

Two Models of Inter-University Competition: Predicted Capacities, Tuition Fees, and Enrollment

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Abstract

This paper compares two models in which universities compete either through fees (under Bertrand competition, the Anglo-Saxon model) or through enrollment numbers (under Cournot competition, the European model) in a setting where their capacity constraint is chosen endogenously. The results show that, under Bertrand competition with sufficiently convex costs, universities choose a low capacity, which minimizes their costs, and set high fees. In this case, introducing a new university is welfare-improving. Conversely, competition over enrollment leads to higher capacities and a greater number of students per university, which may be detrimental to welfare.

Key words: Cournot competition, Bertrand competition, capacity, higher education market.

Code JEL: C7, H52, I22, I23.

1 Introduction

Public authorities have a real interest in ensuring that students can access a broad range of higher education opportunities. It is therefore important to understand how the strategic behavior of universities affects the availability and cost of higher education for students.

In the UK, tuition fees have nearly tripled since their recent deregulation, a reform that also included relaxing controls on the number of students a given university can recruit. Since 2012, the vast majority of UK public universities have been charging tuition fees of £9,000, the maximum allowed under the government-imposed cap. However, this deregulation should have led to a drop in fees. It is tempting to turn to industrial organization theory to explain this paradox. A particularly interesting issue is how competition between universities affects fee levels and enrollment numbers. Another issue which we address here is the influence of the number and size of universities on fees and enrollment: are many small universities preferable to a few large ones?

Many papers have been written on the tension between teaching and research under different competition models, depending on the funding rules and/or the mobility costs of students

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(????). Studies of the effects on the higher education market of the size and number of universities are scarce however. Recently, ? have reported that an extra university is always welfare improving, especially when the outcome is two equal universities. In this paper, we analyze how the respective capacities (sizes) chosen by a set of more than two universities influence tuition fees and admission rates as a function of the number of universities in the market.

To this end, we assume that two different university systems are possible, which differ in terms of the strategic variable made available by policy makers.

- In the *Anglo-Saxon system*, universities can set their own fees (as in the UK, the USA, and Canada, among others). For the same level of requirements (or abilities of students), universities may compete on fees to attract students (as under Bertrand competition),
- In the *European system*, universities are not free to use the variable pricing strategy as a response to their environment (as in France, Spain, Italy, and Germany). Tuition fees are set by public authorities, but not the number of students enrolled; universities therefore compete on enrollment (as under Cournot competition).

This distinction highlights differences in the higher education systems in place in OECD countries. Table 1 shows some interesting stylized facts on which our analysis is constructed. They suggest that:

- (i) the proportion of small universities is higher in Anglo-Saxon than in (continental) European countries.
- (ii) tuition fees are higher in Anglo-Saxon countries.
- (iii) while Anglo-Saxon universities tend to be smaller, total student enrollment in these countries is much higher.

	Average annual fees (€) ^a	Entry rates into higher education ^b	Average no. of students per university	% of small unis. (< 10000)	% of large unis. (> 25000)
Germany	No fees	53.18%	67 621	13%	42,5%
France	[189-261]	40.85%	53 166	19%	33,5%
Italy	[200-1000]	47.2%	43 550	31%	38%
Spain	[1000-2000]	52.1 %	45 960	17,7%	46%
Netherlands	2000	65.27	72 217	21%	21%
Canada	[3000-4000]	-	29 309	54%	22%
UK	[5000-11000]	67.44%	25 854	38%	15,7%
USA	[3500-20000]	71.02%	36 348	58,5%	14,6%

Table 1: Fees and size of public universities in 2014-15

^a National Student Fee and Support Systems 2014/15, European Commission

^b OECD (2014), Education at a glance, OECD, Publishing, Paris, Indicator C3

Standard economic intuition suggests that small universities competing on price should lower their fees. Similarly, competition through admission between universities with a large capacity and low fees should increase the total number of students attending university. How then do we explain the opposite behavior?

This paper focusses on the influence of capacity constraints in determining the outcomes of competition between homogeneous universities. In this analysis, universities fix their enrollment capacity before competing (via fees or enrollment). Using the notion of a "non-rigid" capacity introduced by [Lommerud and Rasmussen \(2011\)](#), [Lommerud and Rasmussen \(2011\)](#) argue that competition between universities that produce both research and teaching may generate a framework in which the cost of providing education is convex beyond a certain threshold capacity. These authors consider the capacity constraint as exogenous. In this paper, we consider two stages. In the first, universities endogenously fix their (subsequently invariable) capacity before competing in terms of tuition fees (as in the Anglo-Saxon system) or enrollment (as in the European system) to satisfy student demand.¹ When universities enroll students above capacity, the cost function is strictly convex. The form of the cost function may be explained in several ways: (i) universities may be committed to satisfying enrollment demand, (ii) having too many students negatively impacts research output by increasing the opportunity cost of providing education services since the available research time decreases, and (iii) the assets used in educational activities (teachers, libraries, rooms, administrative services, etc.) are fixed and cannot be increased to meet the additional demand.

The contribution of the present work may be understood as follows. To the best of our knowledge, no existing theoretical model explains how capacities, fees and enrollment are decided in the context of competition. We focus on a setup in which universities compete on enrollment or fees in the presence of "soft" capacity constraints ([Lommerud and Rasmussen \(2011\)](#)). On positive grounds, the equilibrium outcomes in terms of fees, enrollment, capacity, and social welfare provide a valuable comparison of the two underlying university systems (Anglo-Saxon vs continental European). We also determine the efficient capacity for each university and compare it with the equilibrium outcome. We show that:

- (i) under Bertrand competition, when costs are sufficiently convex, universities adopt a low capacity in the first stage to set high fees in the second; conversely, competition over admission leads to large capacities and a greater number of students enrolled per university.
- (ii) when the number of universities increases, tuition fees increase up to the maximum level under Bertrand competition, while under Cournot competition, the total number of students enrolled increases.
- (iii) the equilibrium under Bertrand competition is more efficient in terms of cost minimization.
- (iv) increasing the number of universities has a positive impact on social welfare under Bertrand competition with sufficiently convex costs, but not necessarily under Cournot competition.

The paper is organized as follows. Section 2 introduces the model. Section 3 analyzes and compares the equilibrium results in the two competition systems in terms of capacities and fees/enrollment. Section 4 compares the equilibrium with the most efficient capacities in both models, and extends the comparison to social welfare. Section 5 concludes.

¹We assume here that students who apply for admission are similar in terms of abilities and grades. Admission in the Anglo-Saxon system is typically selective. In the European system, a simple admission standard in terms of secondary school grades is applied.

2 Model

We assume that the mission of universities is to create (through research, R) and disseminate (via teaching, T) fundamental knowledge. The general form of each university's objective function is thus $U(T, R)$, where U is strictly increasing in both arguments. The separability of the objective functions allows us to consider, as elsewhere in the higher education literature, universities that specialize solely in teaching or research. Following ?, a university's objective function is defined as:

$$\text{Max } U_i(T, R) \quad (1)$$

with $\frac{\partial U_i}{\partial T} > 0$ and $\frac{\partial U_i}{\partial R} > 0$

As suggested by ?, universities compete for students for two basic reasons:

- As inputs, students are required for the production of education. With regard to teaching activities, we assume for convenience that the level of teaching in university i is equal to the number of students enrolled, n_i , weighted by the parameter $0 < \gamma < 1$:

$$T = \gamma n_i$$

where γ quantifies the relative importance granted to the education and research objectives of the university.

- As clients, students provide the funds a university needs to operate, either directly through fees (f) or indirectly via the government. R represents each university's expenditure on research. Here, the research output, S_i , depends only on the money invested in it:

$$R = S_i$$

More students enrolled may therefore imply an increased research budget, particularly when a university is funded through a per-student government subsidy s ($0 < s < 1$).

However, increasing the size of the student population is costly, in particular when the number of students enrolled exceeds the capacity k . The cost supported by each university depends on its capacity, k_i , with a unit cost δ ; beyond capacity though, the cost increases quadratically with the number of students. According to ?, the capacity constraint is soft and education services provided above this limit incur an additional per unit cost (μ).

Thus, the cost function of a university i is given by

$$C_i(n_i, k_i) = \begin{cases} \delta k_i & \text{if } 0 \leq n_i \leq k_i \\ \delta k_i + \mu(n_i - k_i)^2 & \text{if } n_i > k_i \end{cases} \quad (2)$$

where $\mu > 0$. The cost parameters (δ and μ) are similar for all universities and constant.

Our model includes two stages: in the first, universities choose a fixed capacity k_i ; they compete on price (tuition fees) or quantity (number of students enrolled) to satisfy student demand. Note that in the second stage, δk_i represents a sunk (i.e. unavoidable) cost²(???). This implies that, when enrollment is below capacity, the marginal cost of each student is nil

²With avoidable fixed costs, there are situations in which the price equilibrium does not exist

and the average cost per student decreases with n_i . Above k_i , the marginal cost is strictly convex and the average cost increases, yielding the familiar U-shape.³

The optimization problem of a university i is thus defined as:

$$\text{Max } U_i = \gamma n_i + S_i \quad \text{s.t.} \quad S_i + C_i(n_i, k_i) = n_i(f_i + s) \quad (3)$$

We consider a higher education market with m identical universities, namely with no difference in terms of curriculum, or any other non-price dimension (location, teaching/research trade-off, admission standards, financial endowment etc.). Universities may (without obligation) select students (based on past performance or an admission test). If universities have the same admission requirements, the same subset of the student population is admissible to each one. If not, some proportion of the candidates will nevertheless be eligible for enrollment at several universities. In other words, whatever the recruitment process, universities always compete for some share of the student population. Universities are committed to admitting all eligible students in the second stage. The utility students derive from graduating from university i is defined as $u(\theta) = \theta - f_i$, where θ represents a student's willingness to pay to go to university. We assume that potential students do not differ in their willingness to enroll⁴ and that each student implicitly receives one unit of education. Each student is therefore eligible to attend at least one university.

In order to study inter-university competition and the related issue of imperfect competition in higher education, we consider the following two models:

- (i) In the first, universities compete in terms of quantity (*i.e.* under Cournot competition): universities cannot directly adjust their fees but compete on the number of students enrolled. We consider a market with m universities facing an inverse demand function $D^{-1}(N)$ where $N = \sum_{i=1}^m n_i$ is the total number of students enrolled (each student receiving one unit of education):

$$f(n_i, N_{-i}) = 1 - N$$

$$\text{with } N_{-i} = \sum_{\substack{j=2 \\ j \neq i}}^m n_j.$$

- (ii) In the second, universities compete in terms of price (*i.e.* under Bertrand competition). This scenario describes competition among universities with fee-paying students. For a given university course, eligible students choose to enroll at the university with the lowest fees. Since the universities are otherwise identical, the demand for enrollment at a university i is a discontinuous function of its fees:

$$n_i(f_i, f_{-i}) = \begin{cases} 0 & \text{if } f_i > f_{-i}^{min} \\ \frac{N(f_i)}{m} & \text{if } f_i = f_{-i} \\ N(f_i) = 1 - f_i & \text{if } f_i < f_{-i}^{min} \end{cases}$$

³The average cost per student completion in Australian universities provides at least some evidence of a U-shaped long-run average cost curve in this context (?).

⁴As ? point out, disparities in students' willingness to pay would smooth the process of inter-university competition on price.

with f_i : the tuition fees at university i , and $f_{-i} = \{f_1, \dots, f_{i-1}, f_{i+1}, \dots, f_m\}$: the vector of the fees of all universities on the higher education market. We denote $f_{-i}^{\min} = \text{Min}\{f_1, \dots, f_{i-1}, f_{i+1}, \dots, f_m\}$.

3 Two Competition Models involving m Universities

In this section, we stipulate a two-period model in which m universities fix their capacity, k_i , and then compete in terms of quantity (number of students enrolled, Cournot competition) or fees (Bertrand competition), based on the capacity chosen in the first stage. We study how the convexity of costs and the number of universities influences the equilibrium results, before comparing the two models.

3.1 The Cournot Oligopoly Model with Capacity Constraints

3.1.1 Nash Equilibria

Suppose that m identical universities interact in a two-stage game to decide non-cooperatively on both capacity and enrollment:

- (i) In the first stage of the game, each university i chooses its capacity, k_i , taking the threshold capacity, k_{-i} , of other universities into account.
- (ii) In the second, each university i selects n_i students to enroll, for a given level of capacity, k_i , and for a given number of students at rival universities.

A university's strategy involves choosing a capacity level and then enrolling on that basis.

Consider the utility of university i in the second stage, conditional on k_i :

$$\begin{aligned} U_i(n_i, N_{-i}, k_i) &= n_i(f_i(n_i, N_{-i}) + s + \gamma) - C_i(n_i, k_i) \\ &= n_i((1 - n_i - N_{-i}) + s + \gamma) - C_i(n_i, k_i) \end{aligned}$$

$$\text{with } N_{-i} = \sum_{\substack{j=2 \\ j \neq i}}^m n_j.$$

The utility function is strictly concave in n_i and k_i .

In the second stage, each university chooses to enroll the number of students that maximizes its utility function:

$$\begin{aligned} \frac{\partial U_i}{\partial n_i} &= 0 \\ \Leftrightarrow n_i &= \frac{(1 + \gamma + s) - (m - 1)n_j + 2\mu k_i}{2(1 + \mu)} \end{aligned} \tag{4}$$

with $i \neq j$.

The Nash-Cournot equilibrium is then:

$$n_i = \frac{(1 + \gamma + s)(3 + 2\mu - m) + 2\mu(2(\mu + 1)k_i - (m - 1)k_j)}{(3 + 2\mu - m)(1 + 2\mu + m)} \quad (5)$$

with $i \neq j$. In the first stage, each university maximizes its utility, U_i , as a function of k_i , given the capacity of other universities' capacities k_{-i} . By taking the first order condition, $\frac{\partial U_i}{\partial k_i} = 0$, then imposing symmetry, we can find the equilibrium value k_c^* :

$$k_c^*(m) = \frac{2\mu((1 + 2\mu + m)(3 + 2\mu - m) + (m - 1))(1 + \gamma + s) - ((1 + 2\mu + m)^2(3 + 2\mu - m))\delta}{2\mu((m + 1)(1 + 2\mu + m)(3 + 2\mu - m) - 2\mu(m - 1))} \delta \quad (6)$$

with $k_c^*(m) > 0 \Leftrightarrow \delta < \frac{2\mu((1+2\mu+m)(3+2\mu-m)+(m-1))}{(1+2\mu+m)^2(3+2\mu-m)}(1 + \gamma + s)$ and $m > \lceil 3 + 2\mu \rceil$.

By replacing (??) in (??), the unique and symmetric subgame perfect equilibrium values are:

$$n_c^*(m) = \frac{(1 + 2\mu + m)(3 + 2\mu - m)}{(m + 1)(1 + 2\mu + m)(3 + 2\mu - m) - 2\mu(m - 1)}(1 + \gamma + s - \delta) \quad (7)$$

Hence, the total number of students enrolled:

$$N_c^*(m) = \frac{m(1 + 2\mu + m)(3 + 2\mu - m)}{(m + 1)(1 + 2\mu + m)(3 + 2\mu - m) - 2\mu(m - 1)}(1 + \gamma + s - \delta) \quad (8)$$

Lemma 1. *For a sufficient number of universities, $m > \lceil 3 + 2\mu \rceil$, we verify that:*

$$0 < k_c^*(m) \leq n_c^*(m) \quad \text{if and only if} \quad \underline{\delta} \leq \delta < \bar{\delta}$$

$$\text{with } \underline{\delta} = \frac{2\mu(m-1)}{(m+1)(1+2\mu+m)(3+2\mu-m)}(1 + \gamma + s) \text{ and } \bar{\delta} = \frac{2\mu((1+2\mu+m)(3+2\mu-m)+(m-1))}{(1+2\mu+m)^2(3+2\mu-m)}(1 + \gamma + s)$$

The interpretation of the above lemma is that each university opts to enroll beyond capacity, provided that the unit cost of the installed capacity is not too low (below $\underline{\delta}$). If the fixed capacity cost is above this threshold and the marginal cost per supplementary student is relatively low, students are always enrolled above capacity. (We show that it may be difficult to satisfy these conditions when the number of universities increases.) By contrast, if the capacity cost is low and the parameter μ is high, universities choose a large capacity and limit their enrollment.

3.1.2 Comparative Statics

We now examine the influence of the number of universities on the different equilibrium results.

Corollary 1. *We verify that:*

$$\frac{\partial n_c^*(m)}{\partial m} < 0, \quad \frac{\partial N_c^*(m)}{\partial m} > 0, \quad \frac{\partial f_c^*(m)}{\partial m} < 0 \quad \text{and} \quad \frac{\partial k_c^*(m)}{\partial m} < 0.$$

Proof: see Appendix 1. \square

At equilibrium, the number of students enrolled per university decreases when the number of universities increases, but the total number of students (N) may then increase or decrease. The above corollary shows that the total number of students enrolled grows with the number of universities. As in a standard Cournot oligopoly with convex costs, if the number of institutions increases, the equilibrium outcome approaches the competitive equilibrium: tuition fees are low enough to attract more and more students. However, the capacity chosen by each university decreases as the latter become more numerous.

3.2 Bertrand Oligopoly Model with Capacity Constraint

3.2.1 Nash Equilibria

Universities interact in a two-stage game to decide non-cooperatively on their capacity and tuition fees:

- (i) In the first stage of the game, universities optimally choose their fixed capacity, k_i .
- (ii) In the second, the universities compete on price (viz. fees, f) at fixed capacity.

We can now express the utility U_i of each university i as:

$$U_i(f_i, f_{-i}, k_i, m) = f_i n_i(f_i, f_{-i}) - C(n_i(f_i, f_{-i}), k_i)$$

$$U_i(f_i, f_{-i}, k_i) = \begin{cases} -\delta k_i & \text{if } f_i > f_{-i}^{min} \\ (f_i + \gamma + s) \frac{N(f_i)}{m} - C_i(\frac{N(f_i)}{m}, k_i) = U_d & \text{if } f_i = f_{-i} \\ (f_i + \gamma + s) N(f_i) - C_i(N(f_i), k_i) = U_M & \text{if } f_i < f_{-i}^{min} \end{cases}$$

where as before, f_i stands for the fees of university i . We denote $f_{-i}^{min} = \text{Min}\{f_1, \dots, f_{i-1}, f_{i+1}, \dots, f_m\}$. $N(f_i)$ is the total number of students enrolled (at any university) when university i sets f_i .

The function $U_d(f_i, k_i)$ represents the utility of university i when all universities quote the same fees, while the function U_M represents the utility of university i when it quotes the lowest tuition fees and serves all demand. U_M and U_d are strictly concave in f_i and in k_i , that is $\partial^2 U_d(f_i, k_i) / \partial f_i^2 < 0$, $\partial^2 U_d(f_i, k_i) / \partial k_i^2 < 0$, $\partial^2 U_M(f_i, k_i) / \partial f_i^2 < 0$, $\partial^2 U_M(f_i, k_i) / \partial k_i^2 < 0$.

We define $\bar{f}(k_i)$ as the value that solves $U_M(f, k_i, m) = U_d(f, k_i, m)$. Thus, $\bar{f}(k_i)$ represents the fees threshold at which the university is equally inclined to operate in the higher education market alone and with its rivals. After calculations, we obtain:

$$(1 - f_i)(f_i + \gamma + s) - \mu[(1 - f_i) - k_i]^2 = \frac{1}{m}(1 - f_i)(f_i + \gamma + s) - \mu\left[\frac{(1 - f_i)}{m} - k_i\right]^2$$

$$\bar{f}(k_i, m) = \frac{\mu(m + 1) - m(\gamma + s) - 2m\mu k_i}{m + \mu m + \mu} \quad (9)$$

In the second stage, the fixed capacity cost, δk_i , is sunk and the university only quotes a fee if the variable part of the utility is positive, *i.e.* if $U_d(f, k_i) \geq -\delta k_i$. Thus, we also define \hat{f}_i , the lowest fees compatible with enrolling students in the second stage, as the value that solves $U_d(f, k_i) = -\delta k_i$ for a given k_i .

Finally, we define f^* , the fees that maximize the utility of university i when all universities operate in the higher education market. In simple terms, this value can be interpreted as the maximum fees when all universities have chosen the same capacity in the first stage.

$$f^*(k_i, m) = \arg \max_f \{U_i(f_i, k_i, m)\} = \frac{m(1 - \gamma - s) + 2\mu(1 - mk_i)}{2(m + \mu)} \quad (10)$$

It is important to understand how these utility functions U_M and U_d , and fees \hat{f}_i , f^* , and \bar{f} depend on one another.

Lemma 2.

$$\begin{cases} \bar{f}(k_i, m) < f^*(k_i, m) & \text{if } \mu < \frac{m}{m-1} \\ \bar{f}(k_i, m) = f^*(k_i, m) = \tilde{f} & \text{if } \mu = \frac{m}{m-1} \\ \bar{f}(k_i, m) > f^*(k_i, m) & \text{if } \mu > \frac{m}{m-1} \end{cases}$$

In the following proposition, we take a university's fixed capacity as given and look for the Nash equilibrium in fees.

Proposition 1. *In the second stage, (f_1, \dots, f_m) is a pure-strategy Nash equilibrium if and only if $f_1(k_1) = f_2(k_2) = \dots = f_m(k_m) = f^N(k_i)$, such that:*

(i) *If $\mu < \frac{m}{m-1}$, $f^N \in [\hat{f}(k_i), \bar{f}(k_i)]$ and $\bar{f}_1(k_1) = \dots = \bar{f}_m(k_m) = \bar{f}(k_i)$ is a payoff-dominant pure-strategy Nash equilibrium*

(ii) *If $\mu = \frac{m}{m-1}$, $f^N = \bar{f}(k_i) = f^*(k_i) = \tilde{f}(k_i)$*

(iii) *If $\mu > \frac{m}{m-1}$, $f^N \in [\hat{f}(k_i), \bar{f}(k_i)]$ and $f^* \in [\hat{f}, \bar{f}]$ is a payoff-dominant pure-strategy Nash equilibrium*

Proof: see Appendix 2. \square

The following Table (??) presents the outcomes of Bertrand competition in the second stage of the game in different settings.

Table 2: Different Bertrand competition scenarios in the second stage of the game.

If $\mu < \frac{m}{m-1}$	If $\mu = \frac{m}{m-1}$	If $\mu > \frac{m}{m-1}$
$f^N = \bar{f}(k_i, m)$	$f^N = \tilde{f}(k_i, m)$	$f^N = f^*(k_i, m)$
$= \frac{\mu(m+1) - m(\gamma+s) - 2m\mu k_i}{m + \mu m + \mu}$	$= \frac{(m+1) - (m-1)(\gamma+s) - 2mk_i}{2m}$	$= \frac{m(1-\gamma-s) + 2\mu(1-mk_i)}{2(m+\mu)}$
$\bar{n}(k_i, m) = \frac{1+\gamma+s+2\mu k_i}{m+\mu m+\mu}$	$\tilde{n} = \frac{(m-1)(1+\gamma+s) + 2mk_i}{2m^2}$	$n^*(k_i, m) = \frac{1+\gamma+s+2\mu k_i}{2(m+\mu)}$

The predicted Nash equilibrium in the second stage of the game is essentially identical to the one reported by ?. For all fees above \bar{f} , no student enrolls at university i , undermining its finances. When the other universities charge any fee $f \in [\hat{f}, \bar{f}]$, the best response for university i is to quote the same tuition fees so that students split between the institutions. A university can increase its revenue (through higher enrollment) by lowering its fees, but the corresponding costs, strictly convex above capacity, will increase even more, making a fee decrease nonprofitable. For this reason, a continuum of fees above the competitive price ($f \in [\hat{f}, \bar{f}]$) can be sustained at Nash equilibria in pure strategies. A high value of parameter μ means that costs are sufficiently convex (*i. e.* $C''(n_i) \geq \frac{2m}{m-1}(\frac{-1}{D'})$ which is equivalent to $\mu \geq \frac{m}{m-1}$)⁵ to ensure that high fees are sustained as Nash equilibria in pure strategies (?). The symmetry and the payoff dominance criterion are sufficient here to reduce the set of equilibria⁶ and prove the uniqueness of the solution⁷. There are then three fee equilibria: (i) when $\mu < \frac{m}{m-1}$, the symmetric fee $f = \bar{f}$ is the unique (payoff-dominant) pure-strategy Nash equilibrium; (ii) when $\mu = \frac{m}{m-1}$, there exists a unique and symmetric equilibrium such that $f = f^* = \bar{f}$; and (iii) when $\mu > \frac{m}{m-1}$, the unique and symmetric equilibrium is such that $f = f^*$.

Universities set their fixed capacity in anticipation of its effect on the fee equilibria in the second stage.

Each university chooses its capacity non-cooperatively, by maximizing its utility function with respect to k_i :

$$\frac{\partial U_i(f(k_i), k_i)}{k_i} = 0 \quad \Leftrightarrow$$

$$k_i = \frac{f'(k_i)(m(1-\gamma-s) + 2\mu - 2(m+\mu)f) - m(\delta m - 2\mu) - 2m\mu f}{2m\mu(f'(k_i) + m)}$$

For each configuration, depending on μ , a unique and symmetric solution exists, satisfying $\frac{\partial U_i}{\partial k_i} = 0$, for which:

(i) If $\mu < \frac{m}{m-1}$,

$$\begin{cases} \bar{k}_b^*(m) = \frac{4\mu^2 m(1+\gamma+s) - (m+\mu m + \mu)^2 \delta}{2\mu((m+\mu m + \mu)^2 - 4m\mu^2)} > 0 \quad \text{for } \delta < \frac{4\mu^2 m}{(m+\mu m + \mu)^2} (1+s+\gamma) \\ \bar{f}_b^*(m) = 1 - m \frac{(m+\mu m + \mu)(1+\gamma+s-\delta)}{(m+\mu m + \mu)^2 - 4m\mu^2} \\ \bar{n}_b^*(m) = \frac{(m+\mu m + \mu)(1+\gamma+s-\delta)}{(m+\mu m + \mu)^2 - 4m\mu^2} \end{cases}$$

(ii) If $\mu = \frac{m}{m-1}$,

$$\begin{cases} \tilde{k}_b^*(m) = \frac{1+\gamma+s-m\delta}{2m} > 0 \quad \text{for } \delta < \frac{1}{m}(1+s+\gamma) \\ \tilde{f}_b^*(m) = \frac{1-\gamma-s+\delta}{2} \\ \tilde{n}_b^*(m) = \frac{1+\gamma+s-\delta}{2m} \end{cases}$$

⁵This condition is similar to the one in ? p.86, Prop.1.

⁶An equilibrium point is said to be payoff dominant if it is not strictly dominated by another equilibrium point; that is, there exists no other equilibrium in which utilities are higher for all universities (?).

⁷See ? for a proof of this uniqueness.

(iii) If $\mu > \frac{m}{m-1}$,

$$\begin{cases} k_b^*(m) = \frac{\mu(1+\gamma+s)-(m+\mu)\delta}{2m\mu} > 0 & \text{for } \delta < \frac{\mu}{\mu+m}(1+s+\gamma) \\ f_b^*(m) = \frac{1-\gamma-s+\delta}{2} \\ n_b^*(m) = \frac{1+\gamma+s-\delta}{2m} \end{cases}$$

Corollary 2. $\forall \mu, s, \delta > 0, m \geq 3$ and $k > 0$, we verify that at equilibrium:

$$k_b(m) < n_b(m).$$

Proof: see Appendix 3. \square

Competition on tuition fees between m universities is likely to induce a situation in which student demand exceeds capacity. Whatever the values of the parameters in the game and the number of universities, each institution chooses to enroll beyond capacity at a convex marginal cost.

3.2.2 Comparative Statics

We now investigate how the number of universities influences the equilibrium results.

Corollary 3. For $m \geq 3$, we verify the following properties:

(i) If $\mu < \frac{m}{m-1}$:

$$\begin{cases} \frac{\partial \bar{k}_b^*(m)}{\partial m} < 0 \\ \frac{\partial \bar{f}_b^*(m)}{\partial m} > 0 \\ \frac{\partial \bar{n}_b^*(m)}{\partial m} < 0 \end{cases} \text{ if } \mu > \bar{\mu}$$

$$\text{with } \bar{\mu} = \frac{-m(m-1)+2m\sqrt{m(m-1)}}{3m^2-2m-1} \in (0, \frac{m}{m-1})$$

(ii) If $\mu \geq \frac{m}{m-1}$:

$$\begin{cases} \frac{\partial k_b^*(m)}{\partial m} < 0 \\ \frac{\partial f_b^*(m)}{\partial m} = 0 \\ \frac{\partial n_b^*(m)}{\partial m} < 0 \end{cases}$$

Proof: see Appendix 4. \square

The above properties lead to two surprising effects:

- (i) Conventional wisdom suggests that when the number of universities increases under Bertrand competition, the outcome tends to be more competitive. As shown in Section 3.1, under Cournot competition, tuition fees decrease and the total number of students increases as the number of universities (m) increases. Under Bertrand competition however, fees increase with m . This is somewhat counterintuitive.

- (ii) If the cost function is sufficiently convex ($\mu > \frac{m}{m-1}$), f^* is the unique equilibrium and is constant with m . For intermediate marginal costs ($\mu < \frac{m}{m-1}$ but $\mu > \bar{\mu}$), \bar{f} increases toward f^* .

An intuitive explanation of these properties is that the increase in m has two opposite effects on tuition fees. On the one hand, with more universities, each one faces lower marginal costs since the number of students enrolled (n_i) decreases (the demand being shared out evenly). The lower marginal costs (the cost function is less convex) encourage universities to reduce their fees through the "demand effect". On the other hand, the "capacity effect" promotes the opposite behavior: when universities are more numerous, each one has a lower capacity. If costs are more convex, marginal costs tend to be higher, which pushes fees up. Either effect can dominate, depending on the convexity of the cost function. When μ is very low ($< \bar{\mu}$), the demand effect dominates and fees (\bar{f}) fall as m increases. A stronger capacity effect, $\frac{\partial \bar{n}_b^*(m)}{\partial m} - \frac{\partial \bar{k}_b^*(m)}{\partial m} > 0$, implies higher marginal costs such that fees increase with m up to the maximum, f^* . When costs become "too" convex ($C''(n_i) \geq \frac{2m}{m-1}$ or $\mu \geq \frac{m}{m-1}$), each university chooses a capacity that maximizes its revenue and minimizes the average cost per student. This adjustment allows the fees to be maintained at the maximum level, f^* , regardless of the number of universities.

3.3 Comparison between the Two Competition Models

Proposition 2. *If $k_i > 0$, for all parameters $\gamma, s, \delta > 0$, the subgame perfect Nash Equilibrium in terms of capacity is such that:*

- (i) if $\mu < \frac{m}{m-1}$ then $\bar{k}_b^*(m) < k_c^*(m)$
- (ii) if $\mu = \frac{m}{m-1}$ then $\tilde{k}_b^*(m) < k_c^*(m)$
- (iii) if $\mu > \frac{m}{m-1}$ then $k_b^*(m) < k_c^*(m)$

Proof: see Appendix 5. \square

Whatever the convexity of the cost function, university capacities are always higher under Cournot competition than under Bertrand competition. In practice, universities should therefore be smaller in countries where fees have been deregulated, and larger in countries where fees are regulated.

Proposition 3. *If $k_i > 0$, for all parameters $\gamma, s, \delta, \mu > 0$, the subgame perfect Nash equilibrium in terms of tuition fees is such that:*

- (i) if $\mu \leq \underline{\mu}(m)$ then $\bar{f}_b^*(m) \leq f_c^*(m)$ and $\bar{n}_b^*(m) \geq n_c^*(m)$,
- (ii) if $\underline{\mu}(m) < \mu < \frac{m}{m-1}$ then $\bar{f}_b^*(m) > f_c^*(m)$ and $\bar{n}_b^*(m) < n_c^*(m)$,
- (iii) if $\mu = \frac{m}{m-1}$ then $\tilde{f}_b^*(m) > f_c^*(m)$ and $\tilde{n}_b^*(m) < n_c^*(m)$,
- (iv) if $\mu > \frac{m}{m-1}$, $f_b^*(m) > f_c^*(m)$ and $n_b^*(m) < n_c^*(m)$.

with $\underline{\mu} \leq 0.458952$ and $\frac{\partial \underline{\mu}}{\partial m} < 0$.

Proof: see Appendix 5. \square

We show that Bertrand competition can induce both higher and lower fees than those under Cournot competition, depending on the level of the marginal costs μ . If they are sufficiently convex, Bertrand competition leads to high (low) tuition fees (average enrollment), that are (is) higher (lower) than under Cournot competition. Conventional wisdom suggests that the Bertrand model encourages competition when costs are lowly convex. Whatever the values of the parameter governing the game (s , γ , δ and μ), there are fewer students enrolled per university in a deregulated system than in a regulated one.

4 Comparison between Universities' Equilibrium and Efficient Capacities

Under both competition models, the cost function of university i is given by:

$$C(k_i, n_i) = \begin{cases} \delta k_i & \text{if } n_i \leq k_i \\ \delta k_i + \mu(n_i - k_i)^2 & \text{if } n_i > k_i \end{cases} \quad (11)$$

Because of average cost curve is U-shaped in k , there exists an efficient capacity that minimizes the average cost:

$$\text{Min}_{n_i} AC(k_i, n_i) = \text{Min}_{n_i} \left[\frac{C(k_i, n_i)}{n} \right]$$

The capacity that minimizes the average cost for a given number of students enrolled is:

$$k_{\min} = \frac{2\mu n_i - \delta}{2\mu}$$

Assuming that universities adopt the efficient capacity, the results obtained under the two types of competition are presented Tables 3 and 4:

capacity	$k_c^{\min} = \frac{2\mu(1+\gamma+s)-(2\mu+m+1)\delta}{2\mu(m+1)}$
number of students	$n_c^{\min} = \frac{1+\gamma+s-\delta}{m+1}$
fees	$f_c^{\min} = \frac{1-m(\gamma+s-\delta)}{m+1}$

Table 3: Efficient solution in k under Cournot competition

Proposition 4. *Comparison of the equilibrium outcomes of the two competition models with the efficient solution:*

Our results indicate that Cournot competition tends to induce capacities that are too high in terms of minimizing the average cost per student for a given university. Bertrand competition is more efficient in this regard, but only if costs are sufficiently convex.

	$\mu < \frac{m}{m-1}$	$\mu \geq \frac{m}{m-1}$
capacity	$k_b^{\min} = \frac{2\mu(1+\gamma+s)-(m+\mu m+\mu)\delta}{2\mu(m+\mu m-\mu)}$	$k_b^{\min} = \frac{\mu(1+\gamma+s)-(m+\mu)\delta}{2\mu m}$
number of students	$\bar{n}_b^{\min} = \frac{1+\gamma+s-\delta}{m+\mu m-\mu}$	$n_b^{\min} = \frac{1+\gamma+s-\delta}{2m}$
fees	$\bar{f}_b^{\min} = \frac{\mu(m-1)-m(\gamma+s-\delta)}{m+\mu m-\mu}$	$f_b^{\min} = \frac{1-\gamma-s+\delta}{2}$

Table 4: Efficient solution in k under Bertrand competition

	Cournot equilibrium vs efficient solution	Bertrand equilibrium vs efficient solution	
		$\mu < \frac{m}{m-1}$	$\mu \geq \frac{m}{m-1}$
capacity	$k_c^{\min} < k_c^*$	$k_b^{\min} > k_b^*$	$k_b^{\min} = k_b^* = \bar{k}_b$
number of students/university	$n_c^{\min} > n_c^*$	$n_b^{\min} < n_b^*$	$n_b^{\min} = n_b^* = \bar{f}_b$
fees	$f_c^{\min} < f_c^*$	$f_b^{\min} < f_b^*$	$f_b^{\min} = f_b^* = \bar{f}_b$
Total number of students	$N_c^{\min} > N_c^*$	$N_b^{\min} > N_b^*$	$N_b^{\min} = N_b^* = \bar{f}_b$

5 Conclusion

In this paper, we have analyzed a two-stage game between universities that fix their capacity, and then compete on tuition fees or enrollment. Interestingly, our model emphasizes the role of the strategic variable in deciding the outcome of the game and offers a framework that explains some stylized facts. Competition through fees produces a higher-education market in which small universities charge high fees. On the contrary, when competition is conducted through admission, the universities are larger and the tuition fees lower. Bertrand competition leads universities to adopt the efficient capacity, allowing them to minimize their average cost per student; this is not the case under Cournot competition, in which the high capacities may be detrimental to social welfare.

Finally, competition between an increasing number of small universities allows fees to be sustained at their maximum level without reducing either the total number of admissions or social welfare. Under Cournot competition, the increasing number of (too) large universities is detrimental to social welfare. These results may be used as guidelines for higher-education policy-makers who have been encouraging the clustering of universities (particularly in France).

Some of the simplifying assumptions made here merit further discussion. First of all, the particular form of the demand function in this work is critical to the results obtained. We assume that demand is infinitely elastic: this elasticity can have an impact on the different equilibrium outcomes of the two models. The nature of student support policies (financial or other) may also influence our results. Secondly, we assume that the universities are identical in terms of curriculum and all other non-price dimensions. Further investigations are required to include these factors in the two models.

Appendix 1: Proof of Corollary 2

- i) $\frac{\partial n_c^*(m)}{\partial m} = -\frac{(1+2\mu+m)^2((3+2\mu-m)^2-2\mu+8\mu(m-1)(1+\mu))}{((m+1)(1+2\mu+m)(3+2\mu-m)-2\mu(m-1))^2}(1+\gamma+s-\delta) < 0$
- ii) $\frac{\partial N_c^*(m)}{\partial m} = n_c^*(m) + m \frac{\partial n_c^*(m)}{\partial m} =$
- iii) $\frac{\partial f_c^*(m)}{\partial m} = -\frac{\partial N_c^*(m)}{\partial m} < 0$

iv) From Equation ??,

$$k_c^*(m) = \frac{1 + 2\mu + m}{2\mu} n_c^*(m) + \frac{1 + \gamma + s}{2\mu}$$

$$\text{sign} \left(\frac{\partial k_c^*(m)}{\partial m} \right) = \text{sign} \left(n_c^*(m) + (1 + 2\mu + m) \frac{\partial n_c^*(m)}{\partial m} \right) = \text{sign} \left(\frac{(3+2\mu-m)}{((m+1)(1+2\mu+m)(3+2\mu-m)-2\mu(m-1))} + \frac{\partial n_c^*(m)}{\partial m} \right)$$

$$\text{sign} (2\mu(3 + 2\mu - m)(1 + \mu) - 4\mu(m - 1) - 2\mu(3 + 2\mu - m)^2(1 + 2\mu + m)) < 0$$

Appendix 2: Proof of Proposition 1

Let us first investigate symmetric strategy profiles in the interval $[\hat{f}, \bar{f}]$. When a competitor charges fees $f \in [\hat{f}_i; \bar{f}_i]$, the best response for university i is to quote the same amount. When university i quotes the same fees, it obtains $U_d(f)$. We know that for all $f \geq \hat{f}_i$, $U_d(f) \geq -TFC$. If the university deviates (by quoting $f - \epsilon$), it gains $U_d(f - \epsilon)$. We also know that for all $f \in [\hat{f}_i; \bar{f}_i]$, $U_d(f) \geq U_M(f) > U_M(f - \epsilon)$. Since the university must satisfy all the demand it faces, the additional revenue (from higher enrollment) is less than the increase in costs: the university enrolls additional students at an excessive marginal cost. By quoting $f + \epsilon$, university i receives no demand and obtains zero variable utility for its research activities. The optimal strategy is therefore for each university to quote the same fee. There are no incentives to deviate, which proves the implication in Proposition ?. It also proves that all asymmetrical strategy profiles in which at least one firm quotes a price in the interval are not Nash equilibria. We now have to investigate all the other strategy profiles (viz. those in which firms only quote prices outside the interval). It is easy to check that for all symmetric strategic profiles with $f < \hat{f}$, the firm's interest is to increase its tuition fees. Conversely, universities will lower their fees if $f > \bar{f}$. The payoff dominance criterion, the natural criterion given that all actors are presumably fully rational, is sufficient to provide uniqueness in the three configurations. \square

Appendix 3: Proof of Corollary 3

Let us compare the number of students enrolled in a given university with its capacity in different situations:

$$n_b^*(m) > k_b^*(m) \quad \forall \gamma, s, \text{ et } m$$

$$(i) \text{ If } \mu < \frac{m}{m-1} : n_b^*(m) > k_b^*(m) \Leftrightarrow 2\mu(m + m\mu + \mu)(1 + \gamma + s - \delta) > 4\mu^2 m(1 + \gamma + s) - (m + m\mu + \mu)^2 \delta$$

$$\Leftrightarrow 2\mu(m + \mu - m\mu)(1 + \gamma + s) > (m + m\mu + \mu)(\mu - m - m\mu)\delta$$

which is always verified because $m + \mu - m\mu > 0$ and $\mu - m - m\mu < 0$.

$$(ii) \text{ If } \mu = \frac{m}{m-1} : \tilde{n}_b^*(m) > \tilde{k}_b^*(m) \Leftrightarrow \frac{1+\gamma+s-m\delta}{2m} < \frac{1+\gamma+s-\delta}{2m} \quad \forall m \geq 2 \text{ which is always verified.}$$

$$(iii) \text{ If } \mu > \frac{m}{m-1} : n_b^*(m) > k_b^*(m) \Leftrightarrow \frac{m+\mu}{\mu} > 1 \quad \forall m \geq 2 \text{ which is always verified.}$$

Appendix 4: Proof of Corollary 5

Case 1. If $\mu < \frac{m}{m-1}$:

$$\frac{\partial \bar{n}_b^*(m)}{\partial m} = -\frac{(1+\mu)(m+\mu m+\mu)^2 - 4\mu^3}{((m+\mu m+\mu)^2 - 4m\mu^2)^2} (1+s+\gamma-\delta) < 0$$

$$\frac{\partial \bar{k}_b^*(m)}{\partial m} = -\frac{2\mu m^2(1+\mu)^2 - 2\mu^3}{((m+\mu m+\mu)^2 - 4m\mu^2)^2} (1+s+\gamma-\delta) < 0$$

$$\frac{\partial \bar{N}_b^*(m)}{\partial m} = \bar{n}_b^*(m) + m \frac{\partial \bar{n}_b^*(m)}{\partial m} =$$

$$(\mu^2(-3m^2 + 2m + 1) + \mu(2m(1-m)) + m^2) \frac{(1+s+\gamma-\delta)}{((m+\mu m+\mu)^2 - 4m\mu^2)^2} < 0$$

$$\text{if } \bar{\mu} < \mu \leq \frac{m}{m-1}$$

$$\text{with } \bar{\mu} = \frac{-m(m-1)+2m\sqrt{m(m-1)}}{3m^2-2m-1}$$

$$\frac{\partial \bar{f}_b^*(m)}{\partial m} = -\frac{\partial \bar{N}_b^*(m)}{\partial m} > 0$$

- if $\mu > \bar{\mu}$ hence $\frac{\partial \bar{f}_b^*(m)}{\partial m} > 0 \Leftrightarrow \frac{\partial \bar{N}_b^*(m)}{\partial m} < 0, \quad \forall m \geq 3$
- if $\mu = \bar{\mu}$ hence $\frac{\partial \bar{f}_b^*(m)}{\partial m} = 0 \Leftrightarrow \frac{\partial \bar{N}_b^*(m)}{\partial m} = 0 \quad \forall m \geq 3$
- if $\mu < \bar{\mu}$ hence $\frac{\partial \bar{f}_b^*(m)}{\partial m} < 0 \Leftrightarrow \frac{\partial \bar{N}_b^*(m)}{\partial m} > 0 \quad \forall m \geq 3$

$$\text{with } \bar{\mu} = \frac{m-m^2+2\sqrt{m^4-m^3}}{3m^2-2m-1}$$

and we obtain cases 2 ($\mu = \frac{m}{m-1}$) and 3 ($\mu > \frac{m}{m-1}$) with straightforward computations.

Appendix 5: Proof of Propositions 2-3

Case 1 - If $\mu < \frac{m}{m-1}$:

$$n_c^*(m) - \bar{n}_b^*(m) =$$

$$\frac{m^2(5\mu+13\mu^2+12\mu^3+4\mu^4) - m(3+10\mu+10\mu^2+16\mu^3+8\mu^4) + \mu(4\mu^3+4\mu^2-7\mu-3)}{((m+1)(1+2\mu)(3+2\mu)-2\mu(m-1))(\mu^2+m^2-2m\mu+2m^2\mu+\mu^2m^2)} (1+\gamma+s-\delta)$$

We verify that

$$\text{sign}(n_c^*(m) - \bar{n}_b^*(m)) =$$

$$\text{sign}(m^2(5\mu + 13\mu^2 + 12\mu^3 + 4\mu^4) - m(3 + 10\mu + 10\mu^2 + 16\mu^3 + 8\mu^4) + \mu(4\mu^3 + 4\mu^2 - 7\mu - 3))$$

We define $\underline{\mu}(m)$ such that if $\mu > \underline{\mu}(m)$,

$$n_c^*(m) > \bar{n}_b^*(m)$$

The value of $\underline{\mu}(m)$ was solved using the Mathematica program. For $m = 2$, we have $\underline{\mu} = 0.458952$, for $m=3$, $\underline{\mu} = 0.249037$;...for $m = 10$, $\underline{\mu} = 0.06297$.

We conclude that:

- $n_c^*(m) < \bar{n}_b^*(m)$, $f_c^*(m) > \bar{f}_b^*(m)$, and $N_c^*(m) < \bar{N}_b^*(m)$ if $\mu \in]0, \underline{\mu}(m)[$
- $n_c^*(m) = \bar{n}_b^*(m)$, $f_c^*(m) = \bar{f}_b^*(m)$, and $N_c^*(m) = \bar{N}_b^*(m)$ if $\mu = \underline{\mu}(m)$
- $n_c^*(m) > \bar{n}_b^*(m)$, $f_c^*(m) < \bar{f}_b^*(m)$, and $N_c^*(m) > \bar{N}_b^*(m)$ if $\mu \in]\underline{\mu}(m), \frac{m}{m-1}[$

We verify that $\bar{k}_c^*(m) > \bar{k}_b^*(m) \quad \forall \mu \in [0, \frac{m}{m-1}[$.

Case 2 - If $\mu = \frac{m}{m-1}$, we have

$$\begin{aligned} n_c^*(m) - \tilde{n}_b^*(m) &= \frac{(m+1)^2(17m^2-16m+3)}{2m(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) > 0 \\ N_c^*(m) - \tilde{N}_b^*(m) &= \frac{(m+1)^2(17m^2-16m+3)}{m(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) > 0 \\ f_c^*(m) - \tilde{f}_b^*(m) &= -\frac{(m+1)(17m^2-16m+3)}{2(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) < 0 \\ k_c^*(m) - \tilde{k}_b^*(m) &= \frac{(17m-27m^2+11m^3+2m^4-3)}{2m(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) > 0 \end{aligned}$$

Case 3 - If $\mu > \frac{m}{m-1}$:

$$\begin{aligned} n_c^*(m) - n_b^*(m) &= \frac{(m+1)^2(17m^2-16m+3)}{2m(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) > 0 \\ N_c^*(m) - N_b^*(m) &= \frac{(m+1)^2(17m^2-16m+3)}{m(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) > 0 \\ f_c^*(m) - f_b^*(m) &= -\frac{(m+1)(17m^2-16m+3)}{2(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) < 0 \\ \bar{k}_c^*(m) - \bar{k}_b^*(m) &= \frac{(m+2m^2+10\mu(m-1)+4\mu^2(m-1))}{6m(m+1)+2\mu m(6m+5)+4\mu^2 m(4m+1)}(1 + \gamma + s - \delta) > 0 \end{aligned}$$