, and more independent are the chains. The most important feature of this framework arises from the fact that the weights vary over time with r_t , thus rendering the interaction between the two chains dynamical. We illustrate schematically the Dynamical Influence model in Figure 1.

Figure 2: A graphical representation of the Dynamical Influence Model



Note: This is a modified version of Figure 2 from the paper by Pan et al. (2012)

This type of interaction is new in the economic literature. The existing methods based on the modeling of the joint process (S_t^1, S_t^2) allow either for a fixed relation between the chains (in case of static transition probability matrix) or exogenously driven relation (in case of transition probability matrix depending on some covariates). On the contrary, in this model the interaction is designed to be intrinsically dynamical, whether dependent on the covariates or not.

As we will show in the next section, after introduction of a new state variable the model boils down to the classic Hamilton (1989) Markov-switching model. Therefore, once the estimates of the coefficients of (1) and (2), $P^1, P^2, C^1, C^2, Q, R^1, R^2$ are obtained, the standard filtered and smoothed probabilities of each state of each chain can be calculated, including the smoothed probability $P(r_t = j | I_T)$, $j = 1 \dots J$ of being in a particular influence regime j , where $I_\tau = (RF_\tau, FF_\tau)$ is the information available at time τ . On top of that, it would be possible to calculate the joint filtered and smoothed probabilities, which is useful for the purpose of analysis of joint crises in real and financial sectors.

2.2 Granger causality

As we said above, in this framework the two cycles RF_t and FF_t interact on the level of chains. Importantly, the estimated matrices of coefficients R^1 , and R^2 can give us an idea about the causality relation between the two chains S_t^1 and S_t^2 .

Consider a process $\tilde{S}_t = (S_t^1, S_t^2, r_t)$, which is a Markov process with 8 states. We can decompose the transition probabilities as follows:

$$P(S_t^1, S_t^2, r_t | S_{t-1}^1, S_{t-1}^2, r_{t-1}) = P(S_t^1 | S_t^2, r_t, S_{t-1}^1, S_{t-1}^2, r_{t-1}) P(S_t^2 | r_t, S_{t-1}^1, S_{t-1}^2, r_{t-1}) P(r_t | S_{t-1}^1, S_{t-1}^2, r_{t-1}). \quad (17)$$

Using the assumptions (2) and (3) and equation (6), this expression can be simplified:

$$P(S_t^1, S_t^2, r_t | S_{t-1}^1, S_{t-1}^2, r_{t-1}) = P(S_t^1 | r_t, S_{t-1}^1, S_{t-1}^2) P(S_t^2 | r_t, S_{t-1}^1, S_{t-1}^2) P(r_t | r_{t-1}). \quad (18)$$

Now, like Billio and Di Sanzo (2015), we can define Granger non-causality between S_t^1 and S_t^2 :

1. S_{t-1}^2 does not strongly cause S_t^1 one-step ahead given S_{t-1}^1 and r_t if

$$P(S_t^1 | S_{t-1}^1, S_{t-1}^2, r_t) = P(S_t^1 | S_{t-1}^1, r_t) \quad \forall t. \quad (19)$$

2. S_{t-1}^1 does not strongly cause S_t^2 one-step ahead given S_{t-1}^2 and r_t if

$$P(S_t^2 | S_{t-1}^2, S_{t-1}^1, r_t) = P(S_t^2 | S_{t-1}^2, r_t) \quad \forall t. \quad (20)$$

We can also define the independence of two chains as follows:

3. S_t^1 and S_t^2 are independent given r_t if

$$P(S_t^1, S_t^2, r_t | S_{t-1}^1, S_{t-1}^2, r_{t-1}) = P(S_t^1 | r_t, S_{t-1}^1) P(S_t^2 | r_t, S_{t-1}^2) P(r_t | r_{t-1}).$$

Following the approach of Billio and Di Sanzo (2015), for a given parametrization (16), the conditions of the strong one-step ahead non-causality and independence can be derived as restrictions on the parameter space.

The restriction $H_{1 \neq 2}$ of the strong non-causality of S_t^1 towards S_t^2 given r_t implies that the parameter related to S_{t-1}^1 is equal to zero. So, if

$$P(S_t^2 = a | S_{t-1}^1 = b, S_{t-1}^2 = c, r_t) = R_{2,2}^{r_t} \times P_{ca}^2 + R_{1,2}^{r_t} \times C_{ba}^{12}, \quad (21)$$

then

$$H_{1 \neq 2} : R_{1,2}^{r_t} = 0 \quad (22)$$

Under $H_{1 \neq 2}$ S_{t-1}^1 does not cause one-step ahead S_t^2 given S_{t-1}^2 and r_t . Since the terms related to S_{t-1}^1 are excluded from (21), therefore $P(S_t^2 | S_{t-1}^2, S_{t-1}^1, r_t) = P(S_t^2 | S_{t-1}^2, r_t)$.

On the other hand, S_t^2 does not strongly cause one-step ahead S_t^1 given S_{t-1}^1 and r_t if :

$$P(S_t^1 = a | S_{t-1}^1 = b, S_{t-1}^2 = c, r_t) = R_{1,1}^{r_t} \times P_{ba}^1 + R_{2,1}^{r_t} \times C_{ca}^{21}, \quad (23)$$

$$H_{2 \neq 1} : R_{2,1}^{r_t} = 0 \quad (24)$$

The term related to S_{t-1}^2 is excluded from (23), so $P(S_t^1|S_{t-1}^1, S_{t-1}^2, r_t) = P(S_t^1|S_{t-1}^1, r_t)$.

Finally, the restriction of the independence of S_t^1 and S_t^2 given r_t implies that both restrictions (22) and (24) are verified simultaneously:

$$H_{2\perp 1} : R_{2,1}^{r_t} = R_{1,2}^{r_t} = 0 \quad (25)$$

Therefore, the value and significance of the off-diagonal coefficients of the matrices R^1 and R^2 allow to make inference on the causality between the two chains within each regime j . Moreover, since the elements in R^1 and R^2 are not necessarily 0 and 1, we can quantify the relative importance of each of the affecting chains.

Note that the values of C_{ab}^{ij} , $i, j = \{1, 2\}$ give the idea of the global character of Granger causality between the two cycles, defining the channels of interaction irrespective of the current influence regime. At the same time, the conditions on $R_{i,j}^{r_t}$ refer to local changes in Granger causality, and can modify the channel if it exists (the relevant coefficient of C_{ab}^{ij} is non-zero).

3 Maximum Likelihood Estimation

On the basis of observable data, we need to infer the distributions of the underlying latent variables and system parameters for the DI-MS-DFM. If the influence regime were constant, a standard approach to be applied in order to estimate (1)-(16) would be to construct an auxiliary state variable (S_t^1, S_t^2) with 2^2 states:

$$P(S_t^1 = a, S_t^2 = b | S_{t-1}^1 = c, S_{t-1}^2 = d) = (R_{1,1} \times P_{ca}^1 + R_{2,1} \times C_{da}^{2,1})(R_{2,2} \times P_{db}^2 + R_{1,2} \times C_{cb}^{1,2}) \quad (26)$$

However, when different influence regimes come into play, the coefficients of matrices R_1 and R_2 are dependent on r_t , and the transition probability matrix of (S_t^1, S_t^2) becomes Markov-switching itself:

$$P(S_t^1 = a, S_t^2 = b | S_{t-1}^1 = c, S_{t-1}^2 = d, r_t) = (R_{1,1}^{r_t} \times P_{ca}^1 + R_{2,1}^{r_t} \times C_{da}^{2,1})(R_{2,2}^{r_t} \times P_{db}^2 + R_{1,2}^{r_t} \times C_{cb}^{1,2}), \quad (27)$$

so the standard estimation procedures can not be applied. This problem is easily overcome by using the joint state variable $\tilde{S}_t = (S_t^1, S_t^2, r_t)$ with $2^3 = 8$ states instead of (S_t^1, S_t^2) . In this case, the transition probability matrix Π is constant and is computed as follows:

$$\begin{aligned} \Pi = P(\tilde{S}_t | \tilde{S}_{t-1}) &= P(S_t^1 | S_{t-1}^1, S_{t-1}^2, r_t = j) \times P(S_t^2 | S_{t-1}^1, S_{t-1}^2, r_t = j) \times P(r_t = j | r_{t-1} = k) \\ &= P(S_t^1 | S_{t-1}^1, S_{t-1}^2, r_t = j) \times P(S_t^2 | S_{t-1}^1, S_{t-1}^2, r_t = j) \times Q_{k,j}. \end{aligned} \quad (28)$$

Note that, due to the hierarchical structure that we impose on the chains (S_t^1, S_t^2, r_t) , the matrix Π has a more parsimonious representation than a transition matrix of a Markov chain with 8 states would usually have. Indeed, matrix Π contains only 14 parameters instead of 56, which certainly facilitates

the numerical optimization of the likelihood. For notational use, we arrange the eight states of \tilde{S}_t in the following order: $(S_t^1, S_t^2, r_t) = \{(0, 0, 0), (1, 0, 0), (0, 1, 0), (1, 1, 0), (0, 0, 1), (1, 0, 1), (0, 1, 1), (1, 1, 1)\}$.

The classical Hamilton (1989) filter can then be applied. The filter has the filtered probability $P(\tilde{S}_{t-1} = j|I_{t-1})$ as an input, where $I_t = (RF_t, FF_t)$, and produces the filtered probability $P(\tilde{S}_t = j|I_t)$ as an output, giving the likelihood $f(y_t|I_{t-1})$ as a by-product. Once the starting filtered probability $P(\tilde{S}_0 = j|I_0)$ is initiated (we suppose that the probability of starting in any of eight states of \tilde{S}_0 is equal, $P(\tilde{S}_0 = j|I_0) = 1/8, \forall j = 1, \dots, 8$), the filtered probability for steps $t = 1 \dots T$ are calculated by iterating the following:

$$P(\tilde{S}_t = j, \tilde{S}_{t-1} = i|I_{t-1}, \gamma) = P(\tilde{S}_t = j|\tilde{S}_{t-1} = i, \gamma)P(\tilde{S}_{t-1} = i|I_{t-1}, \gamma), \quad (29)$$

$$f(y_t, \tilde{S}_t = j, \tilde{S}_{t-1} = i|I_{t-1}, \gamma) = f(y_t|\tilde{S}_t = j, \tilde{S}_{t-1} = i, I_{t-1}, \gamma)P(\tilde{S}_t = j, \tilde{S}_{t-1} = i|I_{t-1}, \gamma) \quad (30)$$

$$f(y_t|I_{t-1}, \gamma) = \sum_{j=1}^2 \sum_{i=1}^2 f(y_t, \tilde{S}_t = j, \tilde{S}_{t-1} = i|I_{t-1}, \gamma). \quad (31)$$

$$\begin{aligned} P(\tilde{S}_t = j, \tilde{S}_{t-1} = i|I_t, \gamma) &= \frac{f(y_t, \tilde{S}_t = j, \tilde{S}_{t-1} = i|I_{t-1}, \gamma)}{f(y_t|I_{t-1}, \theta)} \\ &= \frac{f(y_t|\tilde{S}_t = j, \tilde{S}_{t-1} = i, I_{t-1}, \theta) \times Pr(\tilde{S}_t = j, \tilde{S}_{t-1} = i|I_{t-1}, \theta)}{f(y_t|I_{t-1}, \gamma)}, \end{aligned} \quad (32)$$

$$f(y_t|\tilde{S}_t = j, \tilde{S}_{t-1} = i, I_{t-1}, \gamma) = (2\pi)^{-1}(\sigma_{S_t^1}^2 \theta_{S_t^2}^2)^{-1/2} \exp\left\{-\frac{1}{2} \frac{(\tilde{R}F_t)^2}{\sigma_{S_t^1}^2} - \frac{1}{2} \frac{(\tilde{F}F_t)^2}{\theta_{S_t^2}^2}\right\}, \quad (33)$$

$$P(\tilde{S}_t = j|I_t) = \sum_{i=1}^2 P(\tilde{S}_t = j, \tilde{S}_{t-1} = i|I_t, \gamma), \quad (34)$$

where

$$y_t = (RF_t, FF_t),$$

$$\gamma = (P, B, C, K, Q, R^1, R^2, \mu_1, \mu_2, \beta_1, \beta_2, \sigma_1^2, \sigma_2^2, \omega_1^2, \omega_2^2, \varphi_1 \dots \varphi_{p_1}, \psi_1 \dots \psi_{p_2}),$$

$$\mu_{S_t^1} = \mu_2(S_t^1 - 1) - \mu_1(S_t^1 - 2),$$

$$\sigma_{S_t^1}^2 = \sigma_2^2(S_t^1 - 1) - \sigma_1^2(S_t^1 - 2),$$

$$\beta_{S_t^2} = \beta_2(S_t^2 - 1) - \beta_1(S_t^2 - 2),$$

$$\theta_{S_t^2} = \theta_2(S_t^2 - 1) - \theta_1(S_t^2 - 2),$$

$$\tilde{R}F_t = RF_t - \mu_{S_t^1} - \varphi(L)RF_t$$

$$\tilde{F}F_t = FF_t - \beta_{S_t^2} - \psi(L)FF_t.$$

As a by-product of the Hamilton filter above, we obtain the log-likelihood function for the whole sample for any given value of γ :

$$\mathcal{L}(y, \gamma) = \ln(f(y_T, y_{T-1}, \dots, y_0 | I_T, \gamma)) = \sum_{t=1}^T \ln(f(y_t | I_{t-1}, \gamma)), \quad (35)$$

where $f(y_t | I_{t-1}, \gamma)$ can be computed using formulas (29) to (34).

Once the filtered probability $P(\tilde{S}_t = j | I_t)$ is obtained for all $t = 1 \dots T$, it is possible to compute the smoothed probability $P(\tilde{S}_t = j | I_T)$ (we refer the reader to Hamilton (1989) for details). The filtered and smoothed probabilities for each chain can be obtained by integrating out the other chains in \tilde{S}_t , i.e.:

$$P(S_t^i = k | I_t) = \sum_{m=1}^2 \sum_{l=1}^2 P(S_t^i, S_t^{-i} = m, r_t = l | I_t), \quad (36)$$

$$P(S_t^i = k | I_T) = \sum_{m=1}^2 \sum_{l=1}^2 P(S_t^i, S_t^{-i} = m, r_t = l | I_T), \quad (37)$$

$$P(r_t = j | I_t) = \sum_{k=1}^2 \sum_{l=1}^2 P(S_t^i = k, S_t^{-i} = m | I_t), \quad (38)$$

$$P(r_t = j | I_T) = \sum_{k=1}^2 \sum_{l=1}^2 P(S_t^i = k, S_t^{-i} = m | I_T). \quad (39)$$

Since the maximum likelihood is obtained with numerical algorithms, this estimation method can be applied only when the number of parameters is not too big. When more interacting chains with more states are involved, or when more influence regimes are allowed for, the optimization algorithms may have difficulties to converge. In this case, the Forward-Backward algorithm and variational EM suggested by Pan et al. (2012) can be used. Pan et al. (2012) have successfully applied this approach to model the interaction between 50 states with 6 latent states each and 3 regimes of influence in order to evaluate flu epidemics.

4 Forecasting

The in-sample analysis tools, such as filtered and smoothed probabilities discussed above, give a posteriori insight into the dating of both financial and business cycles and the types and timing of different interaction regimes. The out-of-sample analysis is a valuable complement, providing a probabilistic draft of future periods.

***H*-step ahead forecast of ergodic probability of the future state.** Since the chain \tilde{S}_t is the Markov chain of order one, it is straightforward that

$$P(\tilde{S}_{t+h}|\tilde{S}_t) = \Pi^h. \quad (40)$$

Then, the h -step ahead forecast for each individual chain can be computed by integrating the other two chains entering \tilde{S}_t out. For example, the h -step ahead forecast for S_{t+h}^1 is:

$$P(S_{t+h}^1 = a|\tilde{S}_t) = \sum_{b=1}^2 \sum_{c=1}^2 P(S_{t+h}^1 = a, S_{t+h} = b, r_{t+h} = c|\tilde{S}_t) = \Pi^h v, \quad (41)$$

where vector v selects the columns of Π^h to be summed. For example, for $P(S_{t+h}^1 = 1|\tilde{S}_t)$ the vector v is $v = (10101010)'$.

H -step ahead forecast of the future state. It is also possible to compute an h -step ahead forecast of the state variable \tilde{S}_t

$$P(\tilde{S}_{t+h}|I_t) = \sum_{i=1}^8 P(\tilde{S}_{t+h}|\tilde{S}_t = i)P(\tilde{S}_t = i|I_t) = P(\tilde{S}_t|I_t)'\Pi^h, \quad (42)$$

where $P(\tilde{S}_t|I_t)$ is the vector of filtered probabilities of being in state $\tilde{S}_t = i$, $i = \{1, \dots, 8\}$.

As in the previous case, the h -step ahead forecast for each chain separately can be calculated by integrating the other chains out. For example, for S_{t+h}^1 we obtain:

$$P(S_{t+h}^1 = a|I_t) = \sum_{b=1}^2 \sum_{c=1}^2 P(S_{t+h}^1, S_{t+h} = b, r_{t+h} = c|I_t) = P(\tilde{S}_t|I_t)'\Pi^h v, \quad (43)$$

where, as before, the vector v selects the columns to sum over. For example, for $P(S_{t+h}^1 = 2|\tilde{S}_t)$ the vector v is $v = (01010101)'$.

H -step ahead forecast of factors. Given equations (1) - (5), the h -step ahead forecasts of the factors are obtained recursively as for regular $AR(p)$ forecasts. For example, if

$$RF_{t+h} = \mu(S_{t+h}^1) + \varphi(L)RF_{t+h-1} + \sigma(S_{t+h}^1)\varepsilon_{t+h}, \quad (44)$$

then

$$\hat{R}F_{t+h} = E(RF_{t+h}|I_t) = E(\mu(S_{t+h}^1)|I_t) + \varphi(L)E(RF_{t+h}|I_t), \quad (45)$$

where the $E(\mu(S_{t+h}^1)|I_t)$ is a known function of $P(S_{t+h}^1 = a|I_t)$ defined in (43), and $\varphi(L)E(RF_{t+h}|I_t)$ can be calculated using the forecasts obtained in the previous iterations, i.e. for $h-1$, $h-2$, etc. The h -step ahead forecast of the financial factor $\hat{F}F_{t+h|t}$ can be obtained in a similar way.

It is important to notice, however, that the DI-MS-DFM is designed for the identification of the latent influence regime and performs poorly in the forecasts of factors. For this reason in the following sections we focus solely on the in-sample and out-of-sample performance for the forecasts of states.

5 In-sample and out-of-sample performance

In this section we evaluate and compare the quality of in-sample and out-of-sample forecasts. We also verify whether the dynamical influence feature, which is obviously a complication to a regular two-factor Markov-switching Dynamic Factor model, actually helps to obtain better forecasts, both in-sample and out-of-sample.

In order to evaluate the performance of the model in terms of identification of the current state of each of the chains, it is difficult to use empirical data since we have no reference dating for the financial cycle and the influence regimes. For this reason, we run a Monte Carlo experiment on the simulated data. We use the data generating process described in equations (1)-(5) with the parameters set to their estimated values that we obtained using data described in the following section (see Table 7.1).⁴ The generated sample has $T = 500$ observations and is simulated 1000 times.

For the analysis of the accuracy of identification of states, we use the following indicators (in all cases $X_t = \{S_t^1, S_t^2, r_t\}$, X_t^* contains the corresponding true values of states, T is the total number of observations, T_1 is the out-of-sample period, indices *is* and *oos* correspond to in-sample and out-of-sample cases, respectively):

1. *QPS*, the quadratic probability score. This indicator is conceptually similar to the mean squared error and is calculated in the following way:

$$QPS_{is}(X) = \frac{\sum_{t=1}^T (P(X_t = 2|I_T, \hat{\gamma}) - X_t^*)^2}{T}, \quad (46)$$

$$QPS_{oos}(X) = \frac{\sum_{t=0}^{T_1-1} (P(X_{T+t+1} = 2|I_{T+t}, \hat{\gamma}) - X_{T+t+1}^*)^2}{T_1}, \quad (47)$$

where $P(X_t = 2|I_T, \hat{\gamma})$ is the smoothed probability of state 2 given in equation (37), $P(X_{T+t+1} = 2|I_{T+t}, \hat{\gamma})$ is the one-step-ahead forecast of probability of state 2 given in equation (43).

2. *FPS*, the false positive score. This indicator gives the proportion of misidentified states and is calculated as

$$FPS_{is}(X) = \frac{\sum_{t=1}^T (I_{P(X_{T+t+1}=2|I_{T+t}, \hat{\gamma}) > 0.5} - X_t^*)^2}{T}, \quad (48)$$

$$FPS_{oos}(X) = \frac{\sum_{t=0}^{T_1-1} (I_{P(X_{T+t+1}=2|I_{T+t}, \hat{\gamma}) > 0.5} - X_{T+t+1}^*)^2}{T_1}, \quad (49)$$

where $I_{\hat{P} > 0.5}$ is the indicator function taking value one when \hat{P} is higher than 0.5, a conventional threshold.

3. *AUROC*, the area under the Receiver Operating Characteristic curve,⁵ measures discrimination, that is, the ability of the model to correctly classify the states of a chain. *AUROC* takes the

⁴The simulations show that model is very sensitive to the difference between the influence regimes. For this reason, to generate our data, we use the estimates with a large difference between \hat{R}_1 and \hat{R}_2 . However, the simulations show that in case the regimes are close, this does not deteriorate the accuracy of the identification of the states of the financial and business cycles (the model performs as good as a one-influence regime Markov-Switching Dynamic Factor model, which is a special case of DI-MS-DFM with $R_1 = R_2$). However, this issue can be solved by using different initial values or setting them as additional parameters of the model.

⁵ROC curve is a graphical plot which juxtaposes the true positive rates (the fraction of correctly identified recessions (or financial disruptions)) and the false positive rates (the fraction of incorrectly identified recessions (or financial disruptions)) as the threshold of the classifier (in this case, the cut-off smoothed probability for a state to be identified as recession state) varies.

value in $[0; 1]$, $AUROC = 1$ meaning that the test is perfect. $AUROC_{is}$ is calculated using the whole available sample, while $AUROC_{oos}$ is calculated using only the out-of-sample period.

4. J , Youden's J statistic, which captures both sensitivity (the proportion of correctly identified recessions) and specificity (the proportion of correctly identified expansions) of the model and measures the probability of an informed decision versus pure chance. For a given threshold level is calculated as

$$J = \frac{TP}{TP + FN} + \frac{TN}{TN + FP} - 1,$$

where TP is the number of true positives (correctly identified recessions or financial disruptions), FN is the number of false negatives (incorrectly identified expansions or financial booms), TN is the number of true negatives (correctly identified expansions or financial booms) and FP is the number of false positives (incorrectly identified recessions or financial disruptions). J takes values in $[-1; 1]$, where $J = 1$ means that there is no false positives or negatives, so the classification is perfect, whereas $J = 0$ means that the proportions of correctly and incorrectly identified states are the same, so the model is useless. For comparability, we set the threshold level h equal to 0.5 for all chains.

QPS can basically be considered as a mean squared error computed for the forecasts (nowcasts) of states, and is informative only in comparison of several classifiers. The other three measures can be used independently, as FPS , $AUROC$ and J - statistic show the relative quality of the model with respect to pure chance.

5.1 In-sample performance

We present below the indicators of the in-sample performance of the DI-MS-DFM. These results are opposed to the ones estimated with the help of a one-regime Markov-Switching Dynamic Factor model (which implies $R^1 = R^2$) (see Table 5.1).

We observe that the DI-MS-DFM performs very well in the identification of the individual cycles - the error rate measured with QPS_{is} and FPS_{is} is low, whereas the classification quality measured with $AUROC_{is}$ and J_{is} is high. The influence regime is more difficult to identify (both QPS_{is} and FPS_{is} are higher, whereas $AUROC_{is}$ and J_{is} are lower), however values of QPS_{is} and FPS_{is} do not exceed the ones usually obtained in the empirical papers for the business cycle.

When comparing the performance of DI-MS-DFM to one-regime MS-DFM (see Table 5.1), one may notice that, all four measures of quality pointing in the same direction, the introduction of dynamic influence regime improves the accuracy of the identification of states of the individual cycles, even though regimes themselves are identified with less precision.

Table 5.1: In-sample performance: smoothed probabilities of the second state

	DI-MS-DFM			
	QPS_{is}	FPS_{is}	$AUROC_{is}$	J_{is}
S^1 - business cycle	0.0182	0.0238	0.9754	0.9222
S^2 - financial cycle	0.0444	0.0567	0.9527	0.6819
r - interaction regimes	0.2034	0.2537	0.7343	0.3951

	One influence regime MS-DFM			
	QPS_{is}	FPS_{is}	$AUROC_{is}$	J_{is}
S^1 - business cycle	0.0213	0.0264	0.9727	0.9199
S^2 - financial cycle	0.0471	0.0598	0.9472	0.6788
r - interaction regimes	-	-	-	-

Note: The table describes the ability of the models to identify state two of each of the chains: “recession” for S_t^1 , “high volatility” for S_t^2 and “Interdependent chains” for r_t .

5.2 Out-of-sample performance

One-step ahead forecasts of states. For the out-of-sample analysis on simulated data, the sample is split into in-sample period with $T_1 = 1, \dots, T - 60$ observations and out-of-sample period with $T_2 = 60$ observations, so that the number of observations in the in-sample period corresponds to the one we used in the real sample (395 observations)). The out-of-sample forecasts $P(\tilde{S}_{T_1+t+1}|I_{T_1+t})$, for $t = 1, \dots, T - T_1 - 1$ are then constructed using the equations (42).

The results of the simulation are given in Table 5.2. The reported values are the averages over 1000 simulations.

As expected, the out-of-sample behavior is inferior compared to in-sample performance. However, the quality is still satisfactory, the values of QPS_{oos} and FPS_{oos} for the business cycle corresponding to the ones usually obtained in the empirical exercises (see, for example, Matas Mir et al., 2008). Similarly to the in-sample performance, the introduction of switches in the influence regime does not deteriorate the quality of the out-of-sample identification of the individual cycles.

6 Data description

We perform our analysis for the business and the financial cycle of the United States. To describe the business cycle, we adopt the Dynamic Factor Model approach by Stock and Watson (1989). We assume that each of the indicators of the real sector of an economy (industrial production, consumption, stock,

Table 5.2: Out-of-sample performance: one-step ahead forecast of the future state

DI-MS-DFM				
	QPS_{oos}	FPS_{oos}	$AUROC_{oos}$	J_{oos}
S^1 - business cycle	0.0615	0.0712	0.8952	0.6574
S^2 - financial cycle	0.1309	0.1279	0.6683	0.2435
r - interaction regimes	0.2857	0.3639	0.6195	0.1628

One influence regime MS-DFM				
	QPS_{oos}	FPS_{oos}	$AUROC_{oos}$	J_{oos}
S^1 - business cycle	0.0672	0.0789	0.8437	0.6376
S^2 - financial cycle	0.1322	0.1383	0.7118	0.1687
r - interaction regimes	-	-	-	-

consumer and business surveys, etc.) can be decomposed into two parts, the first one referring to the comovement of series of the real sector (the real sector), and the idiosyncratic part:

$$x_t = \gamma RF_t + z_t, \quad (50)$$

where x_t is a $K \times 1$ vector of stationarized and deseasonalized economic indicators, RF_t is a univariate common factor of the real sector variables - the business cycle, z_t is a $K \times 1$ vector of idiosyncratic components, which is uncorrelated with RF_t at all leads and lags, γ is a $K \times 1$ vector. Bai (2003), Stock and Watson (2002) showed the consistency of PCA for large N and large T for the estimation of factors, so the common factor RF_t can be approximated with the first principal component of a rich database of macroeconomic variables.

To construct the business cycle indicator we use the Stock-Watson database of indicators from CITIBASE and available in the databank of the Federal Reserve Bank of Saint Louis.⁶ The first principal component explains just 18% of the total variance, however it is highly correlated with the GDP growth, contrary to the other components. In practice, the first component is usually enough to describe the business cycle, the other inclusion of the other components giving only marginal improvement (see Doz and Petronevich (2015), for example).

Unfortunately, there is no commonly used definition of the financial cycle (neither theoretical, nor practical). There is no commonly accepted proxy of the financial cycle indicator. For this reason,

⁶see Stock and Watson (2005, 2006)

we construct a financial cycle indicator using the same approach (50). We thus approximate the financial cycle with the first principal component extracted from the database containing 31 indicators of different segments of the financial sector most used in the empirical papers on financial cycles. In particular, we extended the list of indicators used by Guidolin et al. (2013) with the information on deposits, monetary aggregates, loans, reserve balances and other. The complete list is given in Table A.1 in the Appendix.⁷

All data are seasonally adjusted, stationarized and standardized. The time-span covers the period 1976m06-2014m12.

7 Estimation results

7.1 Characteristics of cycles and identified interaction regimes

In this empirical exercise, we set $\varphi_{p_1} = \psi_{p_2} = 0, \forall p_1, p_2$.⁸ We also impose several technical constraints in order to increase the convergence to the correct local maximum. More specifically, we set $\text{diag}(Q) > 0.5e$ (where e is a vector of ones) to avoid the situations when the influence regimes are not persistent. The initial values of β_0, β_1 and σ_0^2, σ_1^2 are set to the mean and the variance for the business cycle observations above and below 0 (the initial values of μ_0, μ_1 and θ_0^2, θ_1^2 are set similarly for the financial factor). The initial values of the matrices R^1 and R^2 are set to their potential values, for example, $\text{diag}(R^1) = [0.9, 0.9]'$, $\text{diag}(R^2) = [0.9, 0]'$.⁹

The estimation results are given in Table 7.1. According to the estimates, switches in the regime of the business cycle happen mostly in mean, whereas the variance stays relatively stable. On the contrary, the financial factor switches primarily in variance. We also find that expansions in both cycles, as well as recessions of the business cycle, are very persistent ($\hat{P}_{11}^1, \hat{P}_{11}^2, \hat{P}_{22}^1$, are close to one). Recessions in the financial cycle are less persistent (\hat{P}_{22}^2 is below 0.9). These estimates match the findings in the previous literature.

Now consider the parameters characterizing the influence. The business cycle is capable of transmitting both expansion and recession to the financial cycle (the coefficients \hat{C}_{11}^{12} and \hat{C}_{22}^{12} are above 0.9). The transmitting ability is reciprocal, although the financial cycle less likely to transmit expansion to the business cycle (\hat{C}_{11}^{21} is only 0.74). A similar asymmetry of influence between the business cycle and the financial cycle (measured as industrial production growth rate and excess returns, correspondingly) was also detected by Billio and Di Sanzo (2015).

⁷Other datasets were also tested. The corresponding results are available by request.

⁸The number of lags has been chosen according to the information criteria. Inclusion of lags of the dependent variables in equations (1) and (2) does not change the estimates significantly.

⁹Setting the initial values of these parameters to random leads to instability in the results. To solve this problem, we try different plausible values: 1) $\text{diag}(R^1) = [0, 0]'$, $\text{diag}(R^2) = [0.9, 0.9]'$, 2) $\text{diag}(R^1) = [0, 0.9]'$, $\text{diag}(R^2) = [0.9, 0.9]'$, 3) $\text{diag}(R^1) = [0.9, 0]'$, $\text{diag}(R^2) = [0.9, 0.9]'$, 4) $\text{diag}(R^1) = [0.9, 0.9]'$, $\text{diag}(R^2) = [0.9, 0.9]'$, 5) $\text{diag}(R^1) = [0, 0]'$, $\text{diag}(R^2) = [0, 0]'$, 6) $\text{diag}(R^1) = [0.5, 0]'$, $\text{diag}(R^2) = [0.5, 0.5]'$, 7) $\text{diag}(R^1) = [0, 0.5]'$, $\text{diag}(R^2) = [0.5, 0.5]'$. The output obtained with different these sets of initial values are equivalent qualitatively and very similar quantitatively.

Table 7.1: Estimation results

	Business cycle		Financial cycle		
	$\hat{\gamma}$	$\hat{\sigma}_{\hat{\gamma}}$	$\hat{\gamma}$	$\hat{\sigma}_{\hat{\gamma}}$	
$\hat{\mu}_1$	0.5718	(0.0374)	$\hat{\beta}_1$	0.1584	(0.0343)
$\hat{\mu}_2$	-0.8512	(0.0712)	$\hat{\beta}_2$	-0.8587	(0.2806)
$\hat{\sigma}_1^2$	0.3103	(0.0276)	$\hat{\theta}_1^2$	0.2742	(0.0311)
$\hat{\sigma}_2^2$	0.8107	(0.0851)	$\hat{\theta}_2^2$	4.0480	(0.9117)
\hat{P}_{11}^1	0.9897	(0.0156)	\hat{P}_{11}^2	0.9888	(0.0192)
\hat{P}_{22}^1	0.9809	(0.0121)	\hat{P}_{22}^2	0.8799	(0.2985)
\hat{C}_{11}^{12}	0.9059	(0.1311)	\hat{C}_{11}^{21}	0.7481	(0.0121)
\hat{C}_{22}^{12}	0.9899	(0.7864)	\hat{C}_{22}^{21}	0.9889	(0.0029)

Influence regimes				
	“Independent chains”		“Interdependent chains”	
$\hat{R}_{1,1}^1$	0.9815		$\hat{R}_{1,1}^2$	0.8562
$\hat{R}_{2,2}^1$	0.9426		$\hat{R}_{2,2}^2$	0.1853
\hat{q}_{11}	0.9900		\hat{q}_{22}	0.9677

Note: The estimated specification is $RF_t = \mu_{s_t^1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma_{s_t^1}^2)$, $FF_t = \beta_{s_t^2} + \xi_t$, $\xi_t \sim N(0, \theta_{s_t^2}^2)$.

The detected abilities of state transmission are clearly necessary for understanding of the relation between the chains. In our framework these should be considered together with the parameters responsible for the influence regimes. The model identified two distinct and very persistent influence regimes (\hat{q}_{11} and \hat{q}_{22} are above 0.96). LR tests confirm that the two identified influence regimes are not redundant (the value of the likelihood ratio $LR = 150.45$, the hypothesis of one regime is rejected at $\alpha = 0.05$). The values $\hat{R}_{1,1}^1$, $\hat{R}_{2,2}^1$, $\hat{R}_{1,1}^2$, $\hat{R}_{2,2}^2$ suggest that the first and the second regimes can be interpreted as “Independent cycles” and “Interdependent cycles”, correspondingly.

We perform a Likelihood-ratio test in order to find out the direction of causality in each of the regimes. More precisely, we test if the high values of $\hat{R}_{1,1}^1$, $\hat{R}_{2,2}^1$ can be interpreted as the absence of causality in the first regime, and if the high value of $\hat{R}_{1,1}^2$ and the low value of $\hat{R}_{2,2}^2$ actually implies that in the second regime the business cycle leads the financial cycle. We test a joint hypothesis H_0 versus the alternative H_1 , where

$$H_0 : \begin{cases} \hat{R}_{1,1}^1 = 1 \\ \hat{R}_{2,2}^1 = 1 \\ \hat{R}_{1,1}^2 = 1 \\ \hat{R}_{2,2}^2 = 0 \end{cases}, \quad H_1 : \begin{cases} \hat{R}_{1,1}^1 \neq 1 \\ \hat{R}_{2,2}^1 \neq 1 \\ \hat{R}_{1,1}^2 \neq 1 \\ \hat{R}_{2,2}^2 \neq 0 \end{cases}. \quad (51)$$

The value of the test statistics is $LR = 81.91$ and largely overcomes the critical value at 5% of

confidence probability ($\chi_{0.95,4}^2 = 9.49$), so H_0 is not rejected.

7.2 Identifying the periods of recession, financial disruption and high interdependence between the cycles

The estimated smoothed probabilities of recession $P(S_t^1 = 2|I_T)$, financial downturn $P(S_t^2 = 2|I_T)$ and second influence regime $P(r_t = 2|I_T)$ are presented in Figures 3-5. Shaded areas correspond to NBER business cycle recessions and are given to verify the validity of the obtained estimates. On Figure 3 one can see that the model captures all business cycle recessions well. The smoothed probability of recession spikes exactly with the beginning of the NBER recession, without either false signals or missed recessions. Whereas the double-dip crisis of 1980 and 1981-1982 is identified very accurately, the duration of the other three recessions observed in the time-span - the early 1990s recession, the dot-com bubble and the Great Recession - appears to be overestimated by the model. This imprecision might be due to the fact that the US business cycle is reported to have at least three states (recession, expansion and slow growth), one of which we have omitted in this simple specification of the model.¹⁰

The adequacy of the estimated smoothed probabilities of financial disruptions is difficult to evaluate since there is no benchmark dating of financial cycles. To provide at least some reference, we use the dates of the beginning of banking crises as identified by Laeven and Valencia (2008 and 2010) and Reinhart and Rogoff (2008) to pinpoint the gravest events in the US banking sector (September 1988 and July 2007) which certainly correspond to financial crises, even though it is possible that they do not cover all financial crises but only those in the banking sector. Comparing the graphs of the smoothed probability with these reference dates on Figure 4, we can see that the model captures the banking crisis of 2008 with much precision, but foreruns the crisis of 1988 by about 10 months. In general, smoothed probability of financial disruption detects all the major events in the last 40 years: the savings and loans crisis and bank crisis during the double-dip recession of 1980 and 1981-1982, Black Monday of 1987, early 1990s recession, the Russian crisis of 1998, bursting of dot-com bubble in 2001, the global financial crisis of 2008.

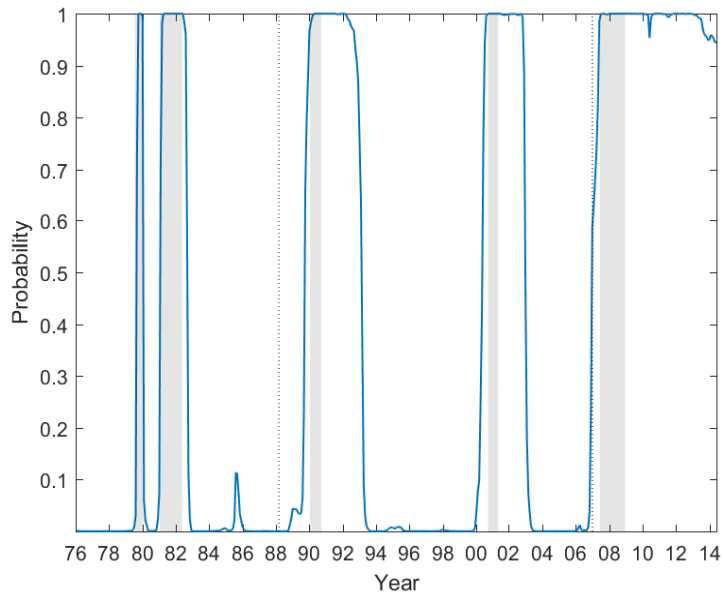
The ongoing influence regime at each point of time is clearly visible from Figure 5. The “Interdependent cycles” regime was active during the double-dip recession and the Great Recession. Both cases (and not during the other two observed recessions during the period under consideration) were marked with increased panic on the stock exchange, which can probably be an explanation of the higher interaction between the financial and the business cycles during these periods. This idea is consistent with the theory of sunspot equilibria: the exogenous random Markov-Switching process r_t can be viewed as an extrinsic variable, influencing the economy through expectations but not affecting the fundamentals. In other words, if the agents’ beliefs are such that the current shock (either financial or economic) is likely to be devastating, they act accordingly on the stock exchange, launching a self-reinforcing mechanism of transition of the shock from the financial sector to the real and back - the economy enters the “Interdependent cycles regime”. Otherwise, if the agents are sure that the shock is temporary (as the Black Monday of 1987, for example), the interaction is just not activated

¹⁰Indeed, the GDP growth rate was recovering much slower during the last three recessions comparing to the preceding ones.

(“Independent cycles” influence regime is on), and the shock does not propagate.

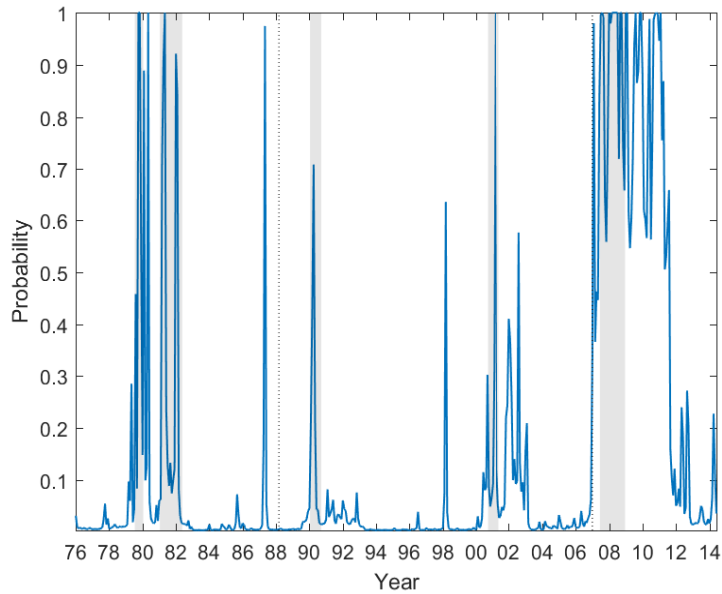
The estimates of the periods of high interaction seem reasonable. However, one may argue that the direction of causality between the two cycles (identified as business cycle leading the financial cycle) might not be the same in 1980-1982 and 2008. If the double-dip recession in the financial cycle might indeed have been provoked by the crisis in the real sector (due to the oil price shock), it seems doubtful that the real sector was leading the financial sector during the Great Recession, when the recession in the real economy was induced by the collapse in the financial sector. This misidentification of causality in the second case might arise from the fact that in this empirical exercise we allow for just two influence regimes. Given the relatively long period of low correlation between the two cycles in the middle of the sample, the model identified the regime of independent cycles and attributed any sort of other relation to the other regime. Therefore, once more influence regimes are allowed for, the model might be able to distinguish different types of interdependence.

Figure 3: Smoothed probability of recession in the business cycle



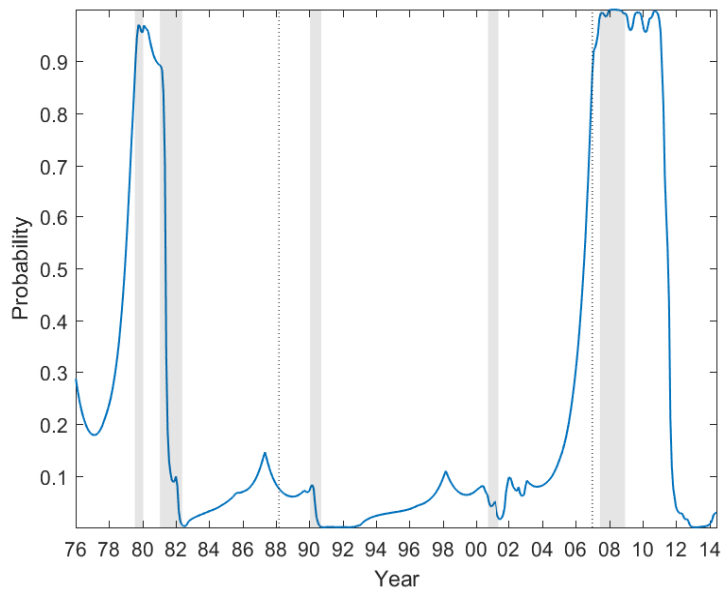
Note: Grey shaded areas correspond to NBER recessions, dotted vertical lines mark the beginning of systemic banking crises as identified by Laeven and Valencia (2008 and 2010) and Reinhart and Rogoff (2008).

Figure 4: Smoothed probability of financial disruption



Note: Grey shaded areas correspond to NBER recessions, while dotted vertical lines mark the beginning of systemic banking crises as identified by Laeven and Valencia (2008 and 2010) and Reinhart and Rogoff (2008).

Figure 5: Smoothed probability of the “Interdependent cycles” regime



Note: Grey shaded areas correspond to NBER recessions, dotted vertical lines mark the beginning of systemic banking crises as identified by Laeven and Valencia (2008 and 2010) and Reinhart and Rogoff (2008).

7.3 Transition probabilities and smoothed probabilities of future states

Table (7.2) contains the estimated one-step ahead transition probabilities for the business cycle and the financial cycle ($P(S_t^i | S_{t-1}^i, S_{t-1}^k, r_{t-1})$, $i, k = \{1, 2\}$, $i \neq k$) calculated using equation (40). These estimates are important since they provide a description of the individual characteristics of each of the cycles. So save space, we report only the probability to switch to expansion (financial boom) $P(S_t^i = 1 | S_{t-1}^i, S_{t-1}^k, r_{t-1})$, the probability of recession (financial disruption) being ($P(S_t^i = 2 | S_{t-1}^i, S_{t-1}^k, r_{t-1}) = 1 - P(S_t^i = 1 | S_{t-1}^i, S_{t-1}^k, r_{t-1})$). Table (7.2) contains the forecasts for all possible combinations of the past values of the chains $S_{t-1}^1, S_{t-1}^2, r_{t-1}$ known at $t - 1$.

Table 7.2: Estimated one-step ahead probability of expansion and financial boom ($P(S_t^i = 1 | S_{t-1}^i, S_{t-1}^k, r_{t-1})$)

	Probability of expansion in the business cycle $P(S_t^1 = 1 S_{t-1}^1, S_{t-1}^2, r_{t-1})$	
	“Independent chains”	“Interdependent chains”
$S_{t-1}^1 = 1, S_{t-1}^2 = 1$	0.99	0.95
$S_{t-1}^1 = 1, S_{t-1}^2 = 2$	0.97	0.85
$S_{t-1}^1 = 2, S_{t-1}^2 = 1$	0.02	0.11
$S_{t-1}^1 = 2, S_{t-1}^2 = 2$	0.01	0.01

	Probability of financial boom $P(S_t^2 = 1 S_{t-1}^1, S_{t-1}^2, r_{t-1})$	
	“Independent chains”	“Interdependent chains”
$S_{t-1}^2 = 1, S_{t-1}^1 = 1$	0.98	0.92
$S_{t-1}^2 = 1, S_{t-1}^1 = 2$	0.93	0.21
$S_{t-1}^2 = 2, S_{t-1}^1 = 1$	0.75	0.87
$S_{t-1}^2 = 2, S_{t-1}^1 = 2$	0.69	0.16

For the business cycle, the probability to switch to expansion depends on the previous state of the business cycle to a large extent. Both expansion and recession states are very persistent (the probability to stay in expansion for any past conditions is above 0.85; similarly, the probability to stay in recession is above 0.89). However, when the "interdependent cycles regime" is active, the impact of the financial cycle is not negligible: financial disruption decreases the probability that the business cycle switches from recession to expansion (from 0.11 to 0.01) confirming the findings of Claessens et al. (2012) who found that disruptions in financial sector tend to make recessions longer. In the same manner, financial disruption reduces chances to stay in expansion in the business cycle (the probability decreases from 0.95 to 0.85).

The probability of financial boom depends both on its past and on the past influence regime. In the "Independent cycles" regime the boom state is very persistent contrary to the disruption state (with

the probability to stay in the state above 0.93 and 0.25 (under any past conditions) correspondingly). In the "Interdependent cycles" regime, the past state of the business cycle plays a decisive role. When the business cycle is in expansion, the probability to stay in financial boom is high and is close to the corresponding one in the "Independent cycles" regime. However, a recession in the business cycle decreases this probability dramatically: from 0.92 to 0.21 (for the probability of staying in financial boom), and from 0.87 down to 0.16 (for the probability to switch from financial disruption to boom).

The findings above indicate that the downturns in the financial cycles are temporary by their nature, as the financial market in the developed economies is flexible enough to absorb the shocks relatively quickly. For this reason, on Figure 4 the episodes of financial instability are presented just as spikes in the smoothed probability during the "Independent cycles" regime. To the contrary, when the financial cycle enters into interaction with the business cycle, the downturn state becomes much more persistent.

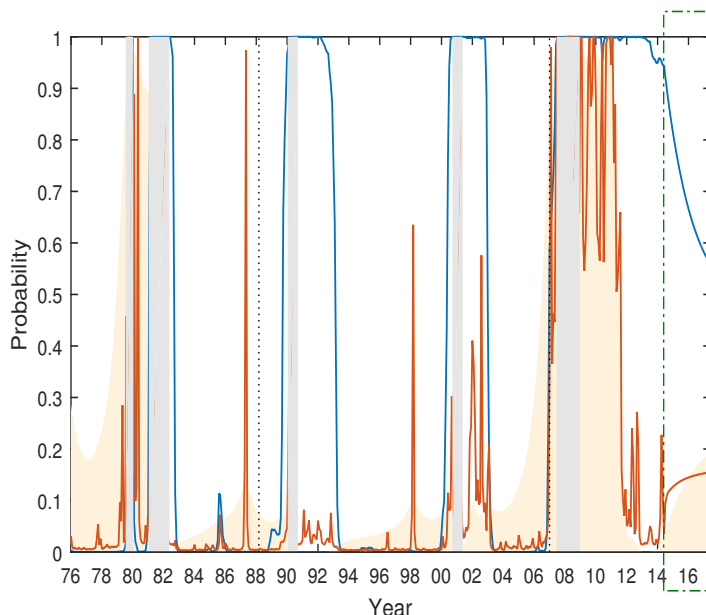
What are the projections of the model for the future? Figure 6 gathers the 36 months ahead forecasts of the smoothed probability of recession (blue line), financial downturn (red line) and "Interdependent cycle" regime (yellow area). The model thus predicts that by 2018 the period of low growth rates in the real sector will be over, financial sector will be stable and the "Independent cycles" regime will dominate.

What sort of implication can this have for policy-makers? Even though at this moment theoretical models do not have an unequivocal answer to the question on linkages between financial and business cycles, the impact of certain instruments of monetary, macro- and microprudential policy, and so do not provide an optimal policy rule, the knowledge of the current state of both cycles as well as the level of their interaction can be helpful for policy adjustments. For example, when the cycles are independent, the spillover effects documented by Zdzienicka et al. (2015), such as the impact of monetary policy on the stability of the financial sector, can be quite limited, which may allow to run more aggressive policies to stimulate either of the cycles. Similarly, the trade-off between financial stability and economic prosperity in the environment of the low interest rates discussed by Coimbra and Rey (2015) and Heider et al. (2017) can be less pronounced. On the contrary, when the cycles are interdependent, the regulator should be prepared to implement large interventions since the recessions appear to be longer and more severe (Claessens et al. (2012)), and the financial sector needs increased support to stabilize.

Given the aggravated character of recessions during the periods of high interaction between the cycles, the set of monetary, fiscal and macroprudential measures should be directed towards the reduction of the procyclicality of the financial sector. Cerutti et al. (2015) find that macroprudential policy is an effective instrument for this purpose and works better during the bust phase of the financial cycle and are more efficient in emergent economies rather than advanced ones. Blanchard et al. (2009) suggest that the monetary policy should take into account the assets price movements, too, however, by now is it not clear how to operationalize this. Another solution for mitigating credit cycles and dramatically reducing the level of government and public debt, proposed by Fischer (1936) and recently rediscovered by Kumhof and Benes (2012), is the radical idea of separation of monetary and credit functions of the banking system, also known as Chicago plan.

Whatever the relevant policy is, given the usual lag between the moment when a problem in an economy is recognized and the moment when the undertaken policy starts giving the first effects, timing is very important. In this concern, the probabilities of the influence regimes and states of individual cycles are of a great use since they provide an operative measure of the current state of the economy and future tendencies, and can be updated as soon as new information arrives. Moreover, once the causality direction is identified for each of the influence regimes, the leading cycle can serve as an early-warning indicator.

Figure 6: 36 months ahead forecast of smoothed probability



Note: Blue line and red line correspond to the smoothed probability of recession in the business cycle and the downturn state in the financial cycle, respectively. Yellow area marks the smoothed probability of being in the "Interdependent cycle" regime. Grey shaded areas correspond to NBER recessions.

8 Conclusion

Previous findings in the literature on business and financial cycles has shown that the cycles evolve, and so does their interaction. In this paper we suggest a flexible econometric framework, the Dynamical Influence Markov Switching Dynamic Factor model (DI-MS-DFM), which allows to capture the changes in this interaction. Contrary to the existing models of the joint dynamics of business and financial cycles, we allow the interaction to be intrinsically dynamical, which implies that there is no need to search for an exogenous variable which could serve as a proxy for the process governing the interaction. Based on the mix of the Dynamical influence model from computer science and the classical Markov-Switching model, the DI-MS-DFM produces a wide range of statistical tools which can be very useful to desing a relevant policy mix for mitigating the effects of downturns in both cycles as well as for reduction of the procyclicality of the financial cycle. More precisely, besides

the characteristics of the individual cycles, the model allows to characterize the existing influence regimes in terms of leading-lagging relation between the two cycles as well as the degree of their interdependence, and to provide a probabilistic indicator of being in a particular regime of interaction at each point of time. Forecasts of the future states and future influence regimes can also be calculated.

We applied the model to the macroeconomic and financial series of the US for the period 1976m06 to 2014m12. The obtained estimates complement the findings in the previous literature. The model clearly identifies two distinct influence regimes, “Independent cycles” and “Interdependent cycles”, the second being active during the double-dip recession in July 1979-November 1981 and the Great Recession in January 2007-January 2012. The periods of higher interaction are well detected, although the results may be even more telling if one allows for three influence regimes.

As any other model, the DI-MS-DFM has several limitations. First, it requires the time span to be long enough in order to make sure that all the regimes of all chains are observed at least once. This implies that the more flexibility one introduces into the model (by increasing the number of chains, individual states, influence regimes), the more data is needed, which can obviously be a problem, especially for the analysis of developing countries. Secondly, the simulations show that the influence regimes are well identified only when they are different enough, however, this does not deteriorate the quality of the estimates the individual behavior of each cycle. Nevertheless, this issue can be solved by a more accurate selection of initial values for the optimization process.

The model can be extended in several ways. The most straightforward direction is the generalization of the model for a larger number of influence regimes and states of each of the cycles. Secondly, it seems appealing to engage more chains into the dynamical interaction, for example, by letting the credit and equity part of the financial market each follow their individual chain. Another interesting application of this kind concerns the interaction of business and financial cycles of several countries (for example, the core countries of the Euro area) which would allow to assess the contribution of each country to the cross-country systemic risk, identify the clusters of interdependence, and construct an indicator of systemic risk in the region. Third, we can let the cycles to interact not only on the level of underlying latent finite-state processes, but also on the level of observations by allowing for cross-correlation in the error terms of the DGPs of the cycles and/or by introducing a VAR structure in equations (1) and (2), which might improve the forecasting ability of the model. In this case the identification issues concerning the distinction between the observation-level and chain-level interaction should be resolved, as well as the causality definition is to be reconsidered.

The Dynamic Influence Markov-Switching Dynamic Factor Model, to our knowledge, is the first instrument for objective and reproducible empirical identification of the regimes of interaction between the real and the financial sectors. Even in its basic form, it appears to produce meaningful inference on individual features of cycles as well as the dynamics of their interaction. All this information can be useful for policy-makers as it enables to adjust the fiscal, monetary and macroprudential policy according to the current influence regime.

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A Appendix

Table A.1: List of financial variables used for describing the financial cycle in the US

Series	Source
Series from Guidolin et al. (2013) database	
Monthly SP500 portfolio returns	FREDII
3mtb. monthly rate	FREDII
10-Year Treasury Constant Maturity Rate	FREDII
2-Year Treasury Constant Maturity Rate	FREDII
Moody's Seasoned Baa Corporate Bond Yield	FREDII
Composite NAREIT	NAREIT
Equity REITs	NAREIT
Mortgage REITs	NAREIT
Excess return on a value-weighted market	FREDII
S&P 500 dividend yield — (12 month dividend per share)/price.	FREDII
Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity	FREDII
10-Year Treasury Constant Maturity Minus 3-Month Treasury Constant Maturity	FREDII
10-Year Treasury Constant Maturity Minus 2-Year Treasury Constant Maturity Rate	FREDII
Unexpected inflation rate	FREDII
Industrial production index	FREDII
Real personal consumption expenditures	FREDII
Other series	
3-month Tbill rate of return minus CPI	FREDII
SP500 PE ratio	FREDII
Federal funds effective rate	FREDII
Monetary Base; Total	FREDII
Total Reserve Balances Maintained with Federal Reserve Banks	FREDII
M1 Money Stock	FREDII
M2 Money Stock	FREDII
Federal Debt: Total Public Debt as Percent of Gross Domestic Product	FREDII
Median Sales Price for New Houses Sold in the United States	FREDII
Total Assets. All Commercial Banks	FREDII
Commercial and Industrial Loans. All Commercial Banks	FREDII
Loans and Leases in Bank Credit. All Commercial Banks	FREDII
Total Savings Deposits at all Depository Institutions	FREDII
Loans to deposits ratio	FREDII
Consumer Credit Outstanding (Levels)	FREDII