# *Intellectual Property Law and Public Sponsorship of Research in a Schumpeterian Growth Model with Fundamental and Applied R&D*

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**Abstract**

We develop a Schumpeterian growth model *à la* Aghion & Howitt (1992), in which we consider simultaneously fundamental and applied research and development (R&D) activities. Then, knowledge accumulation stems from of a series of inventions of new products and from a succession of innovations of existing products. Within this framework, we investigates how intellectual property (IP) law and public sponsorship of R&D can provide appropriate incentives both to *inventors*, who create new products, and to *innovators*, who improve these products.

In order to propose a framework in which Schumpeter's creative destruction mechanism does not deter investments in fundamental R&D, we formalize - within a Schumpeterian decentralized economy - the existence of a design of IP law (like patent law) that enables any inventor to receive a share of the profits realized by each of the following innovator on the sale of the generations of products his invention has made possible.

We show that providing optimal incentives to fundamental R&D requires, not only a design of IP law that ensures a sufficient positive transfer from innovators to inventors, but also public policies in form of subsidies supporting both fundamental and applied R&D activities.

**Keywords:** Endogenous Growth - Schumpeterian Theory - Cumulative Innovations - Applied R&D - Fundamental R&D - R&D Incentives - Intellectual Property Law - Patent - Public Sponsorship of Research

**JEL Classification:** O30 - O31 - O40 - O41

# 1 Introduction

Innovation-based endogenous growth theory underlines the key part played by knowledge accumulation, which results from research and development (R&D) activities. This process has been formalized in two seminal ranges of models. Romer (1990) developed a model based on horizontal product differentiation in which knowledge accumulation is driven by a diversification of the variety of products; long term growth stems from horizontal knowledge accumulation. Grossman & Helpman (1991) and Aghion & Howitt (1992) considered quality-ladders growth models, in which knowledge

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accumulation is driven by quality improving innovations based on stochastic and sequential R&D races; long term growth results from vertical knowledge accumulation.

One of the central issue tackled in the innovation-related literature is the one of the incentives provided to R&D activity to foster the production of knowledge. It has been addressed both by the growth literature (e.g. O'Donoghue & Zweimüller 2004; Acemoglu & Akcigit 2012; Chu, Cozzi & Galli 2012) and by the industrial organization literature (e.g. Scotchmer 1991; Green & Scotchmer 1995; Bessen & Maskin 2009). Focusing on the design of intellectual property rights (IPRs) - in particular of patent law - that would provide appropriate R&D incentives, the seminal papers by Scotchmer (1991) and by Green & Scotchmer (1995) have underlined the difficulties to provide such incentives triggered by the cumulative aspect inherent in the process of knowledge creation. While Scotchmer (1991) investigates how patent protection and cooperative agreements among firms allows to protect incentives for cumulative R&D, Green & Scotchmer (1995) focuses on the division of profits in presence of sequential innovation in a partial equilibrium model considering two-stages innovation. They states that "knowledge and technical progress are cumulative in the sense that products are often the result of several steps of invention, modification, and improvement. Indeed, the 'development' aspect of 'research and development' can be as commercially important as the 'research'. But when innovation occurs in two stages, the first innovator may have insufficient incentives to invest".

Our purpose in this paper is to pursue, *within a dynamic general equilibrium framework of an endogenous growth model*, the study of the problem of R&D incentives raised by the presence of cumulative innovations. More specifically, we investigate the issue of the division of profits between *inventors* and *innovators* in presence of sequential innovations subsequent to an initial discovery made via fundamental research. An important point developed in this paper relates to the distinction between *"inventions"* and *"innovations"*. Even though they both involve the occurrence of knowledge, it is commonly agreed that there is a fine line of difference between the two. While an invention generally refers to the creation of a new product or to the introduction of a process for the first time, an innovation is the improvement of a product or process having already been invented. In accordance with this, we present a Schumpeterian growth model considering simultaneously *fundamental R&D activity*, which generates *inventions* (*i.e.* underlying ideas that will ultimately lead to new products), and *applied R&D activity*, which produces *innovations* (*i.e.* which realizes the potential created by the fundamental R&D, successively increasing the quality of an existing product).

In order to account for these features, we merge the endogenous growth frameworks of horizontal knowledge accumulation (e.g. Romer 1990; Jones 1995) and of vertical knowledge accumulation (e.g. Grossman & Helpman 1991; Aghion & Howitt 1992, 1998; Segerstrom 1998; Young 1998). More precisely, we use the specification of Romer (1990) to consider fundamental R&D activity and the one of Aghion & Howitt (1992, 1998) to consider applied R&D activity. Hence, we consider two possible engines of growth insofar as knowledge accumulation stems both from of a series of inventions of new products and from a succession of innovations of existing products. This framework enables us to study the tradeoff faced by society between engaging in fundamental and in applied R&D activities.

Considering simultaneously inventions and the resulting innovations in a Schumpeterian growth model gives rise to an important matter: how to propose a decentralized economy in which the mechanism that Schumpeter named "creative destruction" does not preclude investments in fundamental research? The issue if the following.

The Schumpeterian equilibrium introduced initially by Aghion & Howitt (1992) involves a fundamental market failure: knowledge is not priced. In order to fund indirectly knowledge creation, one considers assumptions inspired by Schumpeter's creative destruction mechanism. Once an innovation occurs, the resulting knowledge is embodied in an intermediate good; then, the innovator is granted IPRs, like a patent, and monopolizes the production and sale of this private good until replaced by the next innovator. IPRs on rival goods are introduced as a means to provide incentives to invest in R&D. It is well known that, in such a decentralized economy, innovators may have insufficient incentives to invest in R&D. Considering that inventors are at the origin of any sequence of innovation makes the issue even more intricate insofar as it is likely that, without public intervention, inventors have too little incentives to invest in the first place.

In this paper, we adapt the seminal decentralized economy introduced in Aghion & Howitt (1992) to study these issues. In particular, we investigate how to ensure that there are sufficient incentives provided both to fundamental and to applied R&D activities, given the fact that in each sector, these two types of R&D activities are funded via monopoly profits on the same intermediate good. For that purpose, we incorporate in a dynamic general equilibrium model arguments put forward by Scotchmer (1991) along which "in this view of how incentives to innovate are protected, a key role of patent protection is that it sets bargaining positions for the prior agreements and licenses that will form, and therefore determines the division of profit in these contracts." This leads us to construct a Schumpeterian decentralized economy which considers the existence of a design of intellectual property (IP) law (for studying such issues, one generally has in mind patent law) that enables any inventor to receive a share of the profit realized by each of the incumbent monopoly on the sale of the subsequent generations of products his invention has made possible.

By implementing the first-best social optimum in this decentralized economy, we first show that optimality requires that the design of IP law enables inventors to get a strictly positive share of the profits made by its following innovators. Furthermore, we reveal that, even though a design of IP law favorable to inventors is necessary, it is not sufficient to provide optimal incentives to fundamental R&D: in addition to an IP law enabling some division of profit, fundamental R&D needs to be subsidized. This result echoes to arguments put forward, for instance, by Scotchmer (1991). Finally, we show that, in order to compensate for a larger share of profits transferred to inventors, innovators must be given stronger incentives; we underline the fact that public sponsorship of applied R&D is necessary to mitigate the disincentive effect of pro-inventor IP law.

The paper is organized as follows. In Section 2, we present a innovation-based Schumpeterian growth model which explicitly considers fundamental and applied R&D activities, and we provide the first-best social optimum. In Section 3, in order to account both for "creative destruction" and for incentives provided to fundamental R&D activity, we construct a Schumpeterian equilibrium in which the monopoly profit realized in each sector by the incumbent innovator is shared with the inventor at the origin of the sector. Then, we study the agents' behaviors and characterize the equilibrium. In Section 4, we implement the first-best computed in Section 2 within the decentralized economy analyzed in Section 3. This enables us to investigate how IP law design and public sponsorship of applied and fundamental R&D complete each other in providing optimal incentives to innovators and inventors. We conclude in Section 5. All computations are provided in the Appendix, Section 6.

# 2 An Schumpeterian Growth Model with Fundamental and Applied R&D Activities

This section develops an innovation-based Schumpeterian growth model considering simultaneously fundamental and applied R&D activities. In that respect, we merge the seminal endogenous growth frameworks of horizontal knowledge accumulation (as developed by Romer 1990) and of vertical knowledge accumulation (as developed by Aghion & Howitt 1992 and 1998). We present the technologies and the preferences in 2.1 and 2.2; then, we compute the first-best social optimum in 2.3.

## 2.1 Knowledge Accumulation: Inventions & Innovations

Let us start by presenting the key component of any innovation-based endogenous growth model, namely the process of knowledge accumulation. As explained previously, we aim to account for the fact that knowledge creation relies both on fundamental and on applied R&D activities.

In chapter 8 of *Innovations and Incentives* (2005), Scotchmer and Maurer distinguish between "fundamental (or basic) R&D" and "applied (or industrial) R&D". In particular, they argue that applied R&D depends heavily on the underlying basic knowledge. Furthermore, it is commonly agreed to define an *"invention"* as the creation of a new product or new process, and to refer to *"innovations"* when one improves on or makes a significant contribution to an existing product or process. Accordingly, we formalize, within an endogenous growth model, the idea along which innovations build upon inventions: we consider that inventions are the output of fundamental R&D activity while innovations are the output of applied R&D activity. In that respect, we relate to the hereinbefore mentioned seminal endogenous growth models. In models *à la* Romer (1990), knowledge accumulation goes along with the creation of new product varieties. Accordingly, we use Romer's formalization of horizontal knowledge accumulation to consider that *fundamental R&D activity* creates *inventions* (*i.e.* new fundamental knowledge). Besides, in models *à la* Aghion & Howitt (1992 or 1998), knowledge creation results in improvements of the quality of existing products. Therefore, we use their formalization of vertical knowledge accumulation to consider that *applied R&D activity* produces *innovations* (*i.e.* new applied knowledge).

Henceforth, knowledge accumulates both horizontally and vertically. As generally accepted in innovation-based endogenous growth theory, we assume that knowledge (whether fundamental or applied) is produced using two types of inputs: a rival good (labor, but one could also introduce physical capital for instance) and a non rival one (a stock of knowledge previously accumulated).[1]

There is a continuum $[0, Q_t]$ of intermediate sectors. At each date $t$, each sector $i$, $i \in [0, Q_t]$, is characterized by a stock of *applied knowledge* $A_{it}$ and by an intermediate good $i$, produced in quantity $x_{it}$, which embodies this stock of knowledge. Accordingly - assuming that knowledge is homogenous - the *whole stock of knowledge in the economy* at each date $t$, denoted $\mathcal{A}_t$, is define as follows:

$$\mathcal{A}_t = \int_0^{Q_t} A_{it} di \tag{1}$$

We assume that all sectors have an identical initial level of knowledge: $A_{i0} = A_0, \forall i \in [0, Q_t]$.[2] Each sector $i$ has its own R&D activity which produces applied knowledge; in other words, the R&D activity of each sector is dedicated to the creation of *innovations* that improve the quality of the intermediate good produced in this sector. The formalization used for the mechanism at the source of the creation of applied knowledge is derived from standard Schumpeterian growth theory; it relies on two core assumptions. First, the innovation process is uncertain:

**Assumption 1.** *If $l_{it}^A$ is the amount of labor devoted to applied R&D at date $t$ in intermediate sector $i, i \in [0, Q_t]$, to move on to the next quality of intermediate good $i$, innovations occur randomly with a Poisson arrival rate $\lambda l_{it}^A$, $\lambda > 0$.*[3]

Second, we consider that applied R&D activities all draw from a pool of shared technological knowledge (*i.e.* we consider the presence of inter-sectorial knowledge spillovers). For simplicity,

[1] See for instance in Romer (1990), Aghion & Howitt (1992, 1998, 2009), Jones (2005) or Acemoglu (2002, 2009).

[2] Considering some symmetry across sectors is standard in endogenous growth theory. We provide more details on this type of assumption below in 3.3; see the comments of Lemma 3.

[3] It is also possible to consider a more general Poisson arrival rate of innovations in sector $i$: $\lambda(l_{it}^A)$, $\lambda(.)' > 0$. In order to simplify computations, we consider linearity.

we assume that the shared pool of knowledge is $\mathcal{A}_t$, the whole stock of knowledge available in the economy.[4] Formally, we make the following assumption.

**Assumption 2.** *For any sector $i, i \in [0, Q_t]$, if an innovation occurs at date $t$, the increase in knowledge (i.e. the quality improvement of the intermediate good) is $\Delta A_{it} = \sigma \mathcal{A}_t$, $\sigma > 0$.*[5]

From Assumptions 1 and 2, one derives the following lemma in which we provide the production function of innovations (*i.e.* of applied knowledge) in any sector $i$.

**Lemma 1.** *Under Assumptions 1 and 2, the expected applied knowledge in any sector $i$, $i \in [0, Q_t]$, is a differentiable function of time. The law of accumulation of applied knowledge inherent in sector $i$ is*

$$\dot{A}_{it} = \lambda \sigma l_{it}^{A} \mathcal{A}_t, i \in [0, Q_t] \quad (2)$$

**Proof.** See Appendix 6.1.

The law of accumulation of applied knowledge obtained in Lemma 1 is endogenously derived from assumptions made in a stochastic quality ladders model similar, for instance, to the ones of Aghion & Howitt (1998 - Ch. 12), Howitt (1999), or Aghion & Howitt (2009 - Ch. 4). This law of motion exhibits the fact that, in any intermediate sector, applied R&D activity uses the whole knowledge accumulated so far in the economy, capturing the mass of the ideas created in all other sectors.[6] One considers here a formalization of the creation of applied knowledge which is in line with standard endogenous growth theory insofar as it exhibits spillovers in R&D activities.

Regarding the creation of *fundamental knowledge*, we build upon the specifications developed in Romer (1990). Hereinbefore, we have introduced the variable $Q_t$, which stands for the measure of the continuum of intermediate sectors, that is for the "number" of intermediate goods. In Romer's seminal model, this variable is defined as "the aggregate stock of designs". As mentioned above, we want to introduce the fact that applied R&D builds upon fundamental knowledge. In other words, we aim to consider that each invention is the cornerstone for a succession of innovations in a particular sector. In that respect, we propose a complementary interpretation of $Q_t$: we consider that it is also the measure of the set of ideas that have been generated through fundamental R&D activity.[7] Basically, we interpret $Q_t$ as the *aggregate stock of fundamental knowledge* in the economy, and we assume that fundamental knowledge (*i.e.* inventions) is produced along with

$$\dot{Q}_t = \delta L_t^Q Q_t, \delta > 0 \quad (3)$$

where $L_t^Q$ is the amount of labor used in fundamental R&D activity. We normalize the initial stock of fundamental knowledge, $Q_0$, to one. Accordingly, the initial stock of the whole knowledge in the

[4] The significance of the interactions between sectors has universally been underlined. It is a standard assumption in endogenous growth theory (e.g. Romer 1990; Aghion & Howitt 1992, 1998 - Ch. 3, 2009 - Ch. 4; Acemoglu 2002). Besides, several empirical studies stress that R&D performed in one sector may produce positive spillovers effects in other sectors (e.g. Griliches 1992, 1995; Hall, Mairesse & Mohnen 2010). One could consider a more general framework wherein the R&D activity of each sector $i$ draws from a specific pool of knowledge which consists in a subset of the whole knowledge in the economy. This is done for instance in Gray & Grimaud (2016), in which a process of knowledge diffusion is explicitly introduced to take into account the fact that, as stated by Hall, Mairesse & Mohnen (2010), "spillovers are all the more likely and significant as the sender and the receiver are closely related". In this paper we abstract away from these issues by considering that applied knowledge diffuses to the whole economy (*i.e.* global inter-sectorial spillovers).

[5] Similarly to Assumption 1, it is also possible to consider a more general function for the increases in knowledge: $\sigma(\mathcal{A}_t)$, $\sigma(.)' > 0$. Again, in order to simplify computations, we consider linearity.

[6] Introducing explicitly inter-sectorial knowledge diffusion (see footnote 4) would enable to obtain a more general law of knowledge accumulation: knowledge would be created using only part of the mass of ideas developed in other sectors.

[7] A similar interpretation can be found in Gersbach, Sorger & Amon (2009), or in Chu, Cozzi, & Galli (2012).

economy is $\mathcal{A}_0 = A_0$. The law of motion (3) is in fact adapted from the one introduced in Romer (1990). It formalizes the fact that the creation of new fundamental knowledge stems from the use of previously created fundamental knowledge, reflecting the existence of spillovers in fundamental R&D activity and echoing to the idea of "standing on the shoulders of giants".[8]

Differentiating (1) with respect to time gives the law of accumulation of the whole knowledge in the economy:

$$\dot{\mathcal{A}}_t = \int_0^{Q_t} \dot{A}_{it} di + \dot{Q}_t A_{Q_t t} \tag{4}$$

where $\dot{A}_{it}$ and $\dot{Q}_t$ are given by (2) and (3), respectively. From (4), one obtains the growth rate of the whole knowledge in the economy, which is provided in Lemma 2. In the remaining of the paper, we denote by $g_{z_t}$ the rate of growth, $\dot{z}_t/z_t$, of any variable $z_t$.

**Lemma 2.** *The growth rate of the whole knowledge in the economy is*

$$g_{\mathcal{A}_t} = \frac{\dot{\mathcal{A}}_t}{\mathcal{A}_t} = \lambda\sigma L_t^A + \delta L_t^Q \frac{Q_t A_{Q_t t}}{\mathcal{A}_t} \tag{5}$$

**Proof.** Plugging (2) and (3) in (4), one obtains $\dot{\mathcal{A}}_t = \int_0^{Q_t} \lambda\sigma l_{it}^A \mathcal{A}_t di + \delta L_t^Q Q_t A_{Q_t} = \lambda\sigma\mathcal{A}_t \int_0^{Q_t} l_{it}^A di + \delta L_t^Q Q_t A_{Q_t t}$. Hence, one has $g_{\mathcal{A}_t} = \frac{\dot{\mathcal{A}}_t}{\mathcal{A}_t} = \lambda\sigma L_t^A + \delta L_t^Q \frac{A_{Q_t Q_t}}{\mathcal{A}_t}$. $\square$

As anticipated, the growth rate of the whole knowledge in the economy depends both on the quantity of labor devoted to applied R&D and on the one devoted to fundamental R&D. This result follows from the fact that, as shown by the law of motion derived in (4), the whole knowledge in the economy stems from the accumulation and the combination of applied and fundamental knowledge. This formalizes accurately the statement made by Green & Scotchmer (1995) along which "knowledge and technical progress are cumulative in the sense that products are often the result of several steps of invention, modification, and improvement".

## 2.2 Other Technologies and Representative Household's Preferences

Once an invention (*i.e.* new fundamental knowledge) is created, it gives rise to a new line of intermediate good (*i.e.* to the occurrence of a new sector). As mentioned above, the initial quality of this new intermediate good, or equivalently the initial level of applied knowledge in this new sector is $A_0$. Then, the quality of this intermediate good increases as applied knowledge accumulates along with (2). Once invented, each intermediate good $i$ is produced according to

$$x_{it} = \frac{y_{it}}{A_{it}}, i \in [0, Q_t] \tag{6}$$

where $y_{it}$ is the quantity of final good used to produce $x_{it}$ units of intermediate good $i$. This usual technology is often used in Schumpeterian growth theory. It illustrates the increasing complexity in

[8]Note that we aim to merge two seminal frameworks of endogenous growth (Romer 1990 and Aghion & Howitt 1992) while keeping as close as possible to the original models. Remaining close to the formalization of Romer (1990) involves making two implicit assumptions on the process of creation of fundamental knowledge. First, no uncertainty is introduced in this process. Second, fundamental knowledge production does not depend on $\mathcal{A}_t$, the whole knowledge in the economy, but only on $Q_t$, the stock of fundamental knowledge accumulated so far. This means that the production of fundamental knowledge is not enhanced by the use of the applied knowledge. These two assumptions are in line with Romer's equation 3 (p. S83). One could consider a more general law of accumulation of fundamental knowledge, in which the invention process would be stochastic, and/or in which the it would rely on the use of $\mathcal{A}_t$ (*i.e.* including applied knowledge). The study of these extensions are left for further research.

the production of intermediate goods: as the quality of intermediate good $i$ (measured by the stock of knowledge $A_{it}$) increases, the production of this intermediate requires more input (final good here). In other words, the larger the stock of applied knowledge in the sector, the more costly the production of the intermediate embodying this knowledge.

As usual in endogenous growth models, the production of the final good ($Y_t$) requires the use of labor ($L_t^Y$) as well as the use of all available intermediates ($x_{it}, i \in [0, Q_t]$), each of which is associated with its own level of knowledge $A_{it}$. The final good production technology is

$$Y_t = (L_t^Y)^{1-\alpha} \int_0^{Q_t} A_{it}(x_{it})^{\alpha} di, 0 < \alpha < 1 \tag{7}$$

This final good has two competing uses. Besides being used in the production of intermediates (as seen in (6)), it is consumed in quantity $c_t$ by the representative household whose intertemporal preferences are given by

$$\mathcal{U} = \int_0^{\infty} u(c_t)e^{-\rho t}dt, \tag{8}$$

where $\rho$ is the subjective discount rate and $u(c_t)$ is the individual instantaneous utility at date $t$, which is given by $u(c_t) = \ln(c_t)$. At each date $t$, each of the $L$ identical households is endowed with one unit of labor that is supplied inelastically.[9] The total quantity of labor $L$ is used in quantity $L_t^Y$ in the final good production, in quantity $l_{it}^A$ in the applied R&D activity of each sector $i, i \in [0, Q_t]$, and in quantity $L_t^Q$ in fundamental R&D activity. The resulting constraint on the labor market is

$$L = L_t^Y + L_t^A + L_t^Q, \tag{9}$$

where $L_t^A = \int_0^{Q_t} l_{it}^A di$ is the total quantity of labor devoted to applied R&D in the economy. Finally, the constraint on the final good market is

$$Y_t = Lc_t + \int_0^{Q_t} y_{it} di \tag{10}$$

This model relies on assumptions that are standard in innovation-based endogenous growth theory; and it is a generalization of the seminal models of this literature. In particular, the law of motion (4) considers simultaneously horizontal and vertical knowledge accumulation; and it encompasses the laws of knowledge accumulation introduced in models with expanding product-variety and in quality ladders endogenous growth models.[10] Within this framework, growth is driven by the occurrence of inventions at the source of new products and of innovations successively increasing the quality of these products.

[9] The key results of the paper are robust if one considers a more general C.E.S. instantaneous utility function of parameter $\varepsilon$, $u(c_t) = c_t^{1-\varepsilon}/(1-\varepsilon)$, and/or constant population growth.

[10] These two frameworks can be obtained as a limit cases of our double differentiation model. First, a product-variety endogenous growth model *à la* Romer can be obtained by assuming that $\lambda$ and/or $\sigma$ equal to zero (*i.e.* by not considering the possibility to accumulate knowledge vertically). Then, once an intermediate good has been invented, its quality is fixed; normalizing it to one, one has $A_{it} = 1, \forall i \in [0, Q_t], \forall t$. Hence, from (1), one gets $\mathcal{A}_t = Q_t$, which accumulates according to (3). As there is no applied R&D, the constraint (9) writes $L_t = L_t^Y + L_t^Q$. The intermediate good production function (6), the final good production function (7), and the constraint on the final good market (10) all boil down to the usual ones. Second, a quality ladders endogenous growth model can be obtained by assuming that $\delta$ equals to zero: there is no possibility to accumulate knowledge horizontally. Accordingly, the set of intermediate goods $Q_t$ is fixed; normalizing it to one, one gets $\mathcal{A}_t = \int_0^1 A_{it} di$, where $A_{it}$ accumulates according to (2). Thus, the law of motion (4) boils down to $\dot{\mathcal{A}}_t = \int_0^{Q_t} \lambda\sigma l_{it}^A \mathcal{A}_t di = \lambda\sigma L_t^A \mathcal{A}_t$. As there is no fundamental R&D, the constraint (9) writes $L_t = L_t^Y + L_t^A$. The intermediate good production function (6) is unchanged; the final good production function (7), and the constraint on the final good market (10) both boil down to the usual ones.

## 2.3 First-best Social Optimum

The first-best social optimum is the solution of the maximization of the representative household's discounted utility (8) subject to (1), (2), (3), (6), (7), (9) and (10). Proposition 1 gives the optimal quantities and growth rates (we use the superscript "$^o$" for "social optimum").

**Proposition 1.** *In the first-best social optimum,*

- *the partition of labor is*

$$L_t^{Yo} = L^{Yo} = \frac{\delta L}{\lambda\sigma},\ L_t^{Ao} = Q_t^o l_t^{Ao} = L^{Ao} = \frac{\rho}{\delta} - \frac{\delta L}{\lambda\sigma},\ L_t^{Qo} = L^{Qo} = L - \frac{\rho}{\delta}$$

- *The quantity of each intermediate good $i$ is $x_{it}^o = x^o = \alpha^{\frac{1}{1-\alpha}} L^{Yo}, \forall i \in [0, Q_t^o]$*
- *The growth rate of applied knowledge in each sector $i$ is*

$$g_{A_{it}}^o = g_{A_t}^o = \lambda\sigma L^{Ao} = \frac{\lambda\sigma\rho}{\delta} - \delta L, \forall i \in [0, Q_t^o]$$

- *The growth rate of fundamental knowledge is $g_{Q_t}^o = \delta L^{Qo} = \delta L - \rho$*
- *The growth rate of the economy is*

$$g_{c_t}^o = g_{Y_t}^o = g_{\mathcal{A}_t}^o = g_{A_t}^o + g_{Q_t}^o = g^o = \lambda\sigma L^{Ao} + \delta L^{Qo} = \frac{\lambda\sigma\rho}{\delta} - \rho$$

**Proof.** See Appendix 6.2.

The optimal growth rate of the economy does not depend on the size of the population, unlike early endogenous growth models which exhibit this non desirable scale effects property (e.g. Romer 1990, Grossman & Helpman 1991, or Aghion & Howitt 1992). In that sense, our model is in line with "fully endogenous growth models", which eliminate scale effects by allowing for expansion in the number of sectors (e.g. Aghion & Howitt 1998 - Ch. 12, Dinopoulos & Thompson 1998 Peretto 1998, Young 1998, Howitt 1999, Peretto 1999, or Aghion & Howitt 2009 - Ch. 4). For more details on the theoretical and empirical issues involved by the scale effects property and on the various methods introduced to construct scale-invariant models, see for instance Jones (1999), Laincz & Peretto (2006), or Dinopoulos & Sener (2007).

Besides, for both types of R&D activities to exist at the optimum, the parameters of the model must satisfy $L^{Ao} > 0$ and $L^{Qo} > 0$, that is $\frac{\rho}{\delta} - \frac{\delta L}{\lambda\sigma} > 0$ and $L - \frac{\rho}{\delta} > 0$, respectively. These conditions are summarized in the following assumption.

**Assumption 3.** *The parameters of the model satisfy the following conditions:*

$$L > \frac{\rho}{\delta} > \frac{\delta L}{\lambda\sigma} > 0$$

From Assumption 3, one gets $\lambda\sigma > \delta$. This condition guarantees that the optimal growth rate of the economy, $g^o$, is positive. The conditions of Assumption 3 will turn out to be important when we investigate how the first-best can be implemented within the decentralized economy (see Section 4).

By considering simultaneously inventions and their following innovations, our double differentiation Schumpeterian growth model exhibits the tradeoff between devoting resources to fundamental

R&D (that generates the ideas embodied in new products) and applied R&D (that successively upgrade the quality of the existing products by building upon fundamental R&D). This leads us to wonder whether this optimal allocation can be implemented in a Schumpeterian equilibrium. The problem is the following. Since a Schumpeterian decentralized economy relies on that each sector is monopolized by the latest innovator, R&D incentives provided to the inventor at the origin of a sector are likely to be insufficient. Sections 3 and 4 address these issues.

# 3 Schumpeterian Equilibrium and Fundamental R&D Incentives

Let us now investigate whether (and if so, how) it is possible to adapt the Schumpeterian decentralized economy introduced in Aghion & Howitt (1992) so that fundamental R&D activity has sufficient incentives to invest in the creation of inventions; this in spite of the fact that the presence of the "creative destruction" mechanism is likely to preclude such investments.

In 3.1, we construct a considered decentralized economy, adding to the standard Schumpeterian equilibrium *à la* Aghion & Howitt a new feature that enables us to introduce incentives to fundamental R&D. In each sector, we formalize profit sharing between the inventor at the origin of the sector and the following innovators. In 3.2, we study the agents' behaviors. Then, in 3.3, we compute the resulting equilibrium; in particular, we characterize the equilibrium labor partition between final good production, applied R&D activity and fundamental R&D activity.

## 3.1 Decentralized Economy & Division of Profits between Inventors and Innovators

We consider a decentralized economy which is a direct extension of the analysis conducted by Romer (1990) and by Aghion & Howitt (1992). In order to deal with the non-rivalry property of knowledge, these seminal papers focus on a decentralized equilibrium with incomplete markets and imperfect competition. Indeed, in this type of equilibria, a market and a price are specified for intermediate goods that incorporate knowledge, but not for knowledge itself; positive monopoly profits on the sale of intermediate goods - which result from IPRs, like patents, granted to innovators - are used as incentives to invest in the creation of knowledge. This framework, which has become the standard in the endogenous growth literature, provides a realistic decentralized economy in which R&D activity is privately and indirectly funded: the value of innovations stems from the stream of monopoly profits on the sale of intermediate goods embodying knowledge. The key difference between these two framework lies in that the equilibrium *à la* Aghion & Howitt considers Schumpeter's "creative destruction" mechanism.

The main motivation of our study consist in determining how a Schumpeterian decentralized economy *à la* Aghion & Howitt can be adapted to take into account that fundamental R&D activity needs to be given sufficient incentives. For that purpose, we construct a Schumpeterian equilibrium in which each inventor receives a share of the profit realized by the each of the incumbent innovator on the sale of the improved product his invention has made possible. As seen in Section 4 below, this framework will enable us to determine what should be the optimal incentives provided to any inventor and to his following innovators.

The price of the final good is normalized to one; the wage, the interest rate and the price of intermediate good $i$ are denoted by $w_t$, $r_t$ and $p_{it}$ ($i \in [0, Q_t]$), respectively. As usual in endogenous growth theory, the final good market, the labor market and the financial market are perfectly competitive; and there is imperfect competition on each intermediate good market. Let us provide more detail on the latter point.

As explained in Section 2, each intermediate good sector originates from an invention made through fundamental R&D. Once invented, each intermediate good can be modified, improved as the result of several steps of innovation through applied R&D activity. In order to consider a decentralized economy which provides incentives for both types of R&D activities, we rely on the ideas introduced by the seminal innovation-based endogenous growth models of Romer (1990) and of Aghion & Howitt (1992). Romer (1990) developed a decentralized economy which considers infinitely-lived IPRs on the production and sale of intermediate goods embodying newly created knowledge (that he names "new design"). Schumpeterian growth theory (e.g. Aghion & Howitt 1992, 1998) incorporates Schumpeter's "creative destruction" mechanism, considering that "the firm that succeeds in innovating can monopolize the intermediate sector until replaced by the next innovator" (Aghion & Howitt 1998 - Ch. 2): in each sector, there is a monopoly - whose lifespan is finite in average - on the production and sale of the latest quality of intermediate good.

The attempt to unify those two frameworks gives rise to the following issue. How is it possible to reward simultaneously both types of R&D activities while there is creative destruction? The key point lies in that the inventor at the source of a new sector must have sufficient incentives to invest in fundamental R&D in the first place, and that creative destruction tends to reduce these incentives. Such issues have been studied in partial equilibrium. In particular, Green & Scotchmer (1995) show that "in markets with sequential innovation, inventors of derivative improvements might undermine the profit of initial innovators through competition. Profit erosion can be mitigated by broadening the first innovator's patent protection and/or by permitting cooperative agreements between initial innovators and later innovators." In this section, we incorporate some of these features in an innovation-based endogenous growth model considering both fundamental and applied R&D activities. In particular, we consider the existence of a design of IP law (in particular of patent law) that enables the division of profits between initial inventors and later innovators.

Formally, we assume that fundamental R&D activity giving rise to a new intermediate good sector will get a share $\zeta$, $\zeta \in ]0;1[$, of the profits realized by the successive innovators monopolizing this sector. One can think of *ex ante* agreements (e.g. cooperative ventures formed by firms to invest in both types of R&D activities) or of *ex post* licensing (e.g. licensing agreements formed by inventors and innovators after new products have been developed and patents have been awarded). Either way, these depend in particular on the design of IP law; accordingly, $\zeta$ can be interpreted as the measure of the level of protection of IPRs granted to inventors.[11] Henceforth, in each intermediate sector, the inventor at the origin of the sector receives a fraction $\zeta$ of each incumbent's profit for a length which is finite in average (because of the creative destruction mechanism, the lifespan of each monopoly is finite in average). Eventually, the inventor receives an infinite length profit but paid by a succession of monopolies.

In such a decentralized economy, Pareto non-optimality is likely to arise. This follows from the following market failures. The first one results from the presence of monopolies on intermediate goods; it can be corrected by an *ad valorem* subsidy $\psi$ on each intermediate good demand. The second one relates to the externality triggered by market incompleteness: there is no market for knowledge. Here, since we consider two types of knowledge (each of which being produced by a dedicated R&D activity), we introduce two public tools: $\varphi^Q$ and $\varphi^A$, targeted towards fundamental R&D and applied R&D activities, respectively. Let us comment on the two latter tools. In the decentralized economy of Romer (1990), the *laisser faire* R&D effort is sub-optimal.[12] Hence,

[11] The parameter $\zeta$ could also be interpreted as the result of a public regulation aiming at supporting fundamental R&D which would tax firms (here intermediate good producers) in order to fund fundamental R&D. One could also endogenize $\zeta$ by formalizing the mechanisms yielding to cooperative agreements or to licensing. This would somehow extend the analysis of Green & Scotchmer (1995) to an innovation-based growth model that considers a dynamic general equilibrium framework.

[12] Romer's framework can be modified in such a way that the *laisser faire* R&D effort can be sub-optimal or over-

$\varphi^Q$ is assumed to be a subsidy on the profit realized by fundamental R&D activity (in Section 4, we will show that the optimal tool $\varphi^{Qo}$ is indeed a subsidy). It is well known that, in standard Schumpeterian decentralized economies, the *laisser faire* R&D effort can be sub-optimal or over-optimal (see, for instance, Aghion & Howitt 1992, 1998, 2009; Barro & Sala-i-Martin 2003; or Acemoglu 2009). Hence, we assume that the tool $\varphi^A$ can *a priori* consist in a subsidy or in a tax on the profit realized by applied R&D activity. We return on this point in Section 4 below when we implement the first-best social optimum within the decentralized economy. In fact, we will show that, in this new framework considering simultaneously applied and fundamental R&D, when the stock of fundamental knowledge is sufficiently large, optimality requires that each successive innovator is subsidized). Formally, we construct the set of Schumpeterian equilibria as follows.

**Definition 1.** *At each vector of public policy tools* $\left(\zeta, \psi, \varphi^Q, \varphi^A\right)$ *is associated a particular equilibrium. It consists of time paths of set of prices*

$$\left\{\left(w_t\left(\zeta, \psi, \varphi^Q, \varphi^A\right), r_t\left(\zeta, \psi, \varphi^Q, \varphi^A\right), \left\{p_{it}\left(\zeta, \psi, \varphi^Q, \varphi^A\right)\right\}_{i\in[0,Q_t]}\right)\right\}_{t=0}^{\infty}$$

*of labor partition*

$$\left\{\left(L_t^Y\left(\zeta, \psi, \varphi^Q, \varphi^A\right), \left\{l_{\omega t}\left(\zeta, \psi, \varphi^Q, \varphi^A\right)\right\}_{i\in[0,Q_t]}, L_t^Q\left(\zeta, \psi, \varphi^Q, \varphi^A\right)\right)\right\}_{t=0}^{\infty}$$

*of quantities of rival goods*

$$\left\{\left(c_t\left(\zeta, \psi, \varphi^Q, \varphi^A\right), Y_t\left(\zeta, \psi, \varphi^Q, \varphi^A\right), \left\{x_{it}\left(\zeta, \psi, \varphi^Q, \varphi^A\right)\right\}_{i\in[0,Q_t]}\right)\right\}_{t=0}^{\infty}$$

*and of quantities of non-rival goods*

$$\left\{\left(\left\{A_{it}\left(\zeta, \psi, \varphi^Q, \varphi^A\right)\right\}_{i\in[0,Q_t]}, Q_t\left(\zeta, \psi, \varphi^Q, \varphi^A\right)\right)\right\}_{t=0}^{\infty}$$

*such that:*

- *the representative household maximizes his utility; firms maximize their profits;*
- *the final good market, the financial market and the labor market are perfectly competitive and clear;*
- *IP law is designed in such a way that, on each intermediate good market,*
  - ***i)*** *the latest innovator monopolizes the production and sale of the latest-generation intermediate good, until replaced by the next innovator,*
  - ***ii)*** *and the initial inventor receives a share of the resulting monopoly profit;*
- *there is free entry on each R&D activity (the zero profit condition holds).*

In order to simplify notations, we drop the $\left(\zeta, \psi, \varphi^Q, \varphi^A\right)$ arguments for all variables in the following computations. Before studying the agents' behaviors, let us present more formally the division of profits and derive the resulting definitions of the value of inventions and of the value of innovations.

Consider any intermediate good sector $i$, $i \in [0, Q_t]$. The latest successful innovator monopolizes the production and sale of the latest generation of the intermediate good until replaced by the next innovator. Furthermore, given the design of IP law, the inventor at the origin of the sector receives from each incumbent innovator a share $\zeta$ of the incumbent's instantaneous monopoly profit $\pi_\tau^{x_i} =$

optimal. See for instance Benassy (1998), Jones & Williams (2000), or Alvarez-Pelaez & Groth (2005).

$p_{i\tau}x_{i\tau} - y_{i\tau}$, where the quantity of final good $y_{i\tau}$ used to produce the intermediate is given by (6). Then - given the governmental interventions on behalf of fundamental and applied R&D activities ($\varphi^Q$ and $\varphi^A$, respectively) - the design of IPRs implies the following. The inventor receives the net profit $\pi_\tau^{Q_i} = (1+\varphi^Q)\zeta\pi_\tau^{x_i}$ from the date of his invention (*i.e.* from the creation date of the fundamental knowledge from which the sector stems) until infinity. The incumbent innovator, having innovated at date $t$, receives at any date $\tau > t$ the net profit $\pi_\tau^{A_i} = (1+\varphi^A)(1-\zeta)\pi_\tau^{x_i}$ with probability $e^{-\int_t^\tau \lambda l_{iu}^A du}$ (that is provided that there is no innovation between dates $t$ and $\tau$). Accordingly, we define the value of inventions and of innovations as follows.

**Definition 2.** *Consider any sector* $i, i \in [0, Q_t]$.

**a.** *If the* ***invention*** *at the origin of this has occurred at date T, its private value is*

$$\Pi_T^{Q_i} = \int_T^\infty \pi_\tau^{Q_i} e^{-\int_T^\tau r_u du} d\tau \tag{11}$$

**b.** *The private value of an* ***innovation*** *at date t (t > T) in this sector is*

$$\Pi_t^{A_i} = \int_t^\infty \pi_\tau^{A_i} e^{-\int_t^\tau \left(r_u + \lambda l_{iu}^A\right) du} d\tau \tag{12}$$

The private value of an invention is defined as the sum of the present values of the share of the profits received by the inventor at the origin of the sector (taking into account possible subsidies $\varphi^Q$). As argued above, given the presence of the creative destruction mechanism, the inventor at the origin of sector $i$ receives revenues forever (a share of the monopoly profit made by each successive innovator). Note that the expression (11) is similar to the one used by Romer (1990), when he introduce the "price for design" (see equation 6 p. S87). The private value of an innovation in sector $i$ is defined as the sum of the present values of the incumbent's expected net profits on the sale of intermediate good $i$ (taking into account possible subsidies or taxes $\varphi^A$). The term $\lambda l_{iu}^A$ in the discount factor is the rate of creative destruction; as mentioned above, this mechanism implies that, in each sector, the patent owner has a monopoly the lifespan of which is finite in average (*i.e.* the period during which an innovation yields some return is finite in average). Note that the expression (12) appears in Aghion & Howitt (1992), see for instance equation 2.12 (p. 330); it is named the "value of the monopolist's current patent". Somehow, after merging the technologies introduced in Romer (1990) and in Aghion & Howitt (1992), we blend the decentralized economies they considered.

## 3.2 Agents' behaviors

The **representative household** maximizes his intertemporal utility given by (8) subject to his budget constraint, $\dot{b}_t = w_t + r_t b_t - c_t - T_t/L$, where $b_t$ denotes the per capita financial asset and $T_t$ is a lump-sum tax charged by the government in order to finance public policies. This yields the usual Keynes-Ramsey condition:

$$r_t = g_{c_t} + \rho \tag{13}$$

In the **final sector**, the competitive firm maximizes its profit

$$\pi_t^Y = Y_t - w_t L_t^Y - \int_0^{A_t} (1-\psi) p_{it} x_{it} di \tag{14}$$

with respect to $L_t^Y$ and $x_{it}, i \in [0, Q_t]$. Recall that the final good is produced according to (7), $Y_t = (L_t^Y)^{1-\alpha} \int_0^{Q_t} A_{it}(x_{it})^\alpha di$, and that its price is normalized to one. First-order conditions yield

$$w_t = (1-\alpha)(L_t^Y)^{-\alpha} \int_0^{Q_t} A_{it}(x_{it})^\alpha di = (1-\alpha)\frac{Y_t}{L_t^Y} \tag{15}$$

and

$$p_{it} = \frac{\alpha(L_t^Y)^{1-\alpha}A_{it}(x_{it})^{\alpha-1}}{1-\psi}, \forall i \in [0, Q_t] \tag{16}$$

Consider any **intermediate good sector** $i$, $i \in [0, Q_t]$. At each date $t$, the incumbent monopoly maximizes the instantaneous profit $\pi_t^{x_i}$ with respect to $x_{it}$, where the inverse demand for intermediate good $i$ is given in (16). The profit writes $\pi_t^{x_i} = \frac{\alpha(L_t^Y)^{1-\alpha}A_{it}(x_{it})^{\alpha-1}}{1-\psi}x_{it} - A_{it}x_{it}$. Maximization gives the standard symmetric use of intermediate goods in the final good production:

$$\frac{\partial \pi_t^{x_i}}{\partial x_{it}} = 0 \Leftrightarrow \frac{\alpha^2(L_t^Y)^{1-\alpha}A_{it}(x_{it})^{\alpha-1}}{1-\psi} - A_{it} = 0$$

$$\Leftrightarrow x_{it} = x_t = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{1}{1-\alpha}} L_t^Y, \forall i \in [0, Q_t] \tag{17}$$

and the usual mark-up on the price of intermediate goods:

$$p_{it} = \frac{A_{it}}{\alpha}, \forall i \in [0, Q_t] \tag{18}$$

Using (17), the incumbent innovator's net monopoly profit writes

$$\pi_t^{A_i} = (1+\varphi^A)(1-\zeta)\pi_t^{x_i} = (1+\varphi^A)(1-\zeta)\frac{1-\alpha}{\alpha}A_{it}x_t, \forall i \in [0, Q_t] \tag{19}$$

and the inventor's net monopoly profit writes

$$\pi_t^{Q_i} = (1+\varphi^Q)\zeta\pi_t^{x_i} = (1+\varphi^Q)\zeta\frac{1-\alpha}{\alpha}A_{it}x_t, \forall i \in [0, Q_t] \tag{20}$$

From Definition 2, one derives the standard arbitrage conditions in R&D. Differentiating (11) with respect to time, one gets arbitrage condition in any **fundamental R&D activity** at the origin of sector $i$:

$$r_t = \frac{\dot{\Pi}_t^{Q_i}}{\Pi_t^{Q_i}} + \frac{\pi_t^{Q_i}}{\Pi_t^{Q_i}}, \forall i \in [0, Q_t] \tag{21}$$

Similarly, differentiating (12) with respect to time, one gets arbitrage condition in any **applied R&D activity** $i$:

$$r_t + \lambda l_{it}^A = \frac{\dot{\Pi}_t^{A_i}}{\Pi_t^{A_i}} + \frac{\pi_t^{A_i}}{\Pi_t^{A_i}}, \forall i \in [0, Q_t] \tag{22}$$

These two arbitrage conditions illustrate that, at the equilibrium, the rate of return is the same on the financial market, on fundamental R&D activity, and on any applied R&D activity.

Finally, let us explicit the free entry conditions (zero profit conditions) in R&D activities. The unit cost of labor is $w_t$. As seen in (3), the labor cost of producing one invention is $1/Q_t$. Besides, the inventor's revenue when one unit of labor is invested in fundamental R&D is $\Pi_t^{Q_i}$. Accordingly, the free entry condition in fundamental R&D activity is

$$\frac{w_t}{\delta Q_t} = \Pi_t^{Q_i}, \tag{23}$$

Besides, as seen in Assumption 1, innovations occur along with a Poisson arrival rate of parameter $\lambda$; therefore, $\lambda\Pi_t^{A_i}$ is the expected revenue when one unit of labor is invested in applied R&D. Consequently, the free entry condition in any applied R&D activity $i$ is

$$w_t = \lambda\Pi_t^{A_i}, \tag{24}$$

Now that the agents' behaviors have been presented, the equilibrium can be characterized.

### 3.3 Characterization of the Equilibrium

Using the symmetry obtained in (17), together with the definition of the whole knowledge in the economy (1), the final good production function (7) can be rewritten as

$$Y_t = (L_t^Y)^{1-\alpha}(x_t)^\alpha \int_0^{Q_t} A_{it} di = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} L_t^Y \mathcal{A}_t \tag{25}$$

Hence, the wage (15) rewrites

$$w_t = (1-\alpha)\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} \mathcal{A}_t \tag{26}$$

Consequently, using (26) and the free-entry conditions (23) and (24), one derives the private value of the invention at the origin of sector $i$ at date $t$, as defined in (11), and the private value of an innovation in sector $i$ at date $t$, as defined in (12):

$$\Pi_t^{Q_i} = \Pi_t^Q = \frac{1-\alpha}{\delta}\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{\mathcal{A}_t}{Q_t}, \forall i \in [0, Q_t] \tag{27}$$

and

$$\Pi_t^{A_i} = \Pi_t^A = \frac{1-\alpha}{\lambda}\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} \mathcal{A}_t, \forall i \in [0, Q_t] \tag{28}$$

Let us now derive the growth rates of the economy. Log-differentiating (25) with respect to time gives

$$g_{Y_t} = g_{L_t^Y} + g_{\mathcal{A}_t} \tag{29}$$

Furthermore, using (1), (6) and (17), the constraint on the final good market (10) rewrites $Y_t = Lc_t + x_t \int_0^{Q_t} A_{it} di = Lc_t + (\alpha^2/1-\psi)^{\frac{1}{1-\alpha}} L_t^Y \mathcal{A}_t$. Dividing both sides by $Y_t$ and using (25), one gets $Lc_t/Y_t = 1 - \frac{\alpha^2}{1-\psi}$. Log-differentiating this expression gives

$$g_{Y_t} = g_{c_t} \tag{30}$$

From (27), one has $\frac{\dot{\Pi}_t^{Q_i}}{\Pi_t^{Q_i}} = g_{\mathcal{A}_t} - g_{Q_t}$, $\forall i \in [0, Q_t]$. Moreover, from (17), (20) and (27), one obtains $\frac{\pi_t^{Q_i}}{\Pi_t^{Q_i}} = (1+\varphi^Q)\zeta\frac{\delta}{\alpha}\left(\frac{1-\psi}{\alpha^2}\right)^{\frac{\alpha}{1-\alpha}} \frac{Q_t A_{it}}{\mathcal{A}_t} x_t = \frac{(1+\varphi^Q)\zeta\delta\alpha Q_t A_{it}}{(1-\psi)\mathcal{A}_t} L_t^Y, \forall i \in [0, Q_t]$. Hence, the arbitrage condition (21) rewrites

$$r_t = g_{\mathcal{A}_t} - g_{Q_t} + \frac{(1+\varphi^Q)\zeta\delta\alpha Q_t A_{it}}{(1-\psi)\mathcal{A}_t} L_t^Y \tag{31}$$

Similarly, from (28), one has $\frac{\dot{\Pi}_t^{A_i}}{\Pi_t^{A_i}} = g_{\mathcal{A}_t}$, $\forall i \in [0, Q_t]$. Moreover, from (17), (19) and (28), one obtains $\frac{\pi_t^{A_i}}{\Pi_t^{A_i}} = (1+\varphi^A)(1-\zeta)\frac{\lambda}{\alpha}\left(\frac{1-\psi}{\alpha^2}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_{it}}{\mathcal{A}_t} x_t = \frac{(1+\varphi^A)(1-\zeta)\lambda\alpha A_{it}}{(1-\psi)\mathcal{A}_t} L_t^Y, \forall i \in [0, Q_t]$. Hence, the arbitrage condition (22) rewrites

$$r_t + \lambda l_{it}^A = g_{\mathcal{A}_t} + \frac{(1+\varphi^A)(1-\zeta)\lambda\alpha A_{it}}{(1-\psi)\mathcal{A}_t} L_t^Y \tag{32}$$

From (31), one obtains $A_{it} = \mathcal{A}_t, \forall i \in [0, Q_t]$; the stock of applied knowledge is identical across sectors. Consequently, from (32), one gets $l_{it}^A = l_t^A, \forall i \in [0, Q_t]$; the quantity of labor devoted to each applied R&D activity is the same. All results relative to the symmetry of the equilibrium are summarized in the following lemma.

**Lemma 3.** *The equilibrium is characterized by*

- *symmetric uses of intermediate goods:* $x_{it} = x_t = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{1}{1-\alpha}} L_t^Y, \forall i \in [0, Q_t]$
- *symmetric quantities of labor devoted to applied R&D:* $l_{it}^A = l_t^A, \forall i \in [0, Q_t]$
- *symmetric stocks of applied knowledge:* $A_{it} = A_t, \forall i \in [0, Q_t]$

Note that symmetry is in general an usual assumption made in endogenous growth models in order to obtain closed-form solutions (see, for instance, Aghion & Howitt 1992, 1998 - Ch. 3, or Peretto & Smulders 2002). Here we obtain symmetry as an equilibrium result. This point is in line with Peretto (1998, 1999) or Cozzi, Giordani & Zamparelli (2007) in which the relevancy of the symmetric equilibrium is discussed in detail.

From Lemma 3, the total quantity of labor devoted to applied R&D rewrites $L_t^A = Q_t l_t^A$. Furthermore, the whole stock of knowledge in the economy (1) rewrites

$$\mathcal{A}_t = Q_t A_t \tag{33}$$

Accordingly, the law of accumulation of applied knowledge in any sector $i, i \in [0, Q_t]$, (2) is now

$$\dot{A}_t = \lambda\sigma l_t^A Q_t A_t = \lambda\sigma L_t^A A_t \tag{34}$$

One gets the following Proposition.

**Proposition 2.** *Growth rates of knowledge.*

- *The growth rate of applied knowledge in any sector* $i, i \in [0, Q_t]$*, is*

$$g_{A_{it}} = g_{A_t} = \frac{\dot{A}_t}{A_t} = \lambda\sigma L_t^A \tag{35}$$

- *The growth rate of fundamental knowledge in the economy is*

$$g_{Q_t} = \frac{\dot{Q}_t}{Q_t} = \delta L_t^Q \tag{36}$$

- *The growth rate of the whole knowledge in the economy is*

$$g_{\mathcal{A}_t} = \frac{\dot{\mathcal{A}}_t}{\mathcal{A}_t} = g_{A_t} + g_{Q_t}, \text{ where } g_{A_t} \text{ and } g_{Q_t} \text{ are given in (35) and (36), respectively} \tag{37}$$

**Proof.** The growth rate (35) is directly derived from (34). The growth rate (36) results from (3). In Lemma 3, we have shown that $A_{it} = A_t, \forall i \in [0, Q_t]$; in particular, one has $A_{Q_t t} = A_t$. Accordingly, (5) becomes $g_{\mathcal{A}_t} = \lambda\sigma L_t^A + \delta L_t^Q \frac{Q_t A_t}{\mathcal{A}_t}$. Furthermore, from (33), one has $\mathcal{A}_t = Q_t A_t$; hence, on obtains $g_{\mathcal{A}_t} = \lambda\sigma L_t^A + \delta L_t^Q$. Then, from (35) and (36) one obtains (37). □

Given Lemma 3, the arbitrage conditions (31) and (32) rewrite

$$r_t = g_{\mathcal{A}_t} - g_{Q_t} + \frac{(1+\varphi^Q)\zeta\delta\alpha}{(1-\psi)} L_t^Y \tag{38}$$

and

$$r_t + \lambda l_t^A = g_{A_t} + \frac{(1+\varphi^A)(1-\zeta)\lambda\alpha}{(1-\psi)Q_t} L_t^Y \tag{39}$$

Finally, at each date $t$, the equilibrium quantities, growth rates and prices are characterized by (9), (13), (17), (18), (25), (26), (29), (30), (35), (36), (37), (38) and (39). The following proposition provides the characterization of the equilibrium labor partition between its three competing uses, namely the final good production, the applied R&D activity the fundamental R&D activity.

**Proposition 3.** *The labor partition at the equilibrium is characterized as follows.*

$$L = L_t^Y + Q_t l_t^A + L_t^Q \tag{40}$$

$$g_{L_t^Y} + \rho + \delta L_t^Q = \frac{(1+\varphi^Q)\zeta\delta\alpha L_t^Y}{1-\psi} \tag{41}$$

$$g_{L_t^Y} + \rho + \lambda l_t^A = \frac{(1+\varphi^A)(1-\zeta)\lambda\alpha L_t^Y}{(1-\psi)Q_t} \tag{42}$$

**Proof.** Rewriting the labor constraint (9) using the result of symmetry relative to labor devoted to applied R&D activity, one gets (40). Using (29), (30), (35), (36), and (37), one gets

$$g_{c_t} = g_{L_t^Y} + \lambda\sigma L_t^A + \delta L_t^Q \tag{43}$$

From (13), (35), (37) and (38) one gets $g_{c_t} + \rho = \lambda\sigma L_t^A + \frac{(1+\varphi^Q)\zeta\delta\alpha L_t^Y}{1-\psi}$; using (43), one obtains (41). Similarly, from (13), (35), (36), (37) and (39) one gets $g_{c_t} + \rho + \lambda l_t^A = \lambda\sigma L_t^A + \delta L_t^Q + \frac{(1+\varphi^A)(1-\zeta)\lambda\alpha L_t^Y}{(1-\psi)Q_t}$; using (43), one obtains (42). □

As stated in the introduction of the paper, one of our goal was to provide a framework suitable to study the tradeoff faced by society between engaging in fundamental and in applied R&D activities. By characterizing the equilibrium labor partition, Proposition 3 specifies this tradeoff.

# 4 Implementation of the First-best: Optimal Sponsorship to R&D and IP Law

In Section 3, we studied a decentralized economy built on the Schumpeterian equilibrium *à la* Aghion & Howitt. In order to ensure the presence of incentives for both fundamental and applied R&D activities, we considered the existence of a design of patent law from which stems a division of profits between inventors and following innovators. Formally, any inventor giving rise to a new intermediate good sector gets a share $\zeta$ of the profits realized by each of the successive innovators monopolizing this sector. This enables us to overcome the incentives-related issues involved by the fact that such a decentralized economy goes along creative destruction, and that it considers two distinct R&D activities which fund themselves indirectly via monopoly profits on the *same* good.

As explained in 3.1, there are two issues that make Pareto non-optimality likely to arise in this decentralized economy. First, the market failure involved by the presence of a monopoly in each intermediate sector (which can be corrected by $\psi$). Second, the fact that there is no market for knowledge. While the first issue is well known, the second one is more intricate here insofar as it involves in fact two market failures since we consider two types of knowledge. Hence, we introduced two public tools dedicated to mitigate market incompleteness ($\varphi^A$ for applied knowledge and $\varphi^Q$ for fundamental knowledge).

## 4.1 Implementation and Optimal Sponsorship to R&D

In this section, we show how the first-best social optimum exhibited in Proposition 1 can be implemented within the Schumpeterian equilibrium defined and characterized in Section 3. We know that, since there are three markets failure, the first-best can be implement by use of three tools. Proposition 4 exhibits an optimal set of tools that sustains the first-best.

**Proposition 4.** *The first-best social optimum can be implemented in a Schumpeterian equilibrium* à la *Aghion & Howitt, considering division of profits between inventors and innovators, by use of the public tools $\psi^o$, $\varphi^{Qo}$ and $\varphi_t^{Ao}$ characterized as follows.*

$$\psi^o = 1 - \alpha \tag{44}$$

$$(1+\varphi^{Qo})\zeta = \frac{\lambda\sigma}{\delta} \tag{45}$$

$$(1+\varphi_t^{Ao})(1-\zeta) = \frac{1}{\delta L}\left[\sigma\rho Q_t^o + \lambda\sigma\left(\frac{\rho}{\delta} - \frac{\delta L}{\lambda\sigma}\right)\right] \tag{46}$$

*where $Q_t^o = e^{g_{Q_t}^o t}$ with $g_{Q_t}^o = \delta L^{Qo} = \delta L - \rho$.*

**Proof.** The optimal subsidy $\psi^o$ can be obtained by identifying the equilibrium quantities of intermediate goods with the optimal ones. We have shown in (17) that, at the equilibrium, one has $x_{it} = x_t = [\alpha^2/(1-\psi)]^{\frac{1}{1-\alpha}} L_t^Y, \forall i \in [0, Q_t]$. Besides, we have shown in Proposition 1 that at the first-best social optimum, one has $x_{it}^o = x^o = \alpha^{\frac{1}{1-\alpha}} L^{Yo}, \forall i \in [0, Q_t^o]$. Hence, $\psi^o$ must satisfy

$$\left(\frac{\alpha^2}{1-\psi^o}\right)^{\frac{1}{1-\alpha}} L^{Yo} = \alpha^{\frac{1}{1-\alpha}} L^{Yo} \Leftrightarrow \frac{\alpha^2}{1-\psi^o} = \alpha \Leftrightarrow \psi^o = 1-\alpha$$

This proves (44). Let us now derive $\varphi^{Ao}$ and $\varphi^{Qo}$. In Proposition 1, we have shown that at the first-best, the partition of labor is $L_t^{Yo} = L^{Yo} = \frac{\delta L}{\lambda\sigma}$, $L_t^{Ao} = Q_t^o l_t^{Ao} = L^{Ao} = \frac{\rho}{\delta} - \frac{\delta L}{\lambda\sigma}$ and $L_t^{Qo} = L^{Qo} = L - \frac{\rho}{\delta}$. Moreover, at the first-best, one has $g_{L_t^Y}^o = 0$ (see (76) in Appendix 6.2). Besides, in Proposition 3, we have shown that the equilibrium labor partition is characterized by (40), (41) and (42). Rewriting (41) at the first-best gives $g_{L_t^Y}^o + \rho + \delta L_t^{Qo} = \frac{(1+\varphi^{Qo})\zeta\delta\alpha L_t^{Yo}}{1-\psi^o}$. Then, from Proposition 1 and using (44), one obtains

$$\rho + \delta L_t^{Qo} = (1+\varphi^{Qo})\zeta\delta L_t^{Yo} \Leftrightarrow \rho + \delta\left(L - \frac{\rho}{\delta}\right) = (1+\varphi^{Qo})\zeta\delta\frac{\delta L}{\lambda\sigma} \Leftrightarrow (1+\varphi^{Qo})\zeta = \frac{\lambda\sigma}{\delta}$$

This proves (45). Similarly, rewriting (42) at the first-best gives $g_{L_t^Y}^o + \rho + \lambda l_t^{Ao} = \frac{(1+\varphi^{Ao})(1-\zeta)\lambda\alpha L_t^{Yo}}{(1-\psi^o)Q_t^o}$. Then, using Proposition 1 and (44), one gets

$$\rho Q_t^o + \lambda Q_t^o l_t^{Ao} = (1+\varphi^{Ao})(1-\zeta)\lambda L_t^{Yo} \Leftrightarrow \rho Q_t^o + \lambda L^{Ao} = (1+\varphi^{Ao})(1-\zeta)\lambda L^{Yo}$$

$$\Leftrightarrow \rho Q_t^o + \lambda\left(\frac{\rho}{\delta} - \frac{\delta L}{\lambda\sigma}\right) = (1+\varphi^{Ao})(1-\zeta)\lambda\frac{\delta L}{\lambda\sigma} \Leftrightarrow (1+\varphi^{Ao})(1-\zeta) = \frac{1}{\delta L}\left[\sigma\rho Q_t^o + \lambda\sigma\left(\frac{\rho}{\delta} - \frac{\delta L}{\lambda\sigma}\right)\right]$$

Note that $\varphi^{Ao}$ depends on $Q_t^o$. Moreover, one has $Q_t^o = e^{g_{Q_t}^o t}$, where $g_{Q_t}^o$ is given in Proposition 1. Therefore, $\varphi^{Ao}$ depends on time. This proves (46) and concludes the proof of Proposition 4. □

Before studying the part played by the division of profits resulting from IP law and investigating how it intertwines with public sponsorship of R&D, let us first provide some general comments on the optimal tools derived in Proposition 4.

**1.** As seen in (44), the optimal tool used to correct the monopoly distortion, $\psi^o = 1 - \alpha$, consists in the usual subsidy on each intermediate good demand ($\psi^o > 0$ since $0 < \alpha < 1$). This result is analog to the ones found for instance in Romer (1990) and in Aghion & Howitt (1992).

**2.** The optimal tools used to correct the externality triggered by the fact that there are no markets for fundamental and applied knowledge, $\varphi^{Qo}$ and $\varphi^{Ao}$, both clearly depend on $\zeta$. We return on this point below in 4.2.

**3.** It appears clearly in (45) that the optimal tool used to correct the market failure triggered by the fact that there is no market for fundamental knowledge, $\varphi^{Qo} = \frac{\lambda\sigma}{\delta\zeta} - 1$, is a subsidy granted to inventors. Indeed, the parameters of the model are such that $\varphi^{Qo} > 0$.[13] Recall that fundamental R&D has been formalized using the framework of Romer (1990), and that in the decentralized economy considered by Romer, R&D effort is likely to be sub-optimal. Our result thus echoes to Romer's one: fundamental R&D should be subsidized.

**4.** Let us now study the properties of $\varphi_t^{Ao}$, the optimal tool used to correct the market failure triggered by the fact that there is no market for applied knowledge. From (46), one has

$$\varphi_t^{Ao} = \frac{1}{(1-\zeta)\delta L}\left[\sigma\rho Q_t^o + \lambda\sigma\left(\frac{\rho}{\delta} - \frac{\delta L}{\lambda\sigma}\right)\right] - 1$$

The formalization introduced in this paper (considering that knowledge accumulates both horizontally and vertically) introduces a new property to this optimal tool: $\varphi_t^{Ao}$ increases in time since it depends positively on $Q_t^o$ (which measures the number of sector, as well as the stock of fundamental knowledge). Consequently, there exists a date from which the optimal tool targeted at applied R&D will necessarily consist in a subsidy granted to innovators.[14] This result is somehow new with respect to its counterpart found in Aghion & Howitt (1992). Indeed, in the decentralized economy they study, the provision of R&D effort can either be sub-optimal (in which case R&D should be subsidized) or over-optimal (in which case R&D should be taxed). Considering that the number of sectors increases owing to fundamental R&D activity mitigates this result: as $Q_t^o$ increases, the R&D effort devoted to applied R&D activities will perforce become insufficient at some point. The intuition is the following. As the number of sectors increases, the positive externality triggered by the production of applied knowledge becomes more stringent because there are potentially more applications for each new piece of applied knowledge created. Hence, larger subsidies are likely to be required in order to make applied R&D activity internalize the knowledge spillovers it generates.

## 4.2 IP Law and Division of Profit between Applied and Fundamental R&D

The implementation of the first-best within a Schumpeterian decentralized economy considering both the presence of inventors and innovators reveals several key points. In Proposition 4, we have shown that the optimal tools used to correct the externality triggered by the fact that there are no markets for fundamental and applied knowledge both depend on $\zeta$, the degree of protection granted to inventors by IP law, which determines the division of profit between an inventor and its following innovators. Let us investigate how public sponsorship of R&D and IP law intertwine when providing optimal

[13] Assumption 3 implies that one has $L > \frac{\delta L}{\lambda\sigma}$, or equivalently $\frac{\lambda\sigma}{\delta} > 1$. Furthermore, since $\zeta \in ]0;1[$, one has $\frac{\lambda\sigma}{\delta\zeta} > 1$.

[14] From Assumption 3, one has $\frac{\rho}{\delta} > \frac{\delta L}{\lambda\sigma}$; thus, one has $\sigma\rho Q_t^o + \lambda\sigma\left(\frac{\rho}{\delta} - \frac{\delta L}{\lambda\sigma}\right) > \sigma\rho Q_t^o$. Hence, one gets $1 + \varphi_t^{Ao} = \frac{1}{(1-\zeta)\delta L}\left[\sigma\rho Q_t^o + \lambda\sigma\left(\frac{\rho}{\delta} - \frac{\delta L}{\lambda\sigma}\right)\right] > \frac{\sigma\rho}{(1-\zeta)\delta L} Q_t^o$. Moreover, since $\zeta \in ]0;1[$, one obtains $\varphi_t^{Ao} > \frac{\sigma\rho}{\delta L} Q_t^o - 1$. Therefore, since $Q_t^o$ increases with $t$, there exists a date $\bar{t}$ such that $\varphi_t^{Ao} > \frac{\sigma\rho}{\delta L} Q_t^o - 1 > 0, \forall t > \bar{t}$.

incentives simultaneously to inventors and to innovators. In particular, we provides arguments along which optimal R&D incentives to inventors and to innovators require simultaneously a design of IP law which is sufficiently favorable to inventors and public sponsorship of both fundamental and applied R&D activities.

**Incentives to Fundamental R&D.**

Optimal incentives to inventors require both public sponsorship of fundamental R&D and an sufficiently pro-inventor IP law. This paper exhibits two fundamental reasons for this.

**1.** In Proposition 4, we have shown in (45) that the optimal subsidy to inventors, $\varphi^{Qo}$, is decreasing with $\zeta$. This underlines that, the weaker the degree of protection granted to inventors by IP law, the stronger public sponsorship of fundamental R&D should be in order to provide sufficient incentives to inventors. Furthermore, this also shows that the share of the profit that should be received by any inventor is strictly positive. Indeed, if $\zeta$ tends to zero, $\varphi^{Qo}$ tends to infinity.

Here lies one of the key result of the paper: the design of IP law should enables the division of profits between the inventor at the origin of a new product and the following innovators building on this invention.

**2.** Optimal incentives to inventors cannot be provided solely by the IP law, some public sponsorship is necessary. Why is it so? Assume that there is no public policy supporting fundamental R&D activity (*i.e.* assume that $\varphi^{Q} = 0$), one could think that the first-best is still implementable thanks to IP law, provided that it grants a sufficient degree of protection to inventors. Let us determine this degree of protection $\zeta^o$. Rewriting (41) with $\varphi^Q = 0$, one obtains that the optimal degree of protection granted to inventors, $\zeta^o$, must verify $g^o_{L^Y_t} + \rho + \delta L^{Qo}_t = \frac{\zeta^o \delta \alpha L^{Yo}_t}{1-\psi^o}$. From Proposition 1, one has $g^o_{L^Y_t} = 0$, $L^{Qo}_t = L^{Qo} = L - \frac{\rho}{\delta}$ and $L^{Yo}_t = L^{Yo} = \frac{\delta L}{\lambda \sigma}$. From (44), one has $\psi^o = 1 - \alpha$. Hence, one obtains $\rho + \delta \left(L - \frac{\rho}{\delta}\right) = \frac{\zeta^o \delta \alpha}{\alpha} \frac{\delta L}{\lambda \sigma} \Leftrightarrow \zeta^o = \frac{\lambda \sigma}{\delta}$. However, from Assumption 3, one has $\frac{\lambda \sigma}{\delta} > 1$. This means that, if fundamental R&D does not get any public support, in order to implement the first-best, inventors should be able to get more than 100% of the profit realized by each incumbent innovator. This is obviously not satisfying from the perspective of incentives that should be granted to innovators.

These two points underline the idea along which, even if it is necessary, IP law is not sufficient to provide optimal incentives to fundamental R&D: in addition to an IP law enabling some division of profit, fundamental R&D needs to be subsidized. This result echoes to the arguments put forward by Scotchmer (1991). In her own words, "it appears that patent policy is a very blunt instrument trying to solve a very delicate problem. Its bluntness derives largely from the narrowness of what patent breadth can depend on, namely the realized values of the technologies. As a consequence, the prospects for fine-tuning the patent system seem limited, which may be an argument for more public sponsorship of basic research." This issue is precisely the one involved by the fact that two types of R&D activities are funded via monopoly profits on the *same* intermediate good.

**Incentives to Applied R&D.**

Incentives to innovators clearly rely on the public tool $\varphi^A$. Considering simultaneously fundamental and applied R&D activities has proven the requirement for introducing specific incentives to the former in the form of a transfer of profit from the latter. As shown by (46), the tool dedicated to provide optimal incentives to innovators is increasing in $\zeta$, the degree of protection granted to inventors by IP law, which determines the share of profit transferred by each innovator to the inventor thanks to whom he has been able to innovate. In order to compensate for a larger share of the monopoly profit transferred to inventors, innovators must be given stronger incentives via the optimal tool $\varphi_t^{Ao}$ (which - as seen in the comment 4 of Proposition 4 - turns out to necessarily consist in a subsidy at some point).

This result shows the importance of public sponsorship of applied R&D to mitigate the disincentive effect of pro-inventor IP law. Again, one gets a result in line with Scotchmer (1991), who states that "a system of property rights that might seem natural would be to protect the first innovator so broadly that licensing is required from all second generation innovators who use the initial technology, whether in research or in production. But such broad protection can lead to deficient incentives to develop second generation products."

# 5 Conclusion

In this paper, we pursued ideas put forward in the industrial organization literature when considering partial equilibrium frameworks (e.g. Scotchmer 1991; Green & Scotchmer 1995; Bessen & Maskin 2009) in a dynamic general equilibrium Schumpeterian growth model considering both fundamental and applied R&D activities. One particular issue we had to overcome was to construct a Schumpeterian decentralized economy in which fundamental R&D activity was provided sufficient incentives to invest in the creation of inventions, despite the presence of "creative destruction" which tends to deter such incentives. In that respect, we considered the presence of IP law resulting in a division of profits between an inventor and its following innovators.

By implementing the first-best social optimal in a Schumpeterian growth model, in which one distinguishes inventions (produced by fundamental R&D) from innovations (produced by applied R&D), we have shown that providing optimal incentives to fundamental R&D requires not only a design of IP law (such as patent law) that ensure a positive transfer from innovators to inventors, but also public policies in form of subsidies supporting both fundamental and applied R&D activities. The latter point relates to the issue of providing appropriate incentives both for fundamental and applied R&D activities. We have shown that the will to provide sufficient incentives to innovators must not be detrimental to following innovators. The underlying tradeoff is the following. A design of IP law that might seem natural would be to protect the inventor so broadly that licensing is required from all second generation innovators who use the initial technology, whether in R&D or in production. But such broad protection can lead to deficient incentives provided to innovators to develop following generation products. Further research includes to investigate into more detail how to design suitable IP law by taking into account its impact on the negotiations between inventors and innovators yielding to cooperative *ex ante* agreements or to *ex post* licensing.

# 6 Appendix

## 6.1 Law of Knowledge Accumulation - Proof of Lemma 1

Consider any given sector $i$, $i \in [0, Q_t]$, and a time interval $(t, t + \Delta t)$. The level of applied knowledge at date $t$ in this sector is $A_{it}$. Let $k$, $k \in \mathbb{N}$, be the number of innovations that occur during the interval $(t, t + \Delta t)$. Under Assumptions 1 and 2, the level of knowledge at date $t + \Delta t$, $A_{i\,t+\Delta t}$, is a random variable taking the values $\{A_{it} + k\sigma \mathcal{A}_t\}_{k \in \mathbb{N}}$ with associated probabilities $\left\{ \frac{\left(\int_t^{t+\Delta t} \lambda l_{iu}^A du\right)^k}{k!} e^{-\int_t^{t+\Delta t} \lambda l_{iu}^A du} \right\}_{k \in \mathbb{N}}$. Accordingly, the expected level of knowledge at date $t+\Delta t$ is $\mathbb{E}\left[A_{i\,t+\Delta t}\right] = \sum_{k=0}^{\infty} \frac{\left(\int_t^{t+\Delta t} \lambda l_{iu}^A du\right)^k}{k!} e^{-\int_t^{t+\Delta t} \lambda l_{iu}^A du} \left[A_{it} + k\sigma \mathcal{A}_t\right]$.

Rearranging this expression, one gets

$$\mathbb{E}\left[A_{i\,t+\Delta t}\right] = \left[A_{it}\sum_{k=0}^{\infty}\frac{\left(\int_t^{t+\Delta t}\lambda l_{iu}^A du\right)^k}{k!} + \sigma\mathcal{A}_t\left(\int_t^{t+\Delta t}\lambda l_{iu}^A du\right)\sum_{k=1}^{\infty}\frac{\left(\int_t^{t+\Delta t}\lambda l_{iu}^A du\right)^{k-1}}{(k-1)!}\right]e^{-\int_t^{t+\Delta t}\lambda l_{iu}^A du}$$

The MacLaurin series $\sum_{k=0}^{K}\frac{\left(\int_t^{t+\Delta t}\lambda l_{iu}^A du\right)^k}{k!}$ converges to $e^{\int_t^{t+\Delta t}\lambda l_{iu}^A du}$ as $K\to\infty$. Thus, one gets

$$\mathbb{E}\left[A_{i\,t+\Delta t}\right] = \left[A_{it}e^{\int_t^{t+\Delta t}\lambda l_{iu}^A du} + \sigma\mathcal{A}_t\left(\int_t^{t+\Delta t}\lambda l_{iu}^A du\right)e^{\int_t^{t+\Delta t}\lambda l_{iu}^A du}\right]e^{-\int_t^{t+\Delta t}\lambda l_{iu}^A du}$$

$$\Leftrightarrow \mathbb{E}\left[A_{i\,t+\Delta t}\right] = A_{it} + \lambda\sigma\left(\int_t^{t+\Delta t} l_{iu}^A du\right)\mathcal{A}_t$$

Let $\Lambda_{\omega u}$ denote a primitive of $l_{iu}^A$ with respect to the time variable $u$. Rewriting the previous expression, one exhibits Newton's difference quotients of $\mathbb{E}\left[A_{it}\right]$ and of $\Lambda_{it}$: $\frac{\mathbb{E}[A_{i\,t+\Delta t}]-A_{it}}{\Delta t} = \lambda\sigma\frac{\Lambda_{i\,t+\Delta t}-\Lambda_{it}}{\Delta t}\mathcal{A}_t$.

Finally, letting $\Delta t$ tend to zero, one gets $\frac{\partial\mathbb{E}[A_{it}]}{\partial t} \equiv \dot{A}_{it} = \lambda\sigma l_{it}^A\mathcal{A}_t$. This proves that the expected knowledge in any sector $i$ is a differentiable function of time. Its derivative gives the law of motion of the expected knowledge (2) as given in Lemma 1 (the expectation operator is dropped to simplify notations). □

## 6.2 First-best Social Optimum - Proof of Proposition 1

The social planner maximizes the representative household's discounted utility (8) subject to (1), (2), (3), (6), (7), (9) and (10). The maximization program can be written as follows:

$$\begin{array}{l}\text{Max}\ \mathcal{U} = \int_0^\infty \ln(c_t)e^{-\rho t}dt,\ \text{subject to}\\ \{c_t\}_{t\in[0,\infty[}\\ \{L_t^Y\}_{t\in[0,\infty[}\\ \{L_t^Q\}_{t\in[0,\infty[}\\ \{l_{it}^A\}_{t\in[0,\infty[,i\in[0,Q_t]}\\ \{x_{it}\}_{t\in[0,\infty[,i\in[0,Q_t]}\end{array}\quad\begin{cases}Y_t = (L_t^Y)^{1-\alpha}\int_0^{Q_t}A_{it}(x_{it})^\alpha di\\ x_{it} = \frac{y_{it}}{A_{it}}, i\in[0,Q_t]\\ \dot{A}_{it} = \lambda\sigma l_{it}^A\mathcal{A}_t, i\in[0,Q_t]\\ \dot{Q}_t = \delta L_t^Q Q_t\\ \mathcal{A}_t = \int_0^{Q_t}A_{it}di\\ L = L_t^Y + \int_0^{Q_t}l_{it}^A di + L_t^Q\\ Y_t = Lc_t + \int_0^{Q_t}y_{it}di\end{cases}$$

where the control variables are $c_t$, $L_t^Y$, $L_t^Q$, $l_{it}^A$, $x_{it}$, $i\in[0,Q_t]$, and where the state variables of the dynamic optimization problem are $A_{it}, i\in[0,Q_t]$, and $Q_t$. We denote by $\iota_t$, $\mu_t$, $\nu_{it}^A, i\in[0,Q_t]$, and $\nu_t^Q$, the co-state variables associated with the final good resource constraint, with the labor constraint, with the continuum of state variables $A_{it}, i\in[0,Q_t]$, and with the state variable $Q_t$, respectively. After some rearrangement, one can write the Hamiltonian as follows:

$$\mathcal{H} = \ln(c_t)e^{-\rho t} + \iota_t\left[(L_t^Y)^{1-\alpha}\int_0^{Q_t}A_{it}(x_{it})^\alpha di - Lc_t - \int_0^{Q_t}A_{it}x_{it}di\right]$$
$$+\mu_t\left[L - L_t^Y - \int_0^{Q_t}l_{it}^A di - L_t^Q\right] + \int_0^{Q_t}\nu_{it}^A\lambda\sigma l_{it}^A\left(\int_0^{Q_t}A_{jt}dj\right)di + \nu_t^Q\delta L_t^Q Q_t$$

The first-order conditions are $\frac{\partial\mathcal{H}}{\partial c_t} = 0$, $\frac{\partial\mathcal{H}}{\partial L_t^Y} = 0$, $\frac{\partial\mathcal{H}}{\partial l_{it}^A} = 0, i\in[0,Q_t]$ , $\frac{\partial\mathcal{H}}{\partial L_t^Q} = 0$, $\frac{\partial\mathcal{H}}{\partial x_{it}} = 0, i\in[0,Q_t]$, $\frac{\partial\mathcal{H}}{\partial A_{it}} = -\dot{\nu}_{it}^A, i\in[0,Q_t]$, and $\frac{\partial\mathcal{H}}{\partial Q_t} = -\dot{\nu}_t^Q$ (to which the usual transversality conditions are added). They respectively yield:

$$c_t^{-1}e^{-\rho t} = \iota_t L \tag{47}$$

$$\iota_t(1-\alpha)(L_t^Y)^{-\alpha}\int_0^{Q_t}A_{it}(x_{it})^\alpha di = \mu_t \tag{48}$$

$$\nu_{it}^A \lambda\sigma \mathcal{A}_t = \mu_t, \forall i \in [0, Q_t] \tag{49}$$

$$\nu_t^Q \delta Q_t = \mu_t \tag{50}$$

$$\iota_t \left[\alpha (L_t^Y)^{1-\alpha} A_{it}(x_{it})^{\alpha-1} - A_{it}\right] = 0, \forall i \in [0, Q_t] \tag{51}$$

$$\iota_t \left[(L_t^Y)^{1-\alpha}(x_{it})^\alpha - x_{it}\right] + \int_0^{Q_t} \nu_{jt}^A \lambda\sigma l_{jt}^A dj = -\dot{\nu}_{it}^A, \forall i \in [0, Q_t] \tag{52}$$

$$\iota_t \left[(L_t^Y)^{1-\alpha} A_{Q_t t}(x_{Q_t t})^\alpha - A_{Q_t t} x_{Q_t t}\right] - \mu_t l_{Q_t t}^A + \lambda\sigma \left[A_{Q_t t} \int_0^{Q_t} \nu_{it}^A l_{it}^A di + \nu_{Q_t t}^A l_{Q_t t}^A \int_0^{Q_t} A_{jt} dj\right] + \nu_t^Q \delta L_t^Q = -\dot{\nu}_t^Q \tag{53}$$

From (49), one gets $\nu_{it}^A = \nu_t^A, \forall i \in [0, Q_t]$, and from (51), one gets the usual symmetric use of intermediates:

$$x_{it} = x_t = \alpha^{\frac{1}{1-\alpha}} L_t^Y, \forall i \in [0, Q_t] \tag{54}$$

Moreover, as usual in the literature, we consider the symmetric case in which $l_{it}^A = l_t^A$ and $A_{it} = A_t$, $\forall i \in [0, Q_t]$.[15] Accordingly, the whole knowledge (1) now writes

$$\mathcal{A}_t = A_t Q_t \tag{55}$$

Plugging (54) in the final good production function (7) gives

$$Y_t = (L_t^Y)^{1-\alpha} \int_0^{Q_t} A_{it}(\alpha^{\frac{1}{1-\alpha}} L_t^Y)^\alpha di = \alpha^{\frac{\alpha}{1-\alpha}} L_t^Y \int_0^{Q_t} A_{it} di = \alpha^{\frac{\alpha}{1-\alpha}} L_t^Y \mathcal{A}_t$$

Then, using (55), one gets

$$Y_t = \alpha^{\frac{\alpha}{1-\alpha}} L_t^Y A_t Q_t \text{ and thus } g_{Y_t} = g_{L_t^Y} + g_{A_t} + g_{Q_t} \tag{56}$$

Plugging (54) in the final good resource constraint (10), one obtains $Y_t = Lc_t + \int_0^{Q_t} A_{it} \alpha^{\frac{1}{1-\alpha}} L_t^Y di = Lc_t + \alpha^{\frac{1}{1-\alpha}} L_t^Y \mathcal{A}_t$. Dividing both sides of this expression by $Y_t$, using (55) and the expression of $Y_t$ obtained in (56), one obtains $1 = \frac{Lc_t}{Y_t} + \frac{\alpha^{\frac{1}{1-\alpha}} L_t^Y A_t Q_t}{\alpha^{\frac{\alpha}{1-\alpha}} L_t^Y A_t Q_t}$. Thus one has $\frac{Lc_t}{Y_t} = 1 - \alpha$, and thus

$$g_{c_t} = g_{Y_t} \tag{57}$$

Besides, under symmetry, the law of motion of applied knowledge (2), the labor constraint (9) and the first-order conditions (48), (49), (50), (52), (53) can be rewritten as follows

$$\dot{A}_{it} = \dot{A}_t = \lambda\sigma l_t^A A_t Q_t \tag{58}$$

$$L = L_t^Y + l_t^A Q_t + L_t^Q \tag{59}$$

$$\iota_t (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} A_t Q_t = \mu_t \tag{60}$$

$$\nu_t^A \lambda\sigma A_t Q_t = \mu_t \tag{61}$$

$$\nu_t^Q \delta Q_t = \mu_t \tag{62}$$

$$\frac{\iota_t}{\nu_t^A} (1-\alpha)\, \alpha^{\frac{\alpha}{1-\alpha}} L_t^Y + \lambda\sigma l_t^A Q_t = -\frac{\dot{\nu}_t^A}{\nu_t^A} = -g_{\nu_t^A} \tag{63}$$

[15] As detailed in the comments of Lemma 3, considering symmetry across sectors is standard in endogenous growth models as it is often necessary to obtain closed form solutions. In our model, this assumption is necessary to compute the optimum of the model but not to characterize the equilibrium.

$$\frac{\iota_t}{\nu_t^Q}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}L_t^Y A_t - \frac{\mu_t}{\nu_t^Q}l_t^A + 2\frac{\nu_t^A}{\nu_t^Q}\lambda\sigma l_t^A A_t Q_t + \delta L_t^Q = -\frac{\dot{\nu}_t^Q}{\nu_t^Q} = -g_{\nu_t^Q} \tag{64}$$

From (3), one gets the growth rate of fundamental knowledge in the economy

$$g_{Q_t} = \delta L_t^Q \tag{65}$$

From (58), one gets the growth rate of applied knowledge in any sector $i$:

$$g_{A_{it}} = g_{A_t} = \lambda\sigma l_t^A Q_t, i \in [0, Q_t] \tag{66}$$

From (60) and (61), one gets $\frac{\iota_t}{\nu_t^A}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} = \lambda\sigma$. Then, replacing in (63), one obtains

$$\lambda\sigma\left(L_t^Y + l_t^A Q_t\right) = -g_{\nu_t^A} \tag{67}$$

From (60) and (62), one has $\frac{\iota_t}{\nu_t^Q}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}A_t = \delta$; from (61) and (62), one has $\frac{\nu_t^A}{\nu_t^Q}\lambda\sigma A_t = \delta$; and from (62), one has $\frac{\mu_t}{\nu_t^Q} = \delta Q_t$. Replacing in (64), one gets $\delta\left(L_t^Y + l_t^A Q_t + L_t^Q\right) = -g_{\nu_t^Q}$; then, using (59), one has

$$\delta L = -g_{\nu_t^Q} \tag{68}$$

Log-differentiating with respect to time (47), (55), (60), (61) and (62), one gets

$$-g_{c_t} - \rho = g_{\iota_t} \tag{69}$$

$$g_{\mathcal{A}_t} = g_{A_t} + g_{Q_t} \tag{70}$$

$$g_{\iota_t} + g_{A_t} + g_{Q_t} = g_{\mu_t} \tag{71}$$

$$g_{\nu_t^A} + g_{A_t} + g_{Q_t} = g_{\mu_t} \tag{72}$$

$$g_{\nu_t^Q} + g_{Q_t} = g_{\mu_t} \tag{73}$$

Using (67), (69), (71) and (72), one has

$$g_{c_t} + \rho = -g_{\iota_t} = g_{A_t} + g_{Q_t} - g_{\mu_t} = -g_{\nu_t^A} = \lambda\sigma\left(L_t^Y + l_t^A Q_t\right) \tag{74}$$

From (66), (68), (69), (71) and (73), one gets

$$g_{c_t} + \rho = -g_{\iota_t} = g_{A_t} + g_{Q_t} - g_{\mu_t} = g_{A_t} - g_{\nu_t^Q} = \lambda\sigma l_t^A Q_t + \delta L \tag{75}$$

From (74) and (75), one obtains the optimal quantity of labor in the final good sector:

$$L_t^{Yo} = L^{Yo} = \frac{\delta L}{\lambda\sigma}, \forall t \text{ and thus } g^o_{L_t^Y} = 0 \tag{76}$$

Consequently, from (56), (57), (65) and (66), one obtains:

$$g_{c_t} = g_{Y_t} = g_{A_t} + g_{Q_t} = \lambda\sigma l_t^A Q_t + \delta L_t^Q \tag{77}$$

From (75) and (77), one obtains the optimal quantity of labor in fundamental research:

$$L^{Qo} = L - \frac{\rho}{\delta} \tag{78}$$

From (59), (76) and (78), one obtains the optimal quantity of labor in all applied R&D activities:

$$L^{Ao} = l_t^{Ao} Q_t^o = \frac{\rho}{\delta} - \frac{\delta L}{\lambda\sigma} \tag{79}$$

Finally, the first-best social optimum is characterized as follows:

**Optimal partition of labor:** $L^{Yo} = \frac{\delta L}{\lambda\sigma}$, $L^{Ao} = l_t^{Ao} Q_t^o = \frac{\rho}{\delta} - \frac{\delta L}{\lambda\sigma}$ and $L^{Qo} = L - \frac{\rho}{\delta}$

**Optimal quantity of intermediate goods:** $x_i^o = x^o = \alpha^{\frac{1}{1-\alpha}} L^{Yo}, \forall i \in [0, Q_t^o]$

**Optimal growth rates:** $g^o_{A_t} = \lambda\sigma L^{Ao}$, $g^o_{Q_t} = \delta L^{Qo}$, $g_{\mathcal{A}_t} = g^o_{A_t} + g^o_{Q_t}$ and $g^o_{c_t} = g^o_{Y_t} = g^o_{A_t} + g^o_{Q_t}$

This proves Proposition 1. □

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