Multilateral & Costly Linkages between Emissions Trading Systems

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Abstract

Linkages between Emissions Trading Systems (ETSs) are crucial for ensuring cost-effectiveness in the fragmented global climate policy landscape engendered by the Paris Agreement. Research has hitherto focused on the simpler case of bilateral linkages, in part because a rigorous analysis of multilateral linkages poses significant challenges. We propose a language and a theoretical model that allow us to describe and study multilateral linkages between ETSs under uncertainty analytically. We show how every multilateral linkage can be decomposed into its internal bilateral linkages, and demonstrate that linkage is superadditive. We provide a formula for the gains from linkage coalitions of ETSs as a function of the constituent coalitions' sizes and shock characteristics. While the global market is socially efficient, we show that it may not be the most preferred outcome for individual jurisdictions, even in the absence of linkage costs. When we introduce linkage costs which are increasing in both the number and aggregate size of partnering jurisdictions we find that efficient linkage coalition structures may differ from the global market. Finally, we study several alternative cost-sharing rules to check if they render the globally efficient coalition structure a Pareto improvement with respect to autarky. We find that several intuitively appealing rules do not meet this criterion for individual jurisdictions. We demonstrate our theoretical results using a calibrated quantitative example.

Keywords: Emissions Trading, Bilateral linkage, Multilateral linkage, Global Carbon Market, Climate Change.

JEL Classification codes: Q58, H23.

1 Introduction

Markets for emission permits have long been an important climate policy tool in regulating greenhouse gas emissions. A patchwork of jurisdictional emissions trading systems (ETSs) tailored to local circumstances and specific constraints has emerged recently. ETSs are in use in Europe, Switzerland, South Korea, seven Chinese provinces and cities, and several US states and Canadian provinces among other places (ICAP, 2017). More are in the pipeline with China, the world's largest emitter, planning to start a national market in 2017.

Systems integration will be a significant element of the global climate change policy framework in the future (Bodansky et al., 2016). The market provisions contained in Article 6 of the Paris Agreement, adopted by the UN in December 2015, encourage the voluntary integration of emission reduction efforts. Linkages between jurisdictional ETSs is one way this can be done and would generate economic benefits by spreading abatement efforts cost effectively among the participating systems, ultimately generating a uniform linking price. In fact, some jurisdictions are already linked (California and Québec), will link in the near future having completed the required negotiations (Europe and Switzerland), or are contemplating a link with an existing system (Ontario with California and Québec). Against this backdrop, conventional intuition suggests that the *global market* is the most desirable *coalition structure* from a global perspective.[1,2] However, it is not a forgone conclusion that the globally linked market will be adopted when viewed from the perspective of a single jurisdiction, as attested by the few and far between instances of linkage. Moreover, in the presence of costs associated with the formation of linked systems, even the global market may not be globally efficient let alone incentive compatible under alternative cost-sharing arrangements.

Current research examining the determinants of the benefits of linking ETSs has primarily focused on *bilateral linkage*.[3] Other theoretical contributions investigate the effects of forming a global market, e.g. Holtsmark & Midttømme (2015) and Caillaud & Demange (2016) from different perspectives. Using a computable general equilibrium model, Carbone et al. (2009) consider the formation of a single coalition of linked ETSs with endogenous selection of non-cooperative emissions caps. Heitzig (2013) numerically explores the dynamic process

[1] All items in *italics* are formally defined later in the text.

[2] A coalition structure is a partition of the set of jurisdictions. Given the set of five jurisdictions in this paragraph, a possible coalition structure comprises two disjoint coalitions of linked markets, e.g. Europe & Switzerland and California & Québec with Ontario's system under autarky. The global market is the structure consisting of one coalition containing all five jurisdictions.

[3] A fast-growing literature explores economic and political motivations of linking two jurisdictions; see Rehdanz & Tol (2005), Flachsland et al. (2009), Jaffe et al. (2009), Mehling & Haites (2009), Tuerk et al. (2009), Burtraw et al. (2013), Ranson & Stavins (2016) and Doda & Taschini (2016), among others.

of formation of coalitions of linked ETSs where jurisdictions have the possibility to coordinate on emissions cap selection. These last two contributions, however, do not characterize *multilateral linkage* analytically nor do they investigate the determinants of coalition structures. Compared to bilateral linkages, a formal study of multilateral linkages poses numerous challenges, as discussed in Mehling & Görlach (2016), who propose different options for a successful management of these linkages.

In this paper we propose a language and a general theoretical model that allow us to describe and study multilateral linkage between ETSs. We find that any multilateral linkage can be decomposed into its internal bilateral linkages. Second, we show that linkage is superadditive, i.e. the aggregate expected gains from the union of disjoint coalitions of linked ETSs is no less than the sum of separate coalitions' expected gains. Third, we provide an analytical formula for this economic gain as a function of coalitions' sizes and shock characteristics, generalizing the results in Doda & Taschini (2016).

Such a formal approach is useful since the results for bilateral linkages in Doda & Taschini (2016) do not translate easily to multilateral linkages. In a bilateral setting linkages with larger systems are more beneficial, all else constant. In addition, a jurisdiction prefers the permit demand in its partner's market to be variable and weakly correlated with its own.

To build intuition, consider the special case with three jurisdictions where the variance of the shocks affecting each jurisdiction is identical and two jurisdictions have the same size. Let the third jurisdiction be larger. When evaluating possible linkages, the larger jurisdiction has little incentive to link exclusively with a single smaller jurisdiction. Instead, it prefers to be part of the trilaterally linked market. Conversely, because in bilateral links smaller jurisdictions tend to benefit the most, they prefer a bilateral linkage with the larger jurisdiction. In other words, the larger jurisdiction prefers a trilateral linkage whereas the two smaller jurisdictions prefer a bilateral linkage with the larger jurisdiction.[4]

In general, the identification of the outcome of multilateral linkage is not clear when one moves away from special cases. Moreover, the number of possible coalition structures increases exponentially with the number of jurisdictions. For example, with four jurisdictions there are six possible bilateral linkages (with two jurisdictions in autarky), three groups of two bilateral linkages, four trilateral linkages (with one jurisdiction in autarky) and one four-jurisdiction

[4] Similarly, consider three jurisdictions with the same size where one jurisdiction, called NEG, is negatively correlated with the other two jurisdictions, called POS_1 and POS_2, which are positively correlated with each other. When evaluating possible linkages, jurisdiction NEG has little incentive to link bilaterally with POS_1 or POS_2 alone and prefers trilateral linkage. Conversely, since the gains from the link $\{\text{POS}_1,\text{POS}_2\}$ are smaller, POS_1 and POS_2 prefer a bilateral linkage with jurisdiction NEG exclusively.

linkage. That is, combined with *complete autarky* where each system operates independently, we have 15 coalition structures in total. With 10 jurisdictions, there are already 115,975 possible coalition structures. Since we can decompose any multilateral linkage into its internal bilateral linkages, our model can in principle handle and characterize the aggregate and jurisdiction-specific gains in all possible coalition and coalition structures that are generated by any number of jurisdictions.

Empirically, the rare instances of linkages that have occurred so far did so on a bilateral basis between (*i*) jurisdictions with aligned ETSs and thus relatively low linkage costs (California and Québec); (*ii*) one small jurisdiction wishing to join a much larger system, the former thus bearing all the costs associated with the link (Europe and Norway). We take explicit account of this observation in our model and study the effects of both the introduction of linkage costs and of alternative cost-sharing arrangements between jurisdictions. This is our second novel contribution to the literature on linking. Formally, linkage costs have two varaible components: implementation costs that are higher the larger the jurisdictions involved and negotiation costs that are higher the larger the number of partnering jurisdictions. The magnitude of linkage costs is thus endogenous to linkage coalition formation. This reflects the observation that (*i*) it is more costly for large jurisdictions to implement linkage; (*ii*) the larger the number of jurisdictions sitting at the negotiation table, the more difficult to find a compromise (Keohane & Victor, 2016). These considerations have given rise to concepts such as minilateralism (Falkner, 2016) or polycentrism (Ostrom, 2009).

In the presence of costs associated with the formation of linkage coalitions, non-degenerate coalition structures different from the global market may yield the higher aggregate gains net of costs. In particular, such coalition structures may feature some jurisdictions that remain in autarky, unlinked, which we refer to as *incomplete linkage* as well as coexisting linkage coalitions, which we refer to as *polycentric linkage*. In this paper, we take the perspective of a social planner in investigating (*i*) the nature and determinants of efficient linkage coalition structures with linkage costs; (*ii*) how these costs can be shared among participants so that we obtain a Pareto improvement with respect to complete autarky.

Notice that our paper explores the effects of linking under uncertainty by introducing idiosyncratic shocks in each jurisdiction. It is the interaction of these shocks that generates the linkage-related effects on jurisdictional emission levels and welfares. We adopt a combinatorial approach to the analysis of interjurisdictional links to isolate these effects. To this end we assume there is no strategic interaction between cap selection *and* linking decisions of jurisdictions. More specifically, caps are selected non-cooperatively under complete autarky first, and are maintained when considering the merits of linking. This is deliberate because our aim is to understand the determinants of the gains from multilateral linkage and to be able to characterize them analytically.

With invariant caps and in the absence of linkage costs, jurisdictions are always better off in any linkage coalition than under autarky. However, they gain more in some coalitions than others, and our analysis allows us to rank alternative coalitions from a given jurisdiction's perspective. Moreover, linkage costs and various cost-sharing arrangements can be readily incorporated in our model so that jurisdictions can choose among linkage coalitions even in the presence of costs. We argue that accounting for costs associated with linkage brings realism and can address interjurisdictional equity concerns by identifying cost-sharing arrangements that implement the socially efficient outcome. In a world where permit or cash transfers can run into significant political-economy obstacles, this has important practical relevance.

In doing so, our paper deviates from the literature on self-enforcing international environmental agreements (IEA) initiated by Carraro & Siniscalco (1993) and Barrett (1994) in three fundamental ways. First, most of this literature studies a Cartel game where only one single coalition can form and sets aside the question of multiple coalitions. It typically assumes that coalition members choose their emission caps cooperatively (the coalition is a metaplayer).[5] Helm (2003) also identifies the perverse incentives on cap selection that anticipation of linkage can have. Second, we abstract from coalition stability considerations. In general, the literature finds somewhat pessimistic results regarding the size of stable coalitions and identifies a trade-off between efficiency and stability. We also note that the different coalition membership rules and equilibrium concepts in the literature lead to different predictions regarding stability.[6] Note that a recent contribution by Caparrós & Péreau (2017) shows that a sequential negotiation process always leads to the grand coalition even when it is not stable in a multilateral (one-shot) negotiation stage. Third, while we acknowledge that transfers

[5]Absent uncertainty, however, interjurisdictional emissions trading has no effect on the overall emissions level as the effort sharing is already efficient from the coalition's perspective. There are notable exceptions including Finus & Maus (2008) and Carbone et al. (2009). The latter paper, in particular, considers endogenous non-cooperative cap-setting by coalition members so that emissions trading matters in their model.

[6]For instance, Ray & Vohra (1997) study equilibrium binding agreements where coalitions can break up into smaller sub-coalitions, but not *vice versa*. Ray & Vohra (1999) consider some kind of Rubinstein-type bargaining game for coalition formation. Bloch (1995) and Bloch (1996) analyses an alternative-offers bargaining game and an infinite-horizon coalition formation game, respectively, both requiring unanimity for a coalition to form. Yi (1997) considers alternative coalition membership rules, e.g. open membership, unanimity and equilibrium bindingness. In the climate context, Osmani & Tol (2009) analyse farsightedly stable linkage coalitions in the sense of Chwe (1994). Finally, Konishi & Ray (2003) consider a dynamic coalition formation process with farsighted players. With a similar sequential linking process, Heitzig (2013) allows for coalition members to simply link markets but also coordinate on cap selection.

can increase both participation in and stability of linkage coalitions (Nagashima et al., 2009; Lessmann et al., 2015), we approach transfers via alternative linkage cost-sharing rules rather than via alternative permit allocation rules (Altamirano-Cabrera & Finus, 2006).[7]

The paper is organized as follows. Section 2 introduces the model and defines jurisdictional and aggregate gross gains from bilateral linkages. These constitute the basic elements of our subsequent analysis of multilateral linkage. Section 3 introduces the language used in describing multilateral linkage and proposes a general model to study and characterize possible coalition structures. Linkage costs, *globally efficient coalition structure* they induce and several cost-sharing arrangements are introduced in Section 4. The quantitative illustration is in Section 5. Section 6 concludes. An appendix contains the derivations and proofs. All numbered tables and figures are provided at the end.

2 The modelling framework

We consider a standard static model of a perfectly competitive emission permit market that specialises Weitzman (1974) and Yohe (1976) to the sole case of quantity-based policies designed to regulate uniformly mixed pollution in several jurisdictions with independent regulatory authorities. In the model we assume separability between the market for permits and markets for other goods and services. That is, we conduct a partial-equilibrium analysis focusing exclusively on the jurisdictions' regulated emissions and abstract from interactions with the rest of the economy. Second, we assume that the only uncertainty is in the form of additive shocks affecting the jurisdictions' unregulated levels of emissions. These assumptions are somewhat restrictive but relatively standard in the literature. Third, we represent jurisdictions' benefits and damages assuming quadratic functional forms. This is standard, allows for derivation of analytical results and can be viewed as a local approximation of more general functional specifications (Newell & Stavins, 2003). Fourth, the international political economy dimension is omitted. Each jurisdiction has a regulatory authority who can design policies independently of authorities in other jurisdictions with no anticipation of linkage. The model is solved under risk-neutrality.[8]

Jurisdictions. There are n jurisdictions and $\mathcal{I} = \{1, \ldots, n\}$ denotes the set of jurisdictions.

[7] In this respect, note that the globally efficient coalition structure defined in Section 4 is potentially internally stable in the sense of Carraro et al. (2006).

[8] As in Doda & Taschini (2016), risk aversion does not alter our results.

Total benefits from emissions in jurisdiction $i \in \mathcal{I}$ are a function of the level of emissions $q_i \geq 0$ and are subject to jurisdiction-specific shocks θ_i such that, $\forall i \in \mathcal{I}$

$$B_i(q_i;\theta_i) = b_0 + (b_1 + \theta_i)q_i - \frac{b_2}{2\psi_i}q_i^2, \text{ with } b_0, b_1, b_2 \geq 0, \tag{1}$$

where ψ_i may alternatively characterize jurisdiction i's size or abatement technology. A high ψ_i may represent a jurisdiction whose size of regulated emissions is high. To see this, fix $\theta_i = 0$ for all $i \in \mathcal{I}$, i.e. jurisdictions are identical up to the parameter ψ_i's. Then, jurisdiction i's optimal emission level corresponding to an arbitrary permit price $p \in (0; b_1]$ is $q_i^*(p) = \psi_i(b_1 - p)$ and it is proportional to ψ_i. There is an alternative interpretation. The ratio $\frac{b_2}{\psi_i}$ controls the slope of jurisdiction i's linear marginal abatement cost schedule. Hence, a high ψ_i may also represent a jurisdiction who has access to low-cost abatement opportunities at the margin. In what follows we will refer to ψ_i as the size parameter.

For analytical convenience, we assume that jurisdiction-specific shocks are mean-zero and have constant variance.[9] They may be correlated across jurisdictions, i.e. $\forall (i,j) \in \mathcal{I} \times \mathcal{I}_{-i}$

$$\mathbb{E}\{\theta_i\} = 0, \ \mathbb{V}\{\theta_i\} = \sigma_i^2, \text{ and } \mathrm{Cov}\{\theta_i, \theta_j\} = \rho_{ij}\sigma_i\sigma_j \text{ with } \rho_{ij} \in [-1; 1]. \tag{2}$$

Jurisdiction-specific shocks are limited to the intercepts of the marginal benefit schedules. These shocks capture the net effect of stochastic factors that may influence emissions and their associated benefits, e.g. business cycle and technology shocks, jurisdiction-specific events, changes in the price of factors of production, weather fluctuations, etc. To see this, note that absent regulation, jurisdictions emit up to their baseline emissions $(\bar{q}_i)_{i \in \mathcal{I}}$ such that, $\forall i \in \mathcal{I}$

$$\bar{q}_i = \frac{\psi_i}{b_2}(b_1 + \theta_i). \tag{3}$$

For instance, a positive shock realization $\theta_i > 0$ could be seen as a favourable productivity shock that increases benefits from emissions, and correspondingly, baseline emissions levels. In particular, we assume that $\theta_i > -b_1$ for every jurisdiction and every possible shock realization. This guarantees positive baseline emissions and thus that the emission regulation problem at hand is non-trivial.

Because we are considering the case of a uniformly-mixed stock pollutant, environmental damages are a function of aggregate emissions $Q = \sum_{i \in \mathcal{I}} q_i$. For simplicity, we assume that

[9] Considering non mean-zero shocks does not alter our results.

each jurisdiction incurs the same damages from pollution

$$D(Q) = d_0 + d_1 Q + \frac{d_2}{2} Q^2, \text{ with } d_0, d_1, d_2 \geq 0. \tag{4}$$

In sum, jurisdictions are identical up to size and shock.

Cap selection. We assume that risk-neutral jurisdictions do not anticipate linkage and that jurisdictional caps on emissions $(\omega_i)_{i\in\mathcal{I}}$ are set non-cooperatively. That is, jurisdiction $i \in \mathcal{I}$ maximizes its net expected benefits operating its quantity regime under autarky, taking other jurisdictions' cap levels $\Omega_{-i} = \sum_{j\in\mathcal{I}_{-i}} \omega_j$ as given. The Cournot-Nash jurisdictional caps thus satisfy, $\forall i \in \mathcal{I}$

$$\omega_i \doteq \arg\max_{\omega_i \geq 0} \mathbb{E}\Big\{ B_i(\omega_i;\theta_i) - D\left(\omega_i + \Omega_{-i}\right) \Big\}. \tag{5}$$

In particular, jurisdictional caps are proportional to jurisdictional size, such that, $\forall i \in \mathcal{I}$

$$\omega_i = A_1 \cdot \psi_i, \text{ where } A_1 = \frac{b_1 - d_1}{b_2 + d_2 \Psi_{\mathcal{I}}} > 0 \tag{6}$$

where ω_i measures the non-cooperative abatement effort (we assume $b_1 > d_1$) and $\Psi_{\mathcal{I}} = \sum_{i\in\mathcal{I}} \psi_i$ is the aggregate jurisdictional size.[10] Notice that the aggregate cap corresponds to $\Omega = A_1 \cdot \Psi_{\mathcal{I}}$. To facilitate the comparison of outcomes under autarky and linkage, we assume that jurisdictional caps are upheld in both cases, i.e. fixed once and for all, and not part of the linkage negotiation process.[11] Controlling for aggregate emission levels and associated environmental damages allow us to isolate the pure gains from linkage under uncertainty.

The following only considers interior equilibria and below we describe two types of interior equilibria that will serve as references throughout.[12]

Autarkic equilibria. Under autarky, each jurisdiction must comply with its domestic cap ω_i. When jurisdiction i's domestic cap is binding, i.e. when $\theta_i \geq \frac{b_2}{\psi_i}\omega_i - b_1$, the autarkic permit

[10] Appendix C provides the derivations of jurisdictional caps and further shows that our results are unaltered under other cap selection mechanisms (provided that expected jurisdictional autarkic prices are equal across jurisdictions). Appendices are ordered with respect to the natural order of the proofs in the main text.

[11] The effects that the anticipation of linkage can have on cap selection and linkage profitability, first highlighted by Helm (2003), are discussed and illustrated in Appendix C. In game-theoretic terms, we thus consider a game with no spillovers.

[12] By solely considering interior equilibria, our modelling framework is consistent with the standard approach to comparing price and quantity instruments. See Goodkind & Coggins (2015) for extensions accounting for possible corner solutions. Under autarky for instance, when i's domestic cap happens to be slack, i's autarkic permit price is zero and i's emissions level is $\bar{q}_i$.

price in jurisdiction i is $\bar{p}_i = b_1 - b_2 A_1 + \theta_i$. Notice that all jurisdictions face the same expected autarkic permit price $\bar{p} = b_1 - b_2 A_1$. Jurisdictions with positive (resp. negative) shock realizations will face an autarkic price higher (resp. lower) than $\bar{p}$. When jurisdictional autarkic prices differ, the aggregate abatement effort is not efficiently apportioned among jurisdictions. In particular, overall abatement cost-efficiency could be improved upon by allowing some share of the overall abatement effort to be reallocated from relatively high-shock to relatively low-shock jurisdictions. As shown below, linkage is beneficial for all parties as it eliminates the price difference by allowing interjurisdictional reallocation of abatement efforts.

Bilateral linkage equilibria. Bilateral linkages between two jurisdictions constitute the very basic elements of our subsequent analysis of multilateral linkage. Without loss of generality, consider a bilateral link between jurisdictions i and j in $\mathcal{I} \times \mathcal{I}_{-i}$ and call it $\{i, j\}$-linkage. An interior $\{i, j\}$-linkage equilibrium consists of the triple $(p_{\{i,j\}}, q_{\{i,j\},i}, q_{\{i,j\},j})$ where $p_{\{i,j\}}$ is the equilibrium price for fungible permits on the linked market $\{i, j\}$ and $q_{\{i,j\},i}$ denotes equilibrium emission levels in jurisdiction i, and vice versa for j. In particular, the interior $\{i, j\}$-linkage equilibrium price satisfies

$$p_{\{i,j\}} = b_1 - b_2 A_1 + \Theta_{\{i,j\}} = \frac{\psi_i \bar{p}_i + \psi_j \bar{p}_j}{\psi_i + \psi_j}, \text{ where } \Theta_{\{i,j\}} = \frac{\psi_i \theta_i + \psi_j \theta_j}{\psi_i + \psi_j} \tag{7}$$

is the size-averaged shock affecting the linked system $\{i, j\}$. Notice, the linked permit price is the size-weighted average of jurisdictional autarkic prices. In any linkage, $p_{\{i,j\}}$ is therefore closer to the autarkic price of the relatively bigger jurisdiction. The reallocation in abatement efforts consecutive to $\{i, j\}$-linkage are such that jurisdictional marginal benefits are equalized and the aggregate constraint on emissions $\Omega_{\{i,j\}} = \omega_i + \omega_j$ is met. In particular, net jurisdictional demands for permits under $\{i, j\}$-linkage are such that

$$\{i, j\}\text{-linkage:} \begin{cases} q_{\{i,j\},i} - \omega_i = \dfrac{\psi_i}{b_2}(\theta_i - \Theta_{\{i,j\}}) = \dfrac{\psi_i \psi_j}{b_2 (\psi_i + \psi_j)}(\theta_i - \theta_j), \\ q_{\{i,j\},j} - \omega_j = \dfrac{\psi_j}{b_2}(\theta_j - \Theta_{\{i,j\}}) = \dfrac{\psi_i \psi_j}{b_2 (\psi_i + \psi_j)}(\theta_j - \theta_i). \end{cases} \tag{8}$$

Linkage eliminates the post-shock wedge between autarkic prices, the magnitude of which is measured by $|\theta_i - \theta_j|$. In particular, for given shock realizations, the high-shock jurisdiction will 'import' permits since these have higher value there relative to the low shock jurisdiction 'exporting' the permits. In essence, bilateral linkage increases the effective cap in the high-shock jurisdiction and reduces that of the low-shock jurisdiction by the same amount, thereby leaving the aggregate emissions cap $\Omega_{\{i,j\}}$ unchanged.

In this framework, the difference between jurisdictional net benefits under $\{i,j\}$-linkage minus the net benefits under autarky corresponds to the jurisdictional gross gains from the bilateral link. We denote these as $\delta_{\{i,j\},i}$ and $\delta_{\{i,j\},j}$. As shown in Appendix A.2

$$\delta_{\{i,j\},i} = \frac{b_2}{2\psi_i}(q_{\{i,j\},i} - \omega_i)^2, \text{ and } \delta_{\{i,j\},j} = \frac{b_2}{2\psi_j}(q_{\{i,j\},j} - \omega_j)^2, \tag{9}$$

and further plugging in Equation (8) gives

$$\delta_{\{i,j\},i} = \frac{\psi_i \psi_j^2}{2b_2(\psi_i + \psi_j)^2}(\theta_i - \theta_j)^2, \text{ and } \delta_{\{i,j\},j} = \frac{\psi_j \psi_i^2}{2b_2(\psi_i + \psi_j)^2}(\theta_i - \theta_j)^2. \tag{10}$$

Aggregate gross gains from $\{i,j\}$-linkage amounts to

$$\Delta_{\{i,j\}} \doteq \delta_{\{i,j\},i} + \delta_{\{i,j\},j} = \frac{\psi_i \psi_j}{2b_2(\psi_i + \psi_j)}(\theta_i - \theta_j)^2. \tag{11}$$

Taking expectations then yields

$$\mathbb{E}\{\Delta_{\{i,j\}}\} = \frac{\psi_i \psi_j}{2b_2\left(\psi_i + \psi_j\right)}\left(\sigma_i^2 + \sigma_j^2 - 2\rho_{ij}\sigma_i\sigma_j\right) \geq 0. \tag{12}$$

We observe that the aggregate gross gain is (*i*) positive as long as jurisdictional shocks are imperfectly correlated and jurisdictional volatility levels differ, for otherwise the two jurisdictions are identical in our framework and there are no gains from linkage,[13] (*ii*) increasing in both jurisdictional volatilities and sizes, (*iii*) decreasing in interjurisdictional correlation and bigger when jurisdictional shocks are negatively correlated, and (*iv*), for a given aggregate size, maximal when jurisdictions have equal sizes.

We also observe that the aggregate gross gain is apportioned between partnering jurisdictions in inverse proportion to size. Formally, $\mathbb{E}\{\delta_{\{i,j\},i}\}/\mathbb{E}\{\delta_{\{i,j\},j}\} = \psi_j/\psi_i$. This is so because the distance between the autarkic price and the linkage price is relatively larger in the smaller jurisdiction. Considering the alternative interpretation where ψ_i captures jurisdiction i's abatement technology level, the jurisdiction with a high-cost abatement technology gains relatively more from the link.

[13] Notice, when jurisdictional shocks are perfectly correlated, differences in the shocks volatility generate gains from linkage. Then, the bigger the difference in volatility levels, the bigger the gains from the link.

3 Multilateral linkage

3.1 The language of multilateral linkage

Let $\mathbf{C}$ denote the set of possible linkage coalitions in $\mathcal{I}$, where a linkage coalition is defined as any subset of $\mathcal{I}$ with cardinality of at least 2; singletons are not linkage coalitions. Formally, $\mathbf{C} \doteq \{\mathcal{C} : \mathcal{C} \subseteq \mathcal{I}, |\mathcal{C}| \geq 2\}$ with cardinality $|\mathbf{C}| = 2^n - n - 1$. Let also $\mathbf{S}$ be the set of coalition structures, where a coalition structure is defined as a partition of $\mathcal{I}$.[14] That is, $\mathcal{S}$ is a coalition structure i.f.f. $\varnothing \notin \mathcal{S}$, $\cup_{\mathcal{C}\in\mathcal{S}}\mathcal{C} = \mathcal{I}$, and $\forall(\mathcal{C}, \mathcal{C}') \in \mathcal{S} \times \mathcal{S}_{-\mathcal{C}}, \mathcal{C} \cap \mathcal{C}' = \varnothing$.

For instance, with a set of three jurisdictions $\mathcal{I} = \{i, j, k\}$, there are 5 different coalition structures:

$$\underbrace{\{\{i\}, \{j\}, \{k\}\}}_{\text{complete autarky}}\ ,\quad \underbrace{\{\{i, j, k\}\}}_{\text{global market}}$$

and

$$\underbrace{\{\{i, j\}, \{k\}\},\ \{\{i, k\}, \{j\}\},\ \{\{j, k\}, \{i\}\}}_{\text{incomplete linkages}}.$$

The first and second coalition structures are the complete autarky and the global market, respectively. Coalition structures in which there are singletons, i.e. some jurisdictions remain in autarky, are referred to as incomplete linkage, e.g. $\{\{i, k\}, \{j\}\}$.

With a group of four jurisdictions $\mathcal{I} = \{i, j, k, l\}$, richer variation in coalition structures emerge consisting of multiple linkage coalitions, e.g. $\{\{i, j\}, \{k, l\}\}$. Coalition structures in which linkage coalitions coexist are referred to as *polycentric* structures. Notice that polycentric structures may also contain singletons and therefore exhibit incomplete linkage.

Further, let $\mathbf{S}_i$ denote the set of coalition structures containing exactly $i \leq n$ coalitions, whose cardinality is given by the Stirling number of the second kind $\left\{ {n \atop i} \right\}$. The cardinality of $\mathbf{S}$ is thus given by the n^{th} Bell number given n agents, that is

$$|\mathbf{S}| \doteq \sum_{i=1}^{n} |\mathbf{S}_i| = \sum_{i=1}^{n} \left\{ {n \atop i} \right\} = \sum_{i=1}^{n} \frac{1}{i!} \sum_{j=0}^{i} (-1)^{i-j} \binom{i}{j} j^n. \tag{13}$$

As shown in Table 1, the difference in the number of possible linkage coalitions and coalition

[14] For the sake of expositional clarity and consistently with the language of cooperative game theory, coalition structures can only comprise disjoint coalitions. This is without loss of generality and our machinery can characterize situations where jurisdictions belong to several coalitions. In other words, this could represent an *indirect linkage* as defined in Jaffe et al. (2009) and Tuerk et al. (2009).

structures grows exponentially as the number of jurisdictions increases.

Number of jurisdictions	3	4	5	10	15
Number of linkage coalitions	4	11	26	1,013	32,752
Number of coalition structures	5	15	52	115,975	1,382,958,545

Table 1: Number of linkage coalitions and coalition structures

Equipped with this language, we next present our general model of multilateral linkage.

3.2 Multilateral linkage equilibria

For all $\mathcal{C}$ in $\mathbf{C}$, we call $\mathcal{C}$-linkage the formation of a linked market for permits between all jurisdictions in $\mathcal{C}$. By extension, $\mathcal{I}$-linkage corresponds to the global market. An interior $\mathcal{C}$-linkage equilibrium consists of the $(|\mathcal{C}|+1)$-tuple $(p_{\mathcal{C}}, (q_{\mathcal{C},i})_{i\in\mathcal{C}})$, where $p_{\mathcal{C}}$ is the equilibrium price in the linked market and $q_{\mathcal{C},i}$ denotes jurisdiction i's equilibrium emissions level. The equilibrium is fully characterized by the equalization of marginal benefits across partnering jurisdictions (to the $\mathcal{C}$-linkage equilibrium price) and the linked market clearing condition such that, $\forall i \in \mathcal{C}$

$$b_1 + \theta_i - \frac{b_2}{\psi_i} q_{\mathcal{C},i} = p_{\mathcal{C}}, \text{ and } \sum_{i\in\mathcal{C}} q_{\mathcal{C},i} = \Omega_{\mathcal{C}} \doteq A_1 \cdot \Psi_{\mathcal{C}}. \tag{14}$$

After rearranging, the $\mathcal{C}$-linkage equilibrium price can be expressed as the size-weighted average of jurisdictional autarkic prices, that is

$$p_{\mathcal{C}} = b_1 - b_2 A_1 + \Theta_{\mathcal{C}} = \Psi_{\mathcal{C}}^{-1} \sum_{i\in\mathcal{C}} \psi_i \bar{p}_i, \text{ with } \Theta_{\mathcal{C}} \doteq \Psi_{\mathcal{C}}^{-1} \sum_{i\in\mathcal{C}} \psi_i \theta_i. \tag{15}$$

Relative to jurisdictional caps, post-trade deviations in jurisdictional emissions (or jurisdictional net demands for permits) are proportional to both jurisdictional size and the difference between the jurisdictional autarkic price and the prevailing linkage price so that, $\forall i \in \mathcal{C}$

$$q_{\mathcal{C},i} - \omega_i = \frac{\psi_i}{b_2}(\bar{p}_i - p_{\mathcal{C}}). \tag{16}$$

Ex post, jurisdiction i imports permits under $\mathcal{C}$-linkage i.f.f. $\bar{p}_i > p_{\mathcal{C}}$, i.e. the linkage price happens to be lower than its autarkic price – all else equal, this is equivalent to an increase in jurisdiction i's effective cap.

Relative to autarky, the gains from $\mathcal{C}$-linkage accruing to jurisdiction $i \in \mathcal{C}$ are

$$\delta_{\mathcal{C},i} = \frac{b_2}{2\psi_i}\left(q_{\mathcal{C},i} - \omega_i\right)^2 = \frac{\psi_i}{2b_2}(\bar{p}_i - p_{\mathcal{C}})^2, \tag{17}$$

and always non-negative. In other words, every partnering jurisdiction in any $\mathcal{C}$-linkage is always at least as well off as compared to autarky. Formally,

Lemma 3.1. *Under $\mathcal{C}$-linkage, the expected economic surplus accruing to jurisdiction $i \in \mathcal{C}$ is proportional to both its size and the square of the difference in autarkic and $\mathcal{C}$-linkage prices (in expectations) and given by $\mathbb{E}\{\delta_{\mathcal{C},i}\} = \frac{\psi_i}{2b_2}\mathbb{E}\Big\{(\bar{p}_i - p_{\mathcal{C}})^2\Big\}$.*

Proof. The proof is relegated to Appendix A.2. □

Controlling for jurisdictional size, jurisdiction i's gross gain from $\mathcal{C}$-linkage is hence increasing in $\mathbb{E}\left\{(\bar{p}_i - p_{\mathcal{C}})^2\right\}$ and is positive – provided that the shocks' realization is such that its autarkic price differs from the $\mathcal{C}$-linkage price.[15] Controlling for the difference between $\mathcal{C}$-linkage and i's autarkic prices, jurisdiction i's gross gain from $\mathcal{C}$-linkage is proportional to its size. Notice, as long as per-size abatement efforts of all partnering jurisdictions in $\mathcal{C}$ are identical, it holds that

$$\bar{p}_i - p_{\mathcal{C}} = \theta_i - \Theta_{\mathcal{C}}. \tag{18}$$

Inserting the above in (17) and using the definition of $\Theta_{\mathcal{C}}$, we obtain

$$\delta_{\mathcal{C},i} = \frac{\psi_i}{2b_2\Psi_{\mathcal{C}}^2}\left(\sum_{j\in\mathcal{C}_{-i}} \psi_j\left(\theta_i - \theta_j\right)\right)^2. \tag{19}$$

Expanding this and taking expectations then gives

$$\begin{aligned}\mathbb{E}\{\delta_{\mathcal{C},i}\} = \frac{\psi_i}{2b_2\Psi_{\mathcal{C}}^2}\Bigg(&\sum_{j\in\mathcal{C}_{-i}} \psi_j^2\left(\sigma_i^2 + \sigma_j^2 - 2\rho_{ij}\sigma_i\sigma_j\right)\\ &+ \sum_{(j,k)\in\mathcal{C}_{-i}\times\mathcal{C}_{-i}} \psi_j\psi_k\left(\sigma_i^2 + \rho_{jk}\sigma_j\sigma_k - \rho_{ik}\sigma_i\sigma_k - \rho_{ij}\sigma_i\sigma_j\right)\Bigg).\end{aligned} \tag{20}$$

The above expression, however, is relatively cumbersome and does not lend itself to an easy interpretation. In general, when it comes to multilateral linkage, it will be more convenient to express $\mathcal{C}$-linkage quantities as a function of its internal bilateral linkage quantities.

[15] In a recent theoretical study of the optimal scope of price and quantity policies, Caillaud & Demange (2016) observe a similar result but limit their analysis to the analog of our aggregate gross gains.

By an argument of symmetry and with the convention that for all $i \in \mathcal{I}$, $\Delta_{\{i,i\}} = 0$, Appendix A.2 shows that gains from $\mathcal{C}$-linkage accruing to jurisdiction $i \in \mathcal{C}$ write

$$\delta_{\mathcal{C},i} = \Psi_{\mathcal{C}}^{-2} \sum_{j \in \mathcal{C}_{-i}} \left\{ \Psi_{\mathcal{C}_{-i}} (\psi_i + \psi_j) \Delta_{\{i,j\}} - \frac{\psi_i}{2} \sum_{k \in \mathcal{C}_{-i}} (\psi_j + \psi_k) \Delta_{\{j,k\}} \right\}. \tag{21}$$

Summing over all $i \in \mathcal{C}$ yields our first central result

Proposition 3.2. *Any $\mathcal{C}$-linkage can be decomposed into its internal bilateral links, that is*

$$\Delta_{\mathcal{C}} \doteq \sum_{i \in \mathcal{C}} \delta_{\mathcal{C},i} = (2\Psi_{\mathcal{C}})^{-1} \sum_{(i,j) \in \mathcal{C}^2} (\psi_i + \psi_j) \Delta_{\{i,j\}}. \tag{22}$$

The number of such internal bilateral links is triangular and equals $\binom{|\mathcal{C}|+1}{2}$.

Proof. The proof is relegated to Appendix A.3. □

The aggregate surplus from $\mathcal{C}$-linkage thus writes as a size-weighted function of all surpluses from bilateral links between jurisdictions belonging to the linkage coalition $\mathcal{C}$.

Finally, we can make the following observation regarding price volatility in a $\mathcal{C}$-linkage equilibrium, which is a generalization of Proposition 2 in Doda & Taschini (2016).

Observation 1. *Any $\mathcal{C}$-linkage is conducive to reduced size-averaged permit price volatility, i.e. $\mathbb{V}\{p_{\mathcal{C}}\} \leq \Psi_{\mathcal{C}}^{-1} \sum_{i \in \mathcal{C}} \psi_i \mathbb{V}\{\bar{p}_i\}$. This unconditionally reduces volatility in relatively high-volatility jurisdictions but may increase volatility in relatively low-volatility jurisdictions, especially when the latter are relatively small.*

Proof. The proof is relegated to Appendix A.1. □

Let us now characterize the effects of linking two disjoint linkage coalitions.

3.3 Linkage between two coalitions

For all $\mathcal{C} \in \mathbf{C}$ and $\mathcal{C}' \subset \mathcal{C}$, denote by $\mathcal{C}''$ the complement of $\mathcal{C}'$ in $\mathcal{C}$, i.e. $\mathcal{C} = \mathcal{C}' \cup \mathcal{C}''$ and $\mathcal{C}' \cap \mathcal{C}'' = \varnothing$. This is without loss of generality. Unpacking Equation (22) then gives

$$\Delta_{\mathcal{C}} = \Psi_{\mathcal{C}}^{-1} \left(\Psi_{\mathcal{C}'} \Delta_{\mathcal{C}'} + \Psi_{\mathcal{C}''} \Delta_{\mathcal{C}''} + \sum_{(i,j) \in \mathcal{C}' \times \mathcal{C}''} (\psi_i + \psi_j) \Delta_{\{i,j\}} \right). \tag{23}$$

Notice, while in set theoretic terms it holds that $\mathcal{C} = \mathcal{C}' \cup \mathcal{C}''$, in terms of aggregate surplus from system interactions it holds that

$$\Delta_{\mathcal{C}} = \Delta_{\mathcal{C}'} + \Delta_{\mathcal{C}''} + \Delta_{\mathcal{C}' \leftrightsquigarrow \mathcal{C}''}, \tag{24}$$

where $\leftrightsquigarrow$ is a relation operator denoting linkage. That is, the term $\Delta_{\mathcal{C}' \leftrightsquigarrow \mathcal{C}''}$ captures the gains of connecting the two linkage coalitions $\mathcal{C}'$ and $\mathcal{C}''$. Equipped with this definition, one can state our second central result that

Proposition 3.3. *Linkage is a superadditive mechanism satisfying*

$$\mathbb{E}\{\Delta_{\mathcal{C}' \leftrightsquigarrow \mathcal{C}''}\} = \Psi_{\mathcal{C}}^{-1}\left(\sum_{(i,j)\in\mathcal{C}'\times\mathcal{C}''} (\psi_i + \psi_j)\mathbb{E}\{\Delta_{\{i,j\}}\} - \Psi_{\mathcal{C}''}\mathbb{E}\{\Delta_{\mathcal{C}'}\} - \Psi_{\mathcal{C}'}\mathbb{E}\{\Delta_{\mathcal{C}''}\}\right) \geq 0. \tag{25}$$

Proof. The proof is relegated to Appendix A.4. □

In words, the aggregate expected gross gain from the union of disjoint coalitional systems is no less than the sum of the coalitional systems' separate expected gross gains. We provide an intuitive proof for the non-negative sign of Equation (25). Crucially, it follows from the definition and beneficial nature of bilateral linkage, here considered between two groups of interconnected markets for permits, where in particular

$$\mathbb{E}\{\Delta_{\mathcal{C}' \leftrightsquigarrow \mathcal{C}''}\} = \underbrace{\frac{\Psi_{\mathcal{C}'}\Psi_{\mathcal{C}''}}{2b_2\Psi_{\mathcal{C}}}}_{\text{PSE}} \Big(\underbrace{\mathbb{V}\{p_{\mathcal{C}'}\} + \mathbb{V}\{p_{\mathcal{C}''}\}}_{\text{VE}} \underbrace{-2\text{Cov}\{p_{\mathcal{C}'}; p_{\mathcal{C}''}\}}_{\text{DE}}\Big) \geq 0. \tag{26}$$

In comparison with bilateral linkage between two jurisdictions (Doda & Taschini, 2016), the pair size effect (PSE) is of larger magnitude, especially when $\Psi_{\mathcal{C}'} \sim \Psi_{\mathcal{C}''}$. However, this is compensated by lower price volatility effect (VE) as discussed in Observation 1 above. It is more difficult to say something relevant regarding the dependency effect (DE) at this stage.

There are three crucial implications connected to Proposition 3.3 that we illustrate with the help of the following corollaries.

Corollary 3.4. *Multilateral linkage satisfies monotonicity, that is*

$$\forall(\mathcal{C}, \mathcal{C}') \in \mathbf{C}^2,\ \mathcal{C}' \subseteq \mathcal{C} \Rightarrow \mathbb{E}\{\Delta_{\mathcal{C}'}\} \leq \mathbb{E}\{\Delta_{\mathcal{C}}\}. \tag{27}$$

Therefore, $\mathcal{I}$-linkage is the coalition with highest aggregate payoff.

Proof. Follows from superadditivity and since singletons have value zero. □

In words, if one were to choose one and only one linkage coalition, $\mathcal{I}$-linkage would be the most advantageous overall. Superadditivity, in fact, provides the stronger result that

Corollary 3.5. *$\mathcal{I}$-linkage is the socially optimal coalition structure, that is* $\forall \mathcal{S} \in \mathbf{S}$

$$\mathbb{E}\{\Delta_{\mathcal{I}}\} \geq \mathbb{E}\{\Delta_{\mathcal{S}}\}. \tag{28}$$

Proof. Let $\mathcal{S} = \{\mathcal{C}_1, \ldots, \mathcal{C}_{|\mathcal{S}|}\} \in \mathbf{S}_{-\mathcal{I}}$. By definition and linearity, it holds that

$$\Delta_{\mathcal{I}} \doteq \Delta_{\mathcal{S}} + \sum_{1 \leq i < j \leq |\mathcal{S}|} \Delta_{\mathcal{C}_i \leftrightsquigarrow \mathcal{C}_j}, \text{ where } \Delta_{\mathcal{S}} = \sum_{i=1}^{|\mathcal{S}|} \Delta_{\mathcal{C}_i}. \tag{29}$$

Taking expectations then concludes. □

Therefore, $\mathcal{I}$-linkage is the coalition structure conducive to the highest aggregate cost savings in meeting the aggregate constraint on emissions Ω. In words, from a global perspective a single linkage coalition consisting of all jurisdictions linked together outperforms any possible group of disjoint linkage coalitions, and is intimately related to the globally efficient coalition structure we define later.[16]

It is also of interest to characterize the special case where a linkage coalition is linked to an individual jurisdiction (singleton) as it clarifies the distribution of the overall gross gains from the link between partnering jurisdictions. In this respect, one has that

Corollary 3.6. *For all $\mathcal{C} \in \mathbf{C}$ and $i \in \mathcal{I}_{-\mathcal{C}}$, unitary accretion is characterized by*

$$\mathbb{E}\{\Delta_{\mathcal{C} \leftrightsquigarrow \{i\}}\} \doteq \mathbb{E}\{\Delta_{\mathcal{C} \cup \{i\}}\} - \mathbb{E}\{\Delta_{\mathcal{C}}\} = \Psi_{\mathcal{C} \cup \{i\}} \Psi_{\mathcal{C}}^{-1} \mathbb{E}\{\delta_{\mathcal{C} \cup \{i\}, i}\} > \mathbb{E}\{\delta_{\mathcal{C} \cup \{i\}, i}\} \geq 0. \tag{30}$$

Proof. Follows from Equation (25) with $\mathcal{C}' = \mathcal{C}$ and $\mathcal{C}'' = \{i\}$. An alternative direct proof is also presented in Appendix A.5. □

In words, linkage jurisdiction $i \notin \mathcal{C}$ to the linkage coalition $\mathcal{C}$ generates an overall gross gain of $\mathbb{E}\{\delta_{\mathcal{C} \cup \{i\}, i}\} + \psi_i \Psi_{\mathcal{C}}^{-1} \mathbb{E}\{\delta_{\mathcal{C} \cup \{i\}, i}\}$ where the first term accrues to jurisdiction i and the second is shared among jurisdictions in $\mathcal{C}$. Put otherwise, jurisdictions in $\mathcal{C}$ get a share $\psi_i \Psi_{\mathcal{C} \cup \{i\}}^{-1}$ of the overall gross gain $\mathbb{E}\{\Delta_{\mathcal{C} \leftrightsquigarrow \{i\}}\}$. Finally note that our analysis straightforwardly extends to cases where more than two linkage coalitions merge. In particular, we can characterise the gross gains accruing to each separate linkage coalition. For any $\mathcal{C} = \bigcup_i \mathcal{C}_i$ where $\forall i \neq j$,

[16] Linkage costs are key to this definition. See equation (42).

$\mathcal{C}_i \cap \mathcal{C}_j = \varnothing$, surplus accruing to linkage coalition $\mathcal{C}_i$ in forming $\mathcal{C}$ reads

$$\Delta_{\mathcal{C},\mathcal{C}_i} = \Psi_{\mathcal{C}}^{-2} \sum_{\mathcal{C}' \in \mathcal{C}_{-\mathcal{C}_i}} \left\{ \Psi_{\mathcal{C}_{-\mathcal{C}_i}} \Big(\Psi_{\mathcal{C}_i} + \Psi_{\mathcal{C}'} \Big) \Delta_{\mathcal{C}_i \leftrightsquigarrow \mathcal{C}'} - \frac{\Psi_{\mathcal{C}_i}}{2} \sum_{\mathcal{C}'' \in \mathcal{C}_{-\mathcal{C}_i}} \Big(\Psi_{\mathcal{C}'} + \Psi_{\mathcal{C}''} \Big) \Delta_{\mathcal{C}' \leftrightsquigarrow \mathcal{C}''} \right\}. \tag{31}$$

An analytical example with three jurisdictions

There are three jurisdictions denominated by i, j and k and $\mathcal{I} = \{i, j, k\}$. Gross gains from $\mathcal{I}$-linkage write as functions of internal bilateral linkage gross gains, in particular for the aggregate gross gain

$$\Delta_{\mathcal{I}} = \frac{(\psi_i + \psi_j)\Delta_{\{i,j\}} + (\psi_i + \psi_k)\Delta_{\{i,k\}} + (\psi_j + \psi_k)\Delta_{\{j,k\}}}{\psi_i + \psi_j + \psi_k}, \tag{32}$$

as well as for jurisdictional gross gains, e.g. for jurisdiction i

$$\delta_{\mathcal{I},i} = \frac{\psi_j + \psi_k}{(\psi_i + \psi_j + \psi_k)^2} \Big((\psi_i + \psi_j)\Delta_{\{i,j\}} + (\psi_i + \psi_k)\Delta_{\{i,k\}} - \psi_i \Delta_{\{j,k\}} \Big). \tag{33}$$

We consider the following special case (Case 1 in Appendix B) where jurisdictional characteristics are such that jurisdictions are uncorrelated, have same size but different volatilities, that is

$$\text{Case 1:} \begin{cases} \psi_i = \psi_j = \psi_k = \psi > 0, \\ \rho_{ij} = \rho_{ik} = \rho_{jk} = 0, \\ \sigma_k = \sigma > 0, \sigma_i = x\sigma, \sigma_j = y\sigma \; (x, y \geq 0). \end{cases} \tag{34}$$

In this case, aggregate expected gross gain from possible bilateral linkages read

$$\mathbb{E}\{\Delta_{\{i,j\}}\} = \frac{\psi\sigma^2}{4b_2}(x^2 + y^2), \; \mathbb{E}\{\Delta_{\{i,k\}}\} = \frac{\psi\sigma^2}{4b_2}(x^2 + 1), \; \text{and } \mathbb{E}\{\Delta_{\{j,k\}}\} = \frac{\psi\sigma^2}{4b_2}(y^2 + 1), \tag{35}$$

while aggregate gross gains generated by the trilateral link amount to

$$\mathbb{E}\{\Delta_{\mathcal{I}}\} = \frac{\psi\sigma^2}{3b_2}(x^2 + y^2 + 1). \tag{36}$$

It is simple to check that $\mathcal{I}$-linkage is the most advantageous linkage coalition in aggregate. Yet, it is not necessarily the most preferred outcome jurisdictionally speaking. For instance, jurisdiction i is better off linking with j only rather than with both j and k if it holds that

$$\mathbb{E}\{\delta_{\{i,j\},i}\} \geq \mathbb{E}\{\delta_{\mathcal{I},i}\} \; \Leftrightarrow \; 5y^2 \geq 7x^2 + 4, \tag{37}$$

which e.g. holds for $(x, y) = (1, 2)$ or $(2, 3)$. One can thus state that

Observation 2. *Although $\mathcal{I}$-linkage is the most efficient outcome from an aggregate perspective, it may not be the most preferred outcome jurisdictionally speaking.*

Proof. A number of examples are provided in Appendix B. □

By extension to the general case and in the absence of interjurisdictional transfers, the global market may not naturally emerge as some jurisdictions may oppose it and prefer to form smaller linkage coalitions. This, however, requires that jurisdictions' preferences are aligned in terms of linkage coalitions. In our example, $\{i, j\}$-linkage will endogenously form provided that it is also the most preferred option for jurisdiction j, that is if it holds that

$$\mathbb{E}\{\delta_{\{i,j\},j}\} \geq \mathbb{E}\{\delta_{\mathcal{I},j}\} \Leftrightarrow 5x^2 \geq 7y^2 + 4. \tag{38}$$

Notice, however, Equations (37) and (38) cannot hold simultaneously. More generally,

Observation 3. *Jurisdictional preferences in terms of linkage coalitions are non-concordant. For instance, when jurisdiction i prefers $\mathcal{I}$-linkage over $\{i, j\}$-linkage, then jurisdiction j prefers $\{i, j\}$-linkage over $\mathcal{I}$-linkage.*

Proof. The proof is relegated to Appendix B. □

This indicates that one jurisdiction's preferred linkage coalition cannot simultaneously be the favourite coalition for every jurisdiction thereof. To help intuition, fix $(x, y) = (2, 3)$. In this case, jurisdictional linkage preferences are ordered such that

$$\begin{cases} \{i,j\}\text{-linkage} \succ_i \mathcal{I}\text{-linkage} \succ_i \{i,k\}\text{-linkage} \succ_i \{j,k\}\text{-linkage} \\ \mathcal{I}\text{-linkage} \succ_j \{i,j\}\text{-linkage} \succ_j \{j,k\}\text{-linkage} \succ_j \{i,k\}\text{-linkage} \\ \{j,k\}\text{-linkage} \succ_k \mathcal{I}\text{-linkage} \succ_k \{i,k\}\text{-linkage} \succ_k \{i,j\}\text{-linkage} \end{cases} \tag{39}$$

Even without considering linkage costs, this provides a reason why linkage negotiations may fall short of a complete link in the short run (at least when interjurisdictional transfer schemes are infeasible or prove unwieldy).

4 Multilateral linkage with costs

In the presence of costs associated with the formation of linkage coalitions, it might well be that a coalition structure different from the global market yields the highest aggregate payoff, net of costs. This section addresses two issues. First, given linkage costs, we explore the nature of globally efficient coalition structures, or GECS for short. Second, we compare the ability of alternative inter-jurisdictional cost-sharing arrangements in making GECS (or any other desirable coalition structures) Pareto-improving with respect to autarky.

4.1 Efficient coalition structure with linkage costs

Definition of linkage costs. Costs associated with the formation of a linkage coalition have two distinct variable components: (i) linkage implementation costs capturing that the bigger the potential jurisdictions involved, the larger are the implementation-related administrative costs and the costs of harmonizing the rules of the previously independent systems; and (ii) linkage negotiation costs reflecting that costs in forging and establishing climate policy linkage agreements are increasing in the number of participating jurisdictions.[17] These considerations are embedded in the following cost structure such that, $\forall \mathcal{C} \in \mathbf{C}$

$$\kappa\Big(\mathcal{C}; \varepsilon_0, \varepsilon_1\Big) \doteq \varepsilon_0 \cdot \Psi_{\mathcal{C}} + \varepsilon_1 \cdot |\mathcal{C}|^2, \tag{40}$$

where $(\varepsilon_0, \varepsilon_1) \in \mathbb{R}_+^2$ are scaling parameters for the implementation and negotiation costs, respectively. As discussed in the introduction, negotiation costs relate to the minilaterism-polycentrism political dimension of linkage negotiations. We assume that these costs are convex. This dimension is also mechanically captured in the implementation costs via $\Psi_{\mathcal{C}}$, but these further reflect that it is more costly for larger jurisdictions to implement linkage. Let us now characterize GECS for given cost parameters.

Globally Efficient Coalition Structure (GECS). For given linkage cost parameters $(\varepsilon_0, \varepsilon_1) \in \mathbb{R}_+^2$, net aggregate benefits from any coalition structure $\mathcal{S} \in \mathbf{S}$ write

$$\tilde{\Delta}_{\mathcal{S}}(\varepsilon_0, \varepsilon_1) \doteq \Delta_{\mathcal{S}} - K\Big(\mathcal{S}; \varepsilon_0, \varepsilon_1\Big) \tag{41}$$

[17] Fixed per-link sunk costs are not considered as they are blind to both the composition of linkage coalitions and the architecture of coalition structures, thereby unable to discriminate between them.

where by definition, aggregate linkage costs are such that $K\Big(\mathcal{S};\varepsilon_0,\varepsilon_1\Big) \doteq \sum_{\mathcal{C}\in\mathcal{S}} \kappa\Big(\mathcal{C};\varepsilon_0,\varepsilon_1\Big)$. GECS is denoted by $\mathcal{S}^*$ and therefore satisfies

$$\mathcal{S}^*(\varepsilon_0,\varepsilon_1) \doteq \arg\max_{\mathcal{S}\in\mathbf{S}} \left\langle \mathbb{E}\left\{\tilde{\Delta}_{\mathcal{S}}(\varepsilon_0,\varepsilon_1)\right\}\right\rangle . \tag{42}$$

Note that GECS depends on cost parameters. Notice also, while our definition of costs in Equation (40) is exogenously imposed, costs associated with the formation of coalition structures are endogenous to the optimization programme (42). Obviously $\mathcal{S}^*(0,0) = \mathcal{I}$. On the one hand, for a pair of cost parameters low enough, linkage may remain superadditive and GECS correspond to the global market. On the other hand, for a pair of cost parameters high enough, linkage may become subadditive and GECS correspond to complete autarky. In other words, it is precisely the presence of linkage costs that constitutes an impediment to the global market and robs linkage of superadditivity.

On the linkage benefit side, Proposition 3.3 and superadditivity indicate that coalition structures with a small number of large linkage coalitions fare relatively better in aggregate terms as compared to more fragmented linkage structures. On the linkage cost side, we draw the attention to the asymmetric nature of the two cost components: a high (resp. low) $\varepsilon_0/\varepsilon_1$ ratio favours coalition structures consisting of many and small (resp. a few and large) constitutive linkage coalitions. On the face of it, it is hard to tell which architecture of linkage coalitions are most likely to emerge as GECS for given cost parameters, as this depends on economic gains from linkage and ultimately on jurisdictional characteristics. For cost parameters such that linkage is neither superadditive nor subadditive, Section 5 empirically explores the nature of GECS in terms of polycentricity and incompleteness of linkage.

4.2 Alternative inter-jurisdictional cost-sharing arrangements

Definition of cost-sharing arrangements. For any coalition $\mathcal{C} \in \mathbf{C}$, a cost-sharing arrangement is a collection of non-negative weights $\Big(\phi_{\mathcal{C},i}\Big)_{i\in\mathcal{C}}$ such that $\sum_{i\in\mathcal{C}} \phi_{\mathcal{C},i} = 1$ where $\phi_{\mathcal{C},i}$ is the share of the aggregate cost of forming coalition $\mathcal{C}$ incurring to jurisdiction $i \in \mathcal{C}$.[18] For any given inter-jurisdictional cost-sharing arrangement, net jurisdictional gains from forming any coalition $\mathcal{C} \in \mathbf{C}$ write, $\forall i \in \mathcal{C}$

$$\tilde{\delta}_{\mathcal{C},i}(\varepsilon_0,\varepsilon_1) \doteq \delta_{\mathcal{C},i} - \phi_{\mathcal{C},i} \cdot \kappa\Big(\mathcal{C};\varepsilon_0,\varepsilon_1\Big). \tag{43}$$

[18]Notice, cost-sharing arrangements can be assimilated to inter-jurisdictional transfer schemes.

We consider nine alternative interjurisdictional cost-sharing arrangements that are listed in Table 2. $R1$ is an egalitarian rule where all jurisdictions incur the same costs. Linkage costs can also be shared in proportion to size or inverse of size as under $R2$ and $R3$, respectively. $R4$ is a mixed rule where implementation costs are distributed in proportion to size and negotiation costs are evenly apportioned among jurisdictions. Under $R5$ jurisdictions incur costs in proportion to what they gain from the coalition. In a fashion akin to the Shapley value, the more the presence of one jurisdiction is generative of gains for the coalition as a whole, the smaller the share of linkage costs it must incur; this is $R6$. In other words, the more desirable one jurisdiction is, the less it contributes to paying linkage costs. Inversely, $R7$ considers the case where the more desirable one jurisdiction is, the more it contributes to linkage cost payment. Finally, $R8$ and $R9$ replicate $R6$ and $R7$ in terms of *net* contribution.

Is GECS Pareto-improving w.r.t. autarky? Once GECS is known, one must check whether it is incentive-compatible for every jurisdiction. Incentive-compatibility is defined in a weak sense requiring that jurisdictions are at least as well off as under autarky. In other words, incentive-compatibility requires that jurisdictional gains, net of linkage costs, are non-negative for each jurisdiction. Formally, for given linkage cost parameters $(\varepsilon_0, \varepsilon_1) \in \mathbb{R}^2_+$, GECS will be said Pareto-improving with respect to autarky if it holds that $\forall \mathcal{C} \in \mathcal{S}^*(\varepsilon_0, \varepsilon_1)$ and $\forall i \in \mathcal{C}$

$$\mathbb{E}\left\{\tilde{\delta}_{\mathcal{C},i}(\varepsilon_0, \varepsilon_1)\right\} \geq 0. \tag{44}$$

Section 5 compares the nine cost-sharing arrangements in their ability to implement GECS, that is to make GECS Pareto-improving w.r.t. autarky.

An analytical example with three jurisdictions (continued)

As in Section 3.3, $\mathcal{I} = \{i, j, k\}$ and we compare $\mathcal{I}$-linkage with $\{i, j\}$-linkage. To fix ideas, aggregate costs in forming $\mathcal{I}$-linkage and $\{i, j\}$-linkage respectively read

$$\begin{cases} \kappa\Big(\mathcal{I}; \varepsilon_0, \varepsilon_1\Big) = \varepsilon_0 + 3 \cdot (\psi_i + \psi_j + \psi_k) \cdot \varepsilon_1, \\ \kappa\Big(\{i, j\}; \varepsilon_0, \varepsilon_1\Big) = \varepsilon_0 + 2 \cdot (\psi_i + \psi_j) \cdot \varepsilon_1. \end{cases} \tag{45}$$

In Case 1 described in (34), for $x, y > 1$, $\{i, j\}$-linkage brings about higher aggregate gross gains than both $\{i, k\}$-linkage and $\{j, k\}$-linkage, while these three bilateral linkages have identical formation costs. From an aggregate perspective, $\{i, j\}$-linkage is thus the preferred bilateral linkage on net. Moreover, $\{i, j\}$-linkage is preferred to $\mathcal{I}$-linkage from a global perspective when it holds that

$$\mathbb{E}\{\Delta_{\mathcal{I}}\} - \kappa\Big(\mathcal{I};\varepsilon_0,\varepsilon_1\Big) \leq \mathbb{E}\{\Delta_{\{i,j\}}\} - \kappa(\{i,j\};\varepsilon_0,\varepsilon_1) \;\Leftrightarrow\; \sigma^2(x^2+y^2+4) \leq 60 b_2 \varepsilon_1, \tag{46}$$

that is, provided that ε_1 is high enough. Now note that $\{i,j\}$-linkage remains preferable in aggregate provided that

$$\mathbb{E}\{\Delta_{\{i,j\}}\} - \kappa\Big(\{i,j\};\varepsilon_0,\varepsilon_1\Big) \geq 0 \;\Leftrightarrow\; \psi\sigma^2(x^2+y^2) \geq 4b_2(\varepsilon_0 + 4\psi\varepsilon_1). \tag{47}$$

For given cost parameters, when it is the case that $\{i,j\}$-linkage generates the highest aggregate net gains while remaining profitable, it corresponds to GECS. If we fix $(x,y) = (2,3)$ and $\varepsilon_0 = 0$, this is the case when $\frac{17 b_2\sigma^2}{60} \leq \varepsilon_1 \leq \frac{13 b_2 \sigma^2}{16}$.

GECS is Pareto-improving with respect to autarky if no jurisdiction is worse off within the coalition structure as compared to autarky. In turn, this will depend on how aggregate linkage costs are shared among jurisdictions. In the example above, $\{i,j\}$-linkage is implementable if it holds that

$$\begin{cases} \mathbb{E}\{\delta_{\{i,j\},i}\} - \phi_{\{i,j\},i}\cdot\kappa\Big(\{i,j\};\varepsilon_0,\varepsilon_1\Big) \geq 0, \text{ and} \\ \mathbb{E}\{\delta_{\{i,j\},j}\} - \phi_{\{i,j\},j}\cdot\kappa\Big(\{i,j\};\varepsilon_0,\varepsilon_1\Big) \geq 0, \end{cases} \tag{48}$$

where $\phi_{\{i,j\},i} + \phi_{\{i,j\},j} = 1$. For instance, when costs are shared equally or according to size, $\{i,j\}$-linkage is implementable if $\varepsilon_1 \leq \frac{13 b_2\sigma^2}{16}$.

5 Quantitative illustration

5.1 A five-jurisdiction example

In this section we illustrate the quantitative implications of our theory using historical data to discipline the selection of key parameters which determine the value of various linkage arrangements. We adopt a descriptive and combinatorial approach to multilateral linkages among five jurisdictions, namely China (CHN), the United States (USA), the block of European countries who are currently the members of EU-ETS (EUR), Korea (KOR) and Egypt (EGY). We assume that there is a hypothetical ETS in each jurisdiction which covers all carbon emissions in that jurisdiction, and that jurisdictional emission caps are set so that the expected autarkic permit prices are equal across jurisdictions. Our calibration strategy, described in detail in Doda & Taschini (2016), allows us to pin down jurisdiction size and shocks properties. Specifically, Table 3 provides jurisdictional sizes, where China's market size is normalized to 100, and the observed historical volatilities in each market. Table 4 lists the pairwise correlations of the shocks affecting the jurisdictions in the sample.

We emphasize that the sample of jurisdictions we select is deliberate. One of the jurisdictions (CHN) is very large relative to the rest. There are two other jurisdictions which are large and approximately of equal size (USA and EUR). The remaining two are relatively small (KOR and EGY), and substantially more volatile than the larger jurisdictions. Finally, EGY is negatively correlated with all other jurisdictions to varying degrees. Thus, our sample spans the diversity that is present in the data regarding the key determinants of gains from linkage highlighted by our theory. This enables us to illustrate the novel and often unexpected implications of multilateral linkage.

Zero linkage costs

When linkage costs are zero, our theoretical results indicate that the global market generates the largest value.[19] However, as we have shown, the global market may not be the most preferred coalition for each jurisdiction. Table 5 illustrates this result by listing the most and second most preferred coalitions for each jurisdiction.

Table 5 shows that the most preferred coalition of the largest jurisdiction CHN is the global market whereas every other jurisdictions' most preferred coalition is a bilateral linkage with CHN. Jurisdictions' preferences are non-concordant even when considering the second most preferred coalition. Table 5 shows that China's second most preferred coalition does not contain EGY while EGY is a member of every other jurisdiction's second most preferred coalition. In light of the bilateral analysis in Doda & Taschini (2016), one may expect that a jurisdiction's top coalition choices involve partners with high variance and low correlation. This result demonstrates that the mechanism determining jurisdictional gains under multilateral linking is more subtle. The subtlety also applies in the case of size. If the most preferred coalitions of KOR and EGY, i.e. those with CHN, were ruled out exogenously, the second most preferred coalition would include EGY for KOR, and KOR for EGY. Note that this despite the fact that USA and EUR are in principle available to link with. Finally, notice that the most and second most preferred coalitions of USA and EUR include China but exclude each other. These findings indicate that non-concordance of linkage preferences

[19] At this level of abstraction *value* is measured in arbitrary units but its magnitude is comparable across jurisdictions, linkage coalitions, and coalition structures. See Doda & Taschini (2016) for details.

across jurisdictions is likely to be a contributing factor to observed infrequency of linkages in the real world.

More generally, Figure 1 displays jurisdictional gross gains for every possible linkage coalition as a function of coalition's cardinality for CHN, USA and EGY. It is clear that the global market can be far from being the most attractive linkage coalition for a jurisdiction.

Two separate preference clusters can be identified for USA in the left panel of Figure 1: an upper cluster where CHN is in the linkage coalition and a lower cluster where it is not. That is, it is in the interest of USA to form a linkage coalition that contains CHN. Even though EUR is not in Figure 1 it displays a similar pattern with a smaller distance between the two clusters, suggesting 'dispersion' in terms of linkage gain is lower for smaller jurisdictions.

For CHN, three such clusters exist as illustrated in the middle panel of Figure 1: an upper cluster where CHN partners up with the two other large jurisdictions (USA and EUR), a middle cluster where CHN is linked with either USA or EU but not both, and a lower cluster where CHN is only linked to small jurisdictions (KOR and EGY). Potential gains for CHN are dispersed across these linkage coalitions. This suggests that CHN should aim to form linkage coalitions with large partners and ideally the global market.

Notice that no such clusters exist for the smallest jurisdiction EGY as illustrated in the right panel of Figure 1. Even though EGY prefers to link with large jurisdictions, the dispersion of jurisdictional gains across linkage coalitions is rather small. This suggests that a clear ranking of the linkage coalitions for small jurisdictions is harder.

Finally we observe that the linkage coalitions generating high gross gains are also those that are the most costly to form because they involve big partners or many partners, or both. Therefore, it is not obvious prima facie how this trade-off unfolds in determining the most desirable linkage coalitions and coalition structures net of costs.

Positive linkage costs

Accordingly, we introduce linkage costs. This requires us to parametrize the cost function in Equation (40) which is difficult even at this level of abstraction because there is very little empirical guidance to select the pair $(\varepsilon_0, \varepsilon_1)$. To discipline the parametrization, we report three sets of results which are comparable in the sense that the most costly coalition structure, i.e. that where individual jurisdictions negotiate a global market, generates costs equal to 75% of the benefits delivered by the global market. Notice that this does not identify $(\varepsilon_0, \varepsilon_1)$

individually. To pin down a unique pair, we further assume that a share z of the linkage costs are implementation costs, and report results for $z \in \{0, 0.5, 1\}$. In particular, the aggregate gross gain from the global market which includes all five jurisdictions is 0.0473 with cardinality 5 and aggregate size 202.738. Our assumption implies that with $z = 1$ there are only implementation costs of linking and $(\varepsilon_0, \varepsilon_1)$ is uniquely determined $(1.75 \cdot 10^{-4}, 0)$. Conversely, with $z = 0$, there are only negotiation costs of linking and we have the parameter pair $(0, 1.42 \cdot 10^{-3})$. Finally, with $z = 0.5$ the parameter pair is $(8.75 \cdot 10^{-5}, 7.01 \cdot 10^{-4})$.

Table 6 presents the results of combining these cost assumptions with the nine cost-sharing arrangements introduced in Section 4. In particular, we report the GECS ($\mathcal{S}^*$) and the attached expected aggregate *net* gains $\mathbb{E}\{\tilde{\Delta}_{\mathcal{S}^*}\}$. We say that $\mathcal{S}^*$ is blocked by a jurisdiction i under a rule $R\#$, if i receives negative net benefits, i.e. it is worse off under $\mathcal{S}^*$ than autarky. Table 6 also reports the blocking jurisdictions, if any, under a given rule.

We first observe that, when $z = 1$, i.e. only linkage negotiation costs are involved, the globally efficient coalition structure is given by a linked system among the three largest jurisdictions on the one hand, and another system consisting of the remaining two smaller jurisdictions on the other. Notice that the GECS is a polycentric, complete linkage.[20] However, only cost-sharing rules $R3$ and $R5$ are consistent with no jurisdiction blocking this efficient structure. If any other rule were adopted ex ante, KOR would block its linkage coalition with EGY thereby precluding the implementation of GECS. This highlights the fact that the choice of the cost-sharing rule ex ante can determine whether GECS constitutes a Pareto-improvement w.r.t. autarky, i.e. whether GECS is implementable or not.

Second, we observe that when $z = 0.5$, GECS is unchanged. Notice also that although the net gains attached to GECS are half those that obtain with $z = 1$, GECS is now feasible under cost-sharing rules $R1$, $R4$, $R5$ and $R7$. This highlights that high aggregate gains from GECS do not necessarily improve upon its incentive compatibility for jurisdictions involved. Notice also that KOR is no longer the sole blocking jurisdiction, as EUR and CHN might also oppose GECS under certain cost-sharing rules.

Third, we observe that when $z = 0$, i.e. only linkage implementation costs are involved, GECS corresponds to the global market. However, it is only achievable under $R5$, and at least one jurisdiction opposes its implementation depending on the cost-sharing rule chosen ex ante.

It is noteworthy that among the cost specifications and cost-sharing rules considered in

[20] If we were to increase cost parameters further, GECS would display incomplete linkage where it is more beneficial, from an aggregate perspective, that some jurisdictions remain in autarky.

Table 6, $R5$ always renders GECS viable. That is, apportioning linkage costs in proportion to jurisdictional gains from linkage coalitions might facilitate the implementation of efficient coalition structures. In practice, however, determining the associated jurisdictional cost shares might not be as straightforward as for simpler rules, e.g. egalitarian or per size.

5.2 Graphical illustration of a ten-jurisdiction example

Finally, we illustrate some of the analytical results presented in the previous sections, noting that a thorough investigation of multilateral linkage with ten jurisdictions (similar to the five-jurisdiction case above) is currently underway. With ten jurisdictions, Figure 2 displays aggregate *gross* gains for every possible linkage coalition as a function of the coalition's cardinality. The upper and lower envelopes join the maximal and minimal aggregate gains for each coalition's cardinality, respectively.

The monotonicity and curvature of these two envelopes is informative in two respects. First, both envelopes are increasing with coalition's cardinality, which illustrates the monotonicity of linkage (Corollary 3.4). Second, the upper envelope is concave in coalition's cardinality while the lower envelope is convex. This is so because the two envelopes' location is determined by jurisdictional sizes. The upper envelope is obtained by considering linkage coalitions that gradually expand by including the largest jurisdictions, in decreasing size order. The lower envelope represents linkage coalitions first containing the smallest jurisdictions and then gradually expanding by integration of jurisdictions in increasing size order.

Concavity of the upper envelope displays the stepwise aggregate economic gains in gradually expanding the linkage coalition by adding ever smaller jurisdictions inside the coalition. The converse holds for the lower envelope; in particular, notice the significant jump when cardinality increases from 9 to 10 jurisdictions, which corresponds to the inclusion of China. This illustrates by how much, in aggregate terms, China contributes to the global market. Notice finally that while the two envelopes are determined by jurisdictional sizes, variations in aggregate gains between these two envelopes are attributable to a combination of jurisdiction-specific shock properties.

6 Conclusions

Compared to bilateral linkages, a formal study of multilateral linkages poses numerous challenges. In this paper, we advance the theory on multilateral linkages by proposing a language and general theoretical model that allow us to analytically describe multilateral linkages between ETSs and evaluate the corresponding gains. Every multilateral linkage can be decomposed into its internal bilateral linkages. This constitutes our first key result. We also show that the aggregate expected gain from the union of disjoint coalitions of linked ETSs is no less than the sum of separate coalitions' expected gains: linkage is superadditive. Finally, we generalize the results in Doda & Taschini (2016) by providing an analytical formula for this gain as a function of coalitions' sizes and shock characteristics. When we extend the model by introducing linkage costs that are increasing in both the number and aggregate size of partnering jurisdictions, we observe that coalition structures different from the global market may yield higher aggregate gains net of costs. As a consequence, alternative cost-sharing arrangements can make, or break, an efficient coalition structure. This is clearly an area where additional academic and policy work would be useful.

Tables & Figures

Table 2: Interjurisdictional cost-sharing arragements

Rule number	Share of total costs incurred by jurisdiction i $(\phi_{\mathcal{C},i})^\dagger$
$R1$	$\lvert\mathcal{C}\rvert^{-1}$
$R2$	$\psi_i \Psi_{\mathcal{C}}^{-1}$
$R3$	$\Phi_{\mathcal{C}} \psi_i^{-1}$
$R4$	$\varepsilon_0 \cdot \psi_i + \varepsilon_1 \cdot \lvert\mathcal{C}\rvert$
$R5$	$\mathbb{E}\{\delta_{\mathcal{C},i}\}\mathbb{E}\{\Delta_{\mathcal{C}}\}^{-1}$
$R6$	$\left(\mathbb{E}\{\Delta_{\mathcal{C}}\} - \mathbb{E}\{\Delta_{\mathcal{C}_{-i}}\}\right)^{-1} \cdot \left(\sum_{j\in\mathcal{C}} \left(\mathbb{E}\{\Delta_{\mathcal{C}}\} - \mathbb{E}\{\Delta_{\mathcal{C}_{-j}}\}\right)^{-1} \right)^{-1}$
$R7$	$\left(\mathbb{E}\{\Delta_{\mathcal{C}}\} - \mathbb{E}\{\Delta_{\mathcal{C}_{-i}}\}\right) \cdot \left(\lvert\mathcal{C}\rvert \cdot \mathbb{E}\{\Delta_{\mathcal{C}}\} - \sum_{j\in\mathcal{C}} \mathbb{E}\{\Delta_{\mathcal{C}_{-j}}\} \right)^{-1}$
$R8$	$\left(\mathbb{E}\{\tilde{\Delta}_{\mathcal{C}}\} - \mathbb{E}\{\tilde{\Delta}_{\mathcal{C}_{-i}}\}\right)^{-1} \cdot \left(\sum_{j\in\mathcal{C}} \left(\mathbb{E}\{\tilde{\Delta}_{\mathcal{C}}\} - \mathbb{E}\{\tilde{\Delta}_{\mathcal{C}_{-j}}\}\right)^{-1} \right)^{-1}$
$R9$	$\left(\mathbb{E}\{\tilde{\Delta}_{\mathcal{C}}\} - \mathbb{E}\{\tilde{\Delta}_{\mathcal{C}_{-i}}\}\right) \cdot \left(\lvert\mathcal{C}\rvert \cdot \mathbb{E}\{\tilde{\Delta}_{\mathcal{C}}\} - \sum_{j\in\mathcal{C}} \mathbb{E}\{\tilde{\Delta}_{\mathcal{C}_{-j}}\} \right)^{-1}$

Note: $\dagger$: except for $R4$ where the total linkage costs incurring to jurisdiction i are indicated.

Table 3: Calibration results: Size and volatility (ψ_i and σ_i)

	CHN	USA	EUR	KOR	EGY
ψ_i	100	55.038	38.699	6.645	2.356
σ_i	0.028	0.019	0.017	0.034	0.050

Table 4: Calibration results: Pairwise correlation coefficients (ρ_{ij})

	CHN	USA	EUR	KOR	EGY
CHN	1.000				
USA	0.525	1.000			
EUR	0.460	0.652	1.000		
KOR	0.247	0.419	0.277	1.000	
EGY	-0.395	-0.186	-0.101	-0.397	1.000

Table 5: Jurisdictional rankings of linkage coalitions

	Most preferred coalition	Second most preferred coalition
CHN	{CHN,USA,EUR,KOR,EGY}	{CHN,USA,EUR,KOR}
USA	{CHN,USA}	{CHN,USA,EGY}
EUR	{CHN,EUR}	{CHN,EUR,KOR,EGY}
KOR	{CHN,KOR}	{CHN,KOR,EGY}
EGY	{CHN,EGY}	{CHN,KOR,EGY}

Table 6: Multilateral linkage with costs and alternative cost-sharing rules ($x = 0.75$)

	$\mathcal{S}^*$	$\mathbb{E}\{\tilde{\Delta}_{\mathcal{S}^*}\}$	Set of blocking jurisdiction under R#
$z = 1$	{{CHN,USA,EUR},{KOR,EGY}}	0.0221	$R3$ and $R5$: $\emptyset$ $R1$, $R2$, $R4$, $R6$, $R7$, ... $R8$ and $R9$: {KOR}
$z = 0.5$	{{CHN,USA,EUR},{KOR,EGY}}	0.0137	$R1$, $R4$, $R5$ and $R7$: $\emptyset$ $R2$: {KOR} $R3$: {EUR} $R6$ and $R8$: {EUR,KOR} $R9$: {CHN}
$z = 0$	{{CHN,USA,EUR,KOR,EGY}}	0.0118	$R5$: $\emptyset$ $R1$: {KOR} $R2$ and $R4$: {CHN,USA} $R3$ and $R6$: {KOR,EGY} $R7$: {CHN} $R8$: {USA,EUR} $R9$: {EGY}

Note: Cost parameters are set such that linkage costs (i) amount to a share x of the aggregate gains from the global market; (ii) are composed of a share z (resp. $1 - z$) of implementation (resp. negotiation) costs.

Figure 1: Jurisdictional preferences in terms of linkage coalition

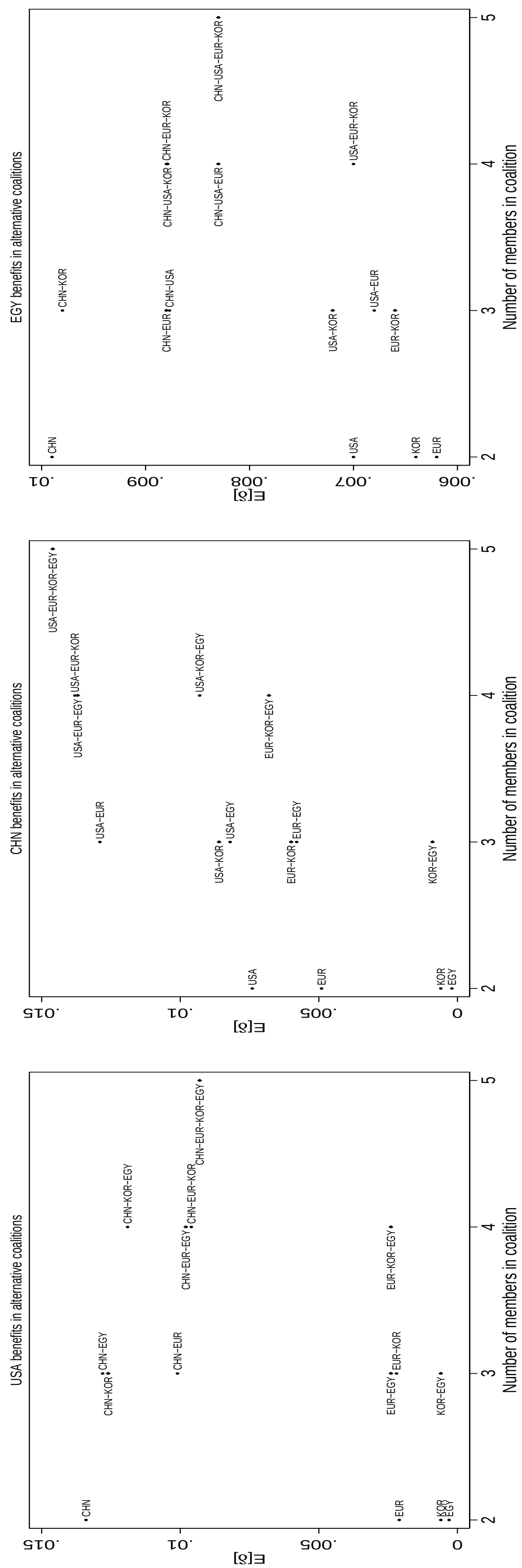
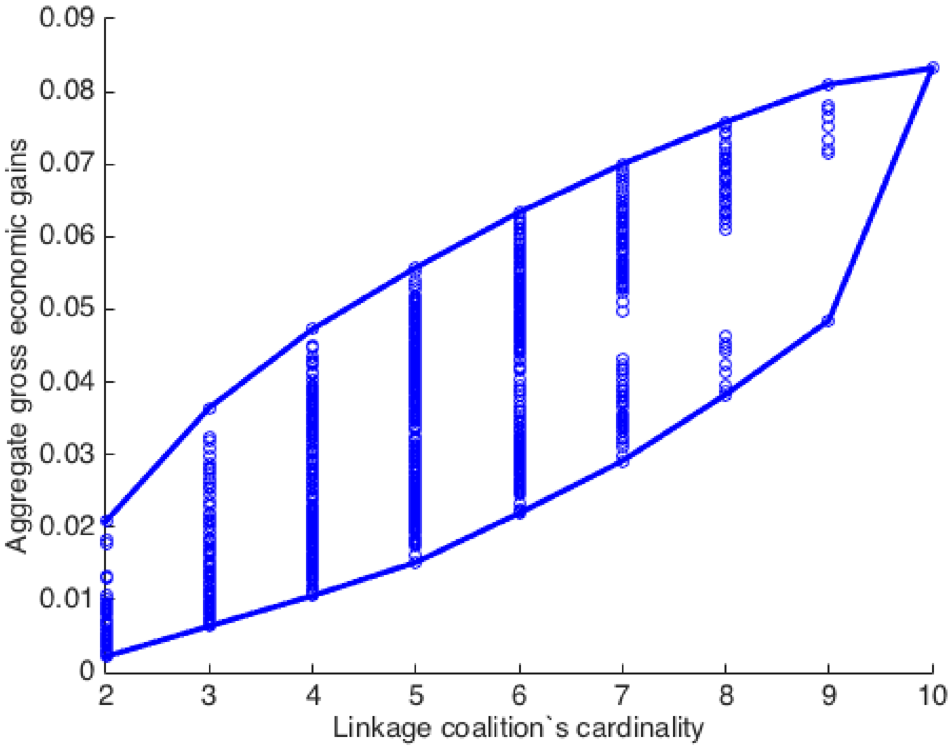


Figure 2: Illustration of monotonicity of linkage



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Appendices

A Collected proofs

Without loss of generality, fix $\mathcal{C} \in \mathbf{C}$ such that $\mathcal{C} = \{1, 2, \dots, m\}$ with $m \in [\![3; n]\!]$.

A.1 Proof of Observation 1

Since the variance is a symmetric bilinear form, it jointly holds that

$$\begin{cases} \mathbb{V}\{p_{\mathcal{C}}\} = \mathbb{V}\{\Theta_{\mathcal{C}}\} = \Psi_{\mathcal{C}}^{-2}\left(\sum_{i=1}^{m} \psi_i^2 \sigma_i^2 + 2 \sum_{1 \le i < j \le m} \psi_i \psi_j \rho_{ij} \sigma_i \sigma_j\right), \\ \Psi_{\mathcal{C}} \sum_{j=1}^{m} \psi_j \mathbb{V}\{\theta_j\} = \sum_{i=1}^{m} \sum_{j=1}^{m} \psi_i \psi_j \sigma_j^2 = \sum_{i=1}^{m} \psi_i^2 \sigma_i^2 + \sum_{1 \le i < j \le m} \psi_i \psi_j (\sigma_i^2 + \sigma_j^2). \end{cases} \tag{A.1}$$

Further noting that $\forall (i,j) \in [\![1; m]\!]^2$, $\sigma_i^2 + \sigma_j^2 \ge 2\rho_{ij}\sigma_i\sigma_j$ and that $\mathbb{V}\{\bar{p}_i\} = \mathbb{V}\{\theta_i\}$ concludes. The statement on price volatility variations at jurisdictional levels can be conducted for a bilateral linkage only as the argument naturally extends to multilateral links. By definition,

$$\mathbb{V}\{\bar{p}_i\} = \sigma_i^2,\ \mathbb{V}\{\bar{p}_j\} = \sigma_j^2,\ \text{and } \mathbb{V}\{p_{\{i,j\}}\} = (\psi_i + \psi_j)^{-2}(\psi_i^2\sigma_i^2 + \psi_j^2\sigma_j^2 + 2\rho_{ij}\psi_i\psi_j\sigma_i\sigma_j). \tag{A.2}$$

Without loss of generality, assume that jurisdiction i (resp. j) is the ex-post low-volatility (resp. high-volatility) jurisdiction, i.e. $\sigma_j \ge \sigma_i$. Then, $\{i,j\}$-linkage reduces price volatility in the high-volatility jurisdiction i.f.f. $\mathbb{V}\{\bar{p}_j\} \ge \mathbb{V}\{p_{\{i,j\}}\}$, which is equivalent to

$$\psi_i(\sigma_j - \sigma_i)\Big(\psi_i(\sigma_i + \sigma_j) + 2\psi_j\sigma_j(1 - \rho_{ij})\Big) \ge 0, \tag{A.3}$$

and unconditionally holds, i.e. for all ψ_i, ψ_j, $\sigma_j \ge \sigma_i$ and $\rho_{ij} \in [-1; 1]$. It might not be the case, however, that $\{i,j\}$-linkage reduces price volatility in the low-volatility jurisdiction. In particular, $\mathbb{V}\{\bar{p}_i\} \ge \mathbb{V}\{p_{\{i,j\}}\}$ holds if and only if

$$\psi_j(\sigma_j - \sigma_i)\Big(\psi_j(\sigma_i + \sigma_j) + 2\psi_i\sigma_i(\rho_{ij} - 1)\Big) \le 0 \ \Leftrightarrow\ \frac{\psi_j}{\psi_i} \le \frac{2\sigma_i(1 - \rho_{ij})}{\sigma_i + \sigma_j}. \tag{A.4}$$

For a given triple $(\sigma_i, \sigma_j, \rho_{ij})$, $\{i,j\}$-linkage effectively reduces volatility in the low-volatility jurisdiction provided that the high-volatility jurisdiction is not too large in comparison.

A.2 Proof of Lemma 3.1 and Equations (17,19,21)

Since damages from pollution are fixed and do not vary with the prevailing linkage coalition structure, the economic surplus from $\mathcal{C}$-linkage accruing to partnering jurisdiction $i \in \mathcal{C}$ is given by the difference between its benefits under $\mathcal{C}$-linkage and autarky, that is

$$\begin{aligned} \delta_{\mathcal{C},i} &= (b_1 + \theta_i - p_{\mathcal{C}})(q_{\mathcal{C},i} - \omega_i) - \frac{b_2}{2\psi_i}(q_{\mathcal{C},i}^2 - \omega_i^2) = \frac{b_2}{\psi_i} q_{\mathcal{C},i}(q_{\mathcal{C},i} - \omega_i) - \frac{b_2}{2\psi_i}(q_{\mathcal{C},i}^2 - \omega_i^2) \\ &= \frac{b_2}{2\psi_i}(q_{\mathcal{C},i} - \omega_i)^2 = \frac{\psi_i}{2b_2}(\bar{p}_i - p_{\mathcal{C}})^2, \end{aligned} \tag{A.5}$$

where the second and fourth equalities obtain via the necessary first-order condition (14) and the net demand for permits (16) under $\mathcal{C}$-linkage, respectively. This is Equation (17) and taking expectations proves Lemma 3.1.

Recalling from Equation (18) that $\bar{p}_i - p_{\mathcal{C}} = \theta_i - \Theta_{\mathcal{C}}$ and applying the definition of $\Theta_{\mathcal{C}}$ further gives Equation (19). Expanding then gives

$$\delta_{\mathcal{C},i} = \frac{\psi_i}{2b_2\Psi_{\mathcal{C}}^2} \sum_{j=1,j\neq i}^{m} \psi_j \left\{ \psi_j(\theta_i - \theta_j)^2 + 2 \sum_{k>j,k\neq i}^{m} \psi_k(\theta_i - \theta_j)(\theta_i - \theta_k) \right\}. \tag{A.6}$$

Now notice that

$$\begin{aligned} 2(\theta_i - \theta_j)(\theta_i - \theta_k) &= (\theta_i - \theta_k + \theta_k - \theta_j)(\theta_i - \theta_k) + (\theta_i - \theta_j)(\theta_i - \theta_j + \theta_j - \theta_k) \\ &= (\theta_i - \theta_j)^2 + (\theta_i - \theta_k)^2 - (\theta_j - \theta_k)^2. \end{aligned} \tag{A.7}$$

Using the above and regrouping sums, we obtain that

$$\delta_{\mathcal{C},i} = \frac{\psi_i}{2b_2\Psi_{\mathcal{C}}^2} \sum_{j=1,j\neq i}^{m} \psi_j \left\{ (\Psi_{\mathcal{C}} - \psi_i)(\theta_i - \theta_j)^2 - \sum_{k>j,k\neq i}^{m} \psi_k(\theta_j - \theta_k)^2 \right\}. \tag{A.8}$$

Noting that $\Psi_{\mathcal{C}_{-i}} = \Psi_{\mathcal{C}} - \psi_i$, Equation (21) then obtains from Equation (11).

A.3 Proof of Proposition 3.2 (bilateral decomposition)

Summing over Equation (21) over all $i \in [\![1; m]\!]$ gives

$$\Delta_{\mathcal{C}} \doteq \sum_{i=1}^{m} \delta_{\mathcal{C},i} = \Psi_{\mathcal{C}}^{-2} \sum_{i=1}^{m} \left\{ \sum_{j=1,j\neq i}^{m} \left\{ \Psi_{\mathcal{C}_{-i}}(\psi_i + \psi_j)\Delta_{\{i,j\}} - \psi_i \sum_{k>j,k\neq i}^{m} (\psi_j + \psi_k)\Delta_{\{j,k\}} \right\} \right\}. \tag{A.9}$$

Regrouping terms by bilateral linkages, the above rewrites

$$\Delta_{\mathcal{C}} = \Psi_{\mathcal{C}}^{-2} \sum_{1 \le i < j \le m} \left\{ \left(\Psi_{\mathcal{C}_{-i}} + \Psi_{\mathcal{C}_{-j}} \right) (\psi_i + \psi_j) \Delta_{\{i,j\}} - \sum_{k=1, k \neq i,j}^{m} \psi_k (\psi_i + \psi_j) \Delta_{\{i,j\}} \right\}, \tag{A.10}$$

which in turn yields

$$\begin{aligned} \Delta_{\mathcal{C}} &= \Psi_{\mathcal{C}}^{-2} \sum_{1 \le i < j \le m} \left\{ \left(\Psi_{\mathcal{C}_{-i}} + \Psi_{\mathcal{C}_{-j}} - \Psi_{\mathcal{C}_{-\{i,j\}}} \right) (\psi_i + \psi_j) \Delta_{\{i,j\}} \right\} \\ &= \Psi_{\mathcal{C}}^{-1} \sum_{1 \le i < j \le m} (\psi_i + \psi_j) \Delta_{\{i,j\}}. \end{aligned} \tag{A.11}$$

This is Equation (22) and proves Proposition 3.2.

A.4 Proof of Proposition 3.3 (superadditivity)

For all $\mathcal{C} \in \mathbf{C}$ and $\mathcal{C}' \subset \mathcal{C}$, denote by $\mathcal{C}''$ the complement of $\mathcal{C}'$ in $\mathcal{C}$, i.e. $\mathcal{C} = \mathcal{C}' \cup \mathcal{C}''$ and $\mathcal{C}' \cap \mathcal{C}'' = \varnothing$. This is without loss of generality. Expanding Equation (22) gives

$$\begin{aligned} \Delta_{\mathcal{C}} &= (2\Psi_{\mathcal{C}})^{-1} \left(\sum_{(i,j) \in \mathcal{C}' \times \mathcal{C}'} (\psi_i + \psi_j) \Delta_{\{i,j\}} + \sum_{(i,j) \in \mathcal{C}'' \times \mathcal{C}''} (\psi_i + \psi_j) \Delta_{\{i,j\}} + 2 \sum_{(i,j) \in \mathcal{C}' \times \mathcal{C}''} (\psi_i + \psi_j) \Delta_{\{i,j\}} \right) \\ &= \Psi_{\mathcal{C}}^{-1} \left(\Psi_{\mathcal{C}'} \Delta_{\mathcal{C}'} + \Psi_{\mathcal{C}''} \Delta_{\mathcal{C}''} + \sum_{(i,j) \in \mathcal{C}' \times \mathcal{C}''} (\psi_i + \psi_j) \Delta_{\{i,j\}} \right). \end{aligned} \tag{A.12}$$

Simple manipulation of Equation (A.12) yields the aggregate payoff from merging the two linkage coalitions $\mathcal{C}'$ and $\mathcal{C}''$

$$\begin{aligned} \Delta_{\mathcal{C}' \leftrightsquigarrow \mathcal{C}''} &\doteq \Delta_{\mathcal{C}} - \Delta_{\mathcal{C}'} - \Delta_{\mathcal{C}''} \\ &= \Psi_{\mathcal{C}}^{-1} \left(\sum_{(i,j) \in \mathcal{C}' \times \mathcal{C}''} (\psi_i + \psi_j) \Delta_{\{i,j\}} + \left(\Psi_{\mathcal{C}'} - \Psi_{\mathcal{C}} \right) \Delta_{\mathcal{C}'} + \left(\Psi_{\mathcal{C}''} - \Psi_{\mathcal{C}} \right) \Delta_{\mathcal{C}''} \right) \\ &= \Psi_{\mathcal{C}}^{-1} \left(\sum_{(i,j) \in \mathcal{C}' \times \mathcal{C}''} (\psi_i + \psi_j) \Delta_{\{i,j\}} - \Psi_{\mathcal{C}''} \Delta_{\mathcal{C}'} - \Psi_{\mathcal{C}'} \Delta_{\mathcal{C}''} \right). \end{aligned} \tag{A.13}$$

As mentioned in the main text $\Delta_{\mathcal{C}' \leftrightsquigarrow \mathcal{C}''}$ also obtains by definition of bilateral linkage, that is

$$\Delta_{\mathcal{C}' \leftrightsquigarrow \mathcal{C}''} = \frac{\Psi_{\mathcal{C}'} \Psi_{\mathcal{C}''}}{2 b_2 \Psi_{\mathcal{C}}} \left(\mathbb{V}\{\Theta_{\mathcal{C}'}\} + \mathbb{V}\{\Theta_{\mathcal{C}''}\} - 2\mathrm{Cov}\{\Theta_{\mathcal{C}'}; \Theta_{\mathcal{C}''}\} \right) \ge 0. \tag{A.14}$$

A.5 Alternative proof of Corollary 3.6

Without loss of generality, fix $i = m$ such that $\mathcal{C}_{-i} = \{1, 2, \ldots, m-1\}$. By subtracting Equation (22), we obtain that

$$\begin{aligned}
\Delta_{\mathcal{C}} - \Delta_{\mathcal{C}_{-i}} &= \Psi_{\mathcal{C}}^{-1} \sum_{1 \leq j < k \leq i} (\psi_j + \psi_k) \Delta_{\{j,k\}} - \Psi_{\mathcal{C}_{-i}}^{-1} \sum_{1 \leq j < k \leq i-1} (\psi_j + \psi_k) \Delta_{\{j,k\}} \\
&= \Psi_{\mathcal{C}}^{-1} \sum_{j=1}^{i-1} (\psi_j + \psi_i) \Delta_{\{j,i\}} - \sum_{1 \leq j < k \leq i-1} (\Psi_{\mathcal{C}_{-i}}^{-1} - \Psi_{\mathcal{C}}^{-1})(\psi_j + \psi_k) \Delta_{\{j,k\}} \\
&= \Psi_{\mathcal{C}}^{-1} \Psi_{\mathcal{C}_{-i}}^{-1} \left(\sum_{j=1}^{i-1} \Psi_{\mathcal{C}_{-i}} (\psi_j + \psi_i) \Delta_{\{j,i\}} - \psi_i \sum_{1 \leq j < k \leq i-1} (\psi_j + \psi_k) \Delta_{\{j,k\}} \right) \\
&= \Psi_{\mathcal{C}} \Psi_{\mathcal{C}_{-i}}^{-1} \delta_{\mathcal{C},i},
\end{aligned} \tag{A.15}$$

where the last line follows from Equation (21). Tacking expectations proves Corollary 3.6. Notice that telescoping Equation (A.15) provides an alternative way of computing the gains from merging two disjoint coalitions.

B Jurisdictional vs. social preferences

Case 1: $\psi_i = \psi_j = \psi_k = \psi$; $\rho_{ij} = \rho_{ik} = \rho_{jk} = 0$; and $\sigma_k = \sigma$, $\sigma_i = x\sigma$, $\sigma_j = y\sigma$ $(x, y \geq 0)$.

$$\mathbb{E}\{\Delta_{\{i,j\}}\} = \frac{\psi\sigma^2}{4b_2}(x^2 + y^2), \ \mathbb{E}\{\Delta_{\{i,k\}}\} = \frac{\psi\sigma^2}{4b_2}(x^2 + 1), \ \text{and } \mathbb{E}\{\Delta_{\{j,k\}}\} = \frac{\psi\sigma^2}{4b_2}(y^2 + 1). \tag{B.1}$$

In terms of jurisdictional preferences, the following holds

$$\begin{cases}
\mathbb{E}\{\delta_{\{i,j\},i}\} \geq \mathbb{E}\{\delta_{\mathcal{I},i}\} \Leftrightarrow 5y^2 \geq 7x^2 + 4 \\
\mathbb{E}\{\delta_{\{i,j\},j}\} \geq \mathbb{E}\{\delta_{\mathcal{I},j}\} \Leftrightarrow 5x^2 \geq 7y^2 + 4 \\
\mathbb{E}\{\delta_{\{i,k\},i}\} \geq \mathbb{E}\{\delta_{\mathcal{I},i}\} \Leftrightarrow 4y^2 + 7x^2 \leq 5 \\
\mathbb{E}\{\delta_{\{j,k\},k}\} \geq \mathbb{E}\{\delta_{\mathcal{I},k}\} \Leftrightarrow 5y^2 \geq 4x^2 + 7 \\
\mathbb{E}\{\delta_{\{i,k\},k}\} \geq \mathbb{E}\{\delta_{\mathcal{I},k}\} \Leftrightarrow 5x^2 \geq 4y^2 + 7
\end{cases} \tag{B.2}$$

It follows that trilateral linkage would be the Condorcet-winning linkage coalition from a global perspective since it ranks first or second for every jurisdiction. As exposed below (with four jurisdictions) there are examples where it is not necessarily the case.

Case 2: $\sigma_i = \sigma_j = \sigma_k = \sigma$; $\rho_{ij} = \rho_{ik} = \rho_{jk} = 0$; and $\psi_k = \psi$, $\psi_i = x\psi$, $\psi_j = y\psi$ $(x, y > 0)$.

$$\mathbb{E}\{\Delta_{\{i,j\}}\} = \frac{xy\psi\sigma^2}{b_2(x+y)},\ \mathbb{E}\{\Delta_{\{i,k\}}\} = \frac{x\psi\sigma^2}{b_2(x+1)},\ \text{and } \mathbb{E}\{\Delta_{\{j,k\}}\} = \frac{y\psi\sigma^2}{b_2(y+1)}. \tag{B.3}$$

In this case, $\mathbb{E}\{\delta_{\{i,j\},i}\} \geq \mathbb{E}\{\delta_{\mathcal{I},i}\} \Leftrightarrow y^3 \geq x(xy + x + 2y)$ which e.g. holds for all $x \geq 5/6$ and $y = 2x$. Note that $\mathbb{E}\{\delta_{\{i,j\},j}\} \geq \mathbb{E}\{\delta_{\mathcal{I},j}\} \Leftrightarrow x^3 \geq y(xy + y + 2x)$ cannot hold simultaneously.

Case 3: $\sigma_i = \sigma_j = \sigma_k = \sigma$; $\psi_i = \psi_j = \psi_k = \psi$; and $\rho_{jk} = \rho \neq 0$, $\rho_{ij} = x\rho$, $\rho ik = y\rho$.

$$\mathbb{E}\{\Delta_{\{i,j\}}\} = \frac{\psi\sigma^2}{2b_2}(1-\rho x),\ \mathbb{E}\{\Delta_{\{i,k\}}\} = \frac{\psi\sigma^2}{2b_2}(1-\rho y),\ \text{and } \mathbb{E}\{\Delta_{\{j,k\}}\} = \frac{\psi\sigma^2}{2b_2}(1-\rho). \tag{B.4}$$

In this case, $\mathbb{E}\{\delta_{\{i,j\},i}\} \geq \mathbb{E}\{\delta_{\mathcal{I},i}\} \Leftrightarrow 8y \geq 4 + x + 3/\rho$.

Proof of Observation 3. We formally show that for any jurisdictional characteristics, both i and j cannot favour $\{i,j\}$-linkage over $\mathcal{I}$-linkage simultaneously. By way of contradiction, assume that $\mathbb{E}\{\delta_{\{i,j\},i}\} \geq \mathbb{E}\{\delta_{\mathcal{I},i}\}$ and $\mathbb{E}\{\delta_{\{i,j\},j}\} \geq \mathbb{E}\{\delta_{\mathcal{I},j}\}$ both hold. It follows that $\mathbb{E}\{\delta_{\{i,j\},i}\} + \mathbb{E}\{\delta_{\{i,j\},j}\} = \mathbb{E}\{\Delta_{\{i,j\}}\} \geq \mathbb{E}\{\delta_{\mathcal{I},i}\} + \mathbb{E}\{\delta_{\mathcal{I},j}\} = \mathbb{E}\{\Delta_{\mathcal{I}}\} - \mathbb{E}\{\delta_{\mathcal{I},k}\}$, which contradicts with Corollary 3.6, i.e. with $\mathbb{E}\{\Delta_{\mathcal{I}}\} - \mathbb{E}\{\Delta_{\{i,j\}}\} > \mathbb{E}\{\delta_{\mathcal{I},k}\}$. The non-concordance of jurisdictional preferences over linkage coalitions extends to the general case.

Empirical examples with four jurisdictions. With a set of four jurisdictions composed of Europe, Japan, Korea and Egypt, jurisdictional preferences for linkage coalitions are non-concordant but four-jurisdiction linkage ranks second for each jurisdiction among all possible linkage coalitions so that it is the Condorcet-winning coalition from a global perspective.

However, if we now consider China, USA, Europe and Japan, four-jurisdiction linkage is the preferred coalition for China but ranks fourth for the other jurisdictions. In particular, there is no unique Condorcet-winning linkage coalition as there is a tie between the four-jurisdiction linkage and two trilateral linkages ({Chine,USA,Europe} and {Chine,USA,Japan}).

C Selection of emissions caps and linkage profitability

Linkage profitability with different jurisdictional abatement efforts. In general, solving for any interior $\mathcal{C}$-linkage equilibrium yields the following permit equilibrium price

$$p_{\mathcal{C}} = b_1 - b_2 \Omega_{\mathcal{C}} \Psi_{\mathcal{C}}^{-1} + \Theta_{\mathcal{C}}, \tag{C.1}$$

and taking the difference with the autarkic price in jurisdiction $i \in \mathcal{C}$ then gives

$$\bar{p}_i - p_{\mathcal{C}} = \theta_i - \Theta_{\mathcal{C}} - b_2 \Big(\Omega_{\mathcal{C}} \Psi_{\mathcal{C}}^{-1} - \omega_i \psi_i^{-1} \Big). \tag{C.2}$$

It holds that $\bar{p}_i - p_{\mathcal{C}} = \theta_i - \Theta_{\mathcal{C}}$ provided that $\Omega_{\mathcal{C}} \Psi_{\mathcal{C}}^{-1} = \omega_i \psi_i^{-1}$, that is jurisdiction i's abatement effort (corrected for size) is equal to the $\mathcal{C}$-average. Our results for $\mathcal{C}$-linkage thus hold provided that all jurisdictions in $\mathcal{C}$ have an emissions cap proportional to size by the same factor, i.e. $\exists A > 0 : \forall i \in \mathcal{C}, \omega_i = A \cdot \psi_i$. When this is not the case, then expected jurisdictional surplus from $\mathcal{C}$-linkage amounts to $\forall i \in \mathcal{C}$

$$\mathbb{E}\{\delta_{\mathcal{C},i}\} = \frac{\psi}{2b_2} \mathbb{E}\{(\theta_i - \Theta_{\mathcal{C}})^2\} + b_2 \psi_i \Big(\Omega_{\mathcal{C}} \Psi_{\mathcal{C}}^{-1} - \omega_i \psi_i^{-1} \Big)^2, \tag{C.3}$$

so that there is an additional shock-independent non-negative term. This implies $\forall i \in \mathcal{C}$

$$\partial_{\omega_i} \mathbb{E}\{\delta_{\mathcal{C},i}\} = 2b_2 \psi_i \Big(\Omega_{\mathcal{C}} \Psi_{\mathcal{C}}^{-1} - \omega_i \psi_i^{-1} \Big) \Big(\Psi_{\mathcal{C}}^{-1} - \psi_i^{-1} \Big) \geq 0 \Leftrightarrow \omega_i \psi_i^{-1} \geq \Omega_{\mathcal{C}} \Psi_{\mathcal{C}}^{-1}. \tag{C.4}$$

In a fashion akin to Helm (2003) and irrespective of the shock structure, jurisdictions with size-adjusted abatement efforts lower than $\mathcal{C}$-average ($\omega_i \psi_i^{-1} \geq \Omega_{\mathcal{C}} \Psi_{\mathcal{C}}^{-1}$) are systematically the seller jurisdictions in expectations ($\mathbb{E}\{\bar{p}_i\} \leq \Psi_{\mathcal{C}}^{-1} \sum_{j \in \mathcal{C}} \psi_j \mathbb{E}\{\bar{p}_j\}$) and thus have an incentive to expand their domestic caps to increase sales. Anticipation of linkage and strategic cap selection is thus an important topic, so far not considered in the paper.

Alternative cap selection mechanisms. Let $\mathcal{C} \in \mathbf{C}$ be a coalition on cap selection: jurisdictions in $\mathcal{C}$ set their caps cooperatively. Denote by $\bar{\mathcal{C}}$ the complement of $\mathcal{C}$ in $\mathbf{C}$: jurisdictions in $\bar{\mathcal{C}}$ behave as singletons w.r.t. cap selection. We assume Stackelberg conjectural variations where $\mathcal{C}$ behaves as the leader. Notice, results would slightly differ under alternative conjectural variations, see e.g. MacKenzie (2011) and Gelves & McGinty (2016). For instance in the paper, we consider a Cournot-Nash solution concept for the non-cooperative outcome.

The aggregate reaction function of singletons to the emissions cap $\Omega_{\mathcal{C}}$ set in $\mathcal{C}$ reads

$$\Omega^r_{\bar{\mathcal{C}}}(\Omega_{\mathcal{C}}) = \frac{\Psi_{\bar{\mathcal{C}}}(b_1 - d_1 - d_2\Omega_{\mathcal{C}})}{b_2 + d_2\Psi_{\bar{\mathcal{C}}}}. \tag{C.5}$$

Coalition $\mathcal{C}$ recognises $\Omega^r_{\bar{\mathcal{C}}}$ when jointly deciding upon $\Omega_{\mathcal{C}}$, that is

$$\max_{(\omega_i)_{i\in\mathcal{C}}} \left\{ \sum_{i\in\mathcal{C}} B_i(\omega_i;\theta_i) - |\mathcal{C}|D\Big(\Omega_{\mathcal{C}} + \Omega^r_{\bar{\mathcal{C}}}(\Omega_{\mathcal{C}})\Big)\right\}. \tag{C.6}$$

Solving and then summing over i in $\mathcal{C}$ gives the aggregate cap

$$\Omega_{\mathcal{C}} = A_{\mathcal{C}} \cdot \Psi_{\mathcal{C}}, \text{ with } A_{\mathcal{C}} \doteq \frac{(b_1 - |\mathcal{C}|d_1)(b_2 + d_2\Psi_{\bar{\mathcal{C}}})^2 - |\mathcal{C}|b_2 d_2(b_1 - d_1)\Psi_{\bar{\mathcal{C}}}}{b_2\Big((b_2 + d_2\Psi_{\bar{\mathcal{C}}})^2 + b_2 d_2|\mathcal{C}|\Psi_{\mathcal{C}}\Big)}, \tag{C.7}$$

and from Equation (C.5) one has for $\bar{\mathcal{C}}$ that

$$\Omega_{\bar{\mathcal{C}}} = A_{\bar{\mathcal{C}}} \cdot \Psi_{\bar{\mathcal{C}}}, \text{ with } A_{\bar{\mathcal{C}}} \doteq \frac{\Psi_{\bar{\mathcal{C}}}\Big(b_1 - d_1 - d_2 A_{\mathcal{C}} \cdot \Psi_{\mathcal{C}}\Big)}{b_2 + d_2\Psi_{\bar{\mathcal{C}}}}. \tag{C.8}$$

Differentiating the above w.r.t. cardinalities of coalitions gives

$$\partial_{|\mathcal{C}|}A_{\mathcal{C}} < 0, \text{ and } \partial_{|\bar{\mathcal{C}}|}A_{\bar{\mathcal{C}}} = -\frac{d_2\Psi_{\mathcal{C}}}{b_2 + d_2\Psi_{\bar{\mathcal{C}}}}\partial_{|\mathcal{C}|}A_{\mathcal{C}} > 0. \tag{C.9}$$

The first inequality means that the higher the number of cooperating jurisdictions, the more pollution externalities are internalised, the higher partnering jurisdictions' individual abatement efforts. The second inequality corresponds to carbon leakage: all else equal, in response to higher abatement efforts in $\mathcal{C}$, jurisdictions in $\bar{\mathcal{C}}$ will lower theirs.

Notice, $\mathcal{C} = \mathcal{I}$ corresponds to full-cooperation with the common abatement effort factor $A_n = \frac{b_1 - nd_1}{b_2 + nd_2\Psi_{\mathcal{I}}}$ and we assume $b_1 > nd_1$. Similarly, when $\bar{\mathcal{C}} = \mathcal{I}$ corresponds to the non-cooperative cap-selection mechanism described in the body of the paper, with common abatement effort $A_1 = \frac{b_1 - d_1}{b_2 + d_2\Psi_{\mathcal{I}}}$. Note that $A_1 < A_n$ since jurisdictions do not internalise the negative externality generated by their polluting activities on the other $n - 1$ jurisdictions.

Two comments are now in order in comparing $\mathcal{C}$-linkage equilibria under full cooperation and no cooperation. In the following, the superscript fc (nc) indicates that caps are selected cooperatively (non cooperatively). First, in terms of equilibrium prices, $\forall \mathcal{C} \in \mathbf{C}$

$$\bar{p}_{\mathcal{C}}^{fc} = b_1 - b_2 A_n + \Theta_{\mathcal{C}} > \bar{p}_{\mathcal{C}}^{nc} = b_1 - b_2 A_1 + \Theta_{\mathcal{C}}, \tag{C.10}$$

that is, autarkic and $\mathcal{C}$-linkage are higher due to more stringent abatement objectives. Now notice that $\forall \mathcal{C} \in \mathbf{C}, \forall i \in \mathcal{C}$

$$q_{\mathcal{C},i}^{fc} - \omega_i^{fc} = q_{\mathcal{C},i}^{nc} - \omega_i^{nc} = \frac{\psi_i}{b_2}(\theta_i - \Theta_{\mathcal{C}}), \tag{C.11}$$

which is independent of the abatement effort factor A, that is the amount of permits traded by any jurisdiction in any $\mathcal{C}$-linkage is the same irrespective of the two cap-selection alternatives and the results of the paper are preserved.