# Impact of displacements costs on a spatially scattered labor market A theoretical approach

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March 6, 2017

### Abstract

We develop in this paper a theoretical model used for explaining the mechanism through which the homeworkplace trips generalized costs impact employment in a spatially scattered labor market. One of the innovative aspects of the paper consists on building a micro-geographic founded matching function that links the parameters of the passengers transport system to those of employment. This function can be used to evaluate the impact of a changing in the transport system parameters on the mismatch between vacant positions and job seekers, at both a local and a global level, in cities whose spatial structures are complex.

### 1 Introduction

Facilitating access to employment for job seekers is an objective stated by politicy makers when they engage in the provision of new transport infrastructures or enhancing existing public transport services. The basic idea is that by enhancing the accessibility, the generalized displacement costs<sup>1</sup> decrease, thus unemployed workers can enlarge their job seeking scope and increase their chances to get out of unemployment.

This need of enhancing accessibility comes out from the spatial disconnection between homes and job opportunities that has long been highlighted in urban economics, particularly in the literature on the spatial mismatch hypothesis (SMH). Born in the USA, this literature focuses on this spatial disconnection, attibuting it mainly to the relocation of unskilled jobs to the suburbs and its consequences on the low-skilled black workers, who are locked in the city centers and cannot move closer to the suburban jobs. The transport system plays here a central role : the long durations and the high costs of the trips between city centers and suburbs are an explanatory factor of the bad labor outcomes of these populations.

The spatial mismatch hypothesis emphasizes the role of the spatial structure, especially the transport system, in the matching process between job seekers and firms with vacancies. Nevertheless, if there are urban models trying to explain the consequences of spatial mismatch,

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<sup>&</sup>lt;sup>1</sup>their monetary costs and / or their durations

to the best of our knowledge, there is no theoretical paper linking directly the parameters of the transport system to employment outcomes. The purpose of the present work is to bridge this gap. Indeed, one of the main contributions we present here consists on building a matching function that has three innovative features. The first is putting space in the heart of the matching process by considering a large city split into multiple districts interconnected via a transport system; the second is building this function from an explicit mechanism based upon a job search process; and the third is taking account of the transport system parameters and the accessibility levels of residence and workplaces locations. Built in this manner, our matching function can be used to evaluate the impact of a change in the parameters of the transport system on the mismatch between vacant positions and job seekers, at both a local and a global level, in cities whose spatial structures are complex.

The present paper suggests a new theoretical approach to addressing the SMH. We go beyond the simple and usual geographical framework of the monocentric city by considering a city consisting on many districts and assuming that job seekers and job opportunities can be located in any of them. Using this spatial structure, we analyze the impact of the transport infrastructures and services on the labor market outcomes at both the city level and the district level.

The model we present is three stages. The first consists in building for each couple of districts a matching function giving the number of successful linkages between the job seekers of a district and the vacancies available elsewhere. The aggregation of all these functions generates the matching function giving the number of successful linkages in the whole city. The second stage consists in developing an extension of the basic job search model à la Stigler in which we take the space and the transport infrastructures parameters <sup>2</sup> into account. This extension is used to show how the accessibility of each district impacts the reservation wage of the workers who live in. The third and last stage consists in completing the matching mechanism by a mechanism of jobs death that feeds the stocks of jobless workers and a mechanism of job creations. The objective is to come up with a global formal representation of the labor market making us able to show how the macroeconomic equilibria <sup>3</sup> are influenced by the transport infrastructures and services.

### 2 Theoretical background

When we analyze the harmful impact of the spatial disconnection between homes and job opportunities on employment we deal with an issue that relates to the spatial mismatch hypothesis, to the job search theory, and to the matching theory. In this section, we give a short overview of the main contributions to these theories.

#### 2.1 The Spatial Mismatch Hypothesis

The harmful impact of the spatial separation between the workers residencies and the job opportunities was highlighted for the first time in the late 1960s by Kain. In a seminal article [11], he argues that the main reason of the adverse labor market outcomes for the black American workers is the disconnection between the inner districts where they live and the suburban areas in which the largest number of unskilled jobs are created.

 $<sup>^{2}</sup>$ The generalized displacement costs (monetary + time cost) between each couple of districts

<sup>&</sup>lt;sup>3</sup>global market and locals

On the aftermath of Kain's article, many empirical papers tested the relevance of the SMH<sup>4</sup>. The empirical evidence suggests that the inhabitants of isolated areas with bad access to employment centers face difficulties in gathering information about vacancies and suffer from higher job-search costs. As a consequence, their job seeking is less intense and less efficient. Based on these results, various welfare policies have been recommended by economists. Among their suggestions, we mention the following ones : helping vulnerable populations to relocate near employment centers, attract jobs to areas suffering from high unemployment rates, helping deprived populations to buy cars and improving the transport connections between isolated areas and job centers.

The first theoretical papers modeling the mechanisms behind the empirical results have only been published in the late 1990's. Gobillon et al. (2007) [9] propose a review of this literature. According to the authors, the mechanisms explaining the spatial mismatch can be classified in two categories : explanations focussing on the firm's point of view (ex: Zenou and Boccard (2000) [29] <sup>5</sup>; Zenou (2002) [25] <sup>6</sup>) and explanations focussing on the worker's point of view. In turn, the later category can be divided in two sub-categories: a first series of papers highlight the role of racial restrictions preventing black households from relocating to the suburbs (Bruckner and Martin (1997 [2]; Bruckner and Zenou [3]); and a second series of papers focus on the decrease in the quality of information and in the search intensity when the distance from employment centers increases. In the second sub-category, the search-matching model à la Mortensen and Pissarides is taken as a starting point, adding the spatial dimension by taking into account the locations of households and employment centers, and by considering the impact of the distance on search efficiency. Among the most important papers falling within this strand of literature we find Coulson and al. (2001 [4]) and Wasmer and Zenou (2002 [26]), where the standard macroeconomic matching function is directly used; and Smith and Zenou (2003 [21] and [22]) where a well behaved spatial matching function is built using a micro scenario.

Being inspired by the American environment, the literature on the SMH has many limitations. First, the large majority of the papers consider a monocentric city where all the job opportunities are concentrated in the CBD or shared between the CBD and a SBD. Second, the backdrop inherent to the developments of this literature is the racial discrimination faced by the blacks in the housing market or the labor market. The objective of the theoretical papers proposed so far is to come up with a model that generates a steady state characterized by the fact that blacks live in the city-centers and suffer from high unemployment and whites live in the suburbs and enjoy low unemployment<sup>7</sup>. Third, in the models proposed so far, addressing the issue of transport infrastructures and services is not a priority. In fact, displacement costs are -only- introduced in order to get a bid rent function faced by people when they choose the location of their home<sup>8</sup>. Fourth, the trip durations are never taken into account, despite their importance. And last but not least, in all the papers known to us, the workers are never considered as taking decisions concerning their participation in the labor market. They are willing to work whatever the proposed wage is, they all have the same productivity and are hired by

<sup>&</sup>lt;sup>4</sup>Most of them focus on the largest US cities, many recent ones deal with Chinese cities (Suhong et al. [24]), while the studies concerning the European cities remain too scarce (Duguet et al. [5]).

 $<sup>^{5}\</sup>mathrm{The}$  employer discrimination on the basis of the applicant's residential location

<sup>&</sup>lt;sup>6</sup>The low productivity level of workers whose homes are located too far away from the firm site

<sup>&</sup>lt;sup>7</sup>Some works where the racial dimension is dropped exist. However the assumptions about the shape of the city (Monocentric city) and the wages (High paid and low paid workers) lead to a steady state which is barely different from the one with the racial discrimination.

<sup>&</sup>lt;sup>8</sup>Displacement costs are considered to increase linearly with the distance, which is problematic particularly when we need to analyze the impact of public transports.

the firms randomly.

#### 2.2 Theory of job search

Job search models started with Stigler in the early 1960's. In these models, job seekers are economic agents operating in an uncertain environment where they must collect information about job opportunities and use it in order to make a rational choice between carrying on job search <sup>9</sup>or stopping it and thus accepting the best current job proposal. Among the main contributions of search theory to labor economics is to provide a rationale for the concept of reservation wage when job seekers in a stochastic world where the information is not perfect. A first paper by Mortensen (1986) and a second one by Mortensen and Pissarides (1999) give an assessment of the most important theoretical extensions of the basic job search model. The listed extensions are made for analyzing unemployment spell durations, job turnover behavior, the impact of experience and personal learning on wage growth etc. ...

The spatial dimension of job search and its impact on the job seeker's behavior and decisions is ignored in this literature, though its great importance. The famous islands model of Lucas and Prescott (1974) is among the very few exceptions. The main assumption of the paper is that the economic activity occurs in an archipelago. Each firm is located in an island and cannot change its location while the workers are free to move and settle in any island. The wage rate is different from an island to another. Each worker has to choose an island to reside in, trying to enjoy the highest wage. Communication across islands being imperfect, it is costly to get information about the wage in an island different from the residence. The paper it takes account of space implicitly only, and the story of the islands is just an illustration of the difficulty to get information about the job market of an area which is not the one of residence.

#### 2.3 Matching theory

The labor market is characterized by the coexistence of job vacancies and unemployed workers. The classical Walrasian view of the economy based on the balance of supply and demand is unable to explain such a stylized fact. To provide explanations to this phenomenon, the economists came up in the late 1970's and the early 1980's with two new views of the labor market.

The first view was put forward by the school of disequilibrium in the 1980s (E. Malinvaud, J. P. Benassy). The idea is that, without frictions in the labor market, the employment level for a wage rate w is equal to the minimum of labor supply O(w) and labor demand D(w),  $E(w) = \min(O(w), D(w))$ . However, as the frictions are inherent to the functioning of the economy, this value is never reached, and thus we have  $E(w) = \Phi(O, D)$ , where  $\Phi$  is a function such as for each couple (O, D) the inequality  $\Phi(O(w), D(w)) < \min(O(w), D(w))$  is met. As a consequence,  $D - \Phi(O, D)$  unemployed workers and  $D - \Phi(O, D)$  vacancy coexist, even for the equilibrium wage rate characterized by the equality O(w) = D(w). The stronger the frictions are, the larger the difference between  $\min(O(w), D(w))$  and  $\Phi(O, D)$ . The second view relies on the concept of the matching function à la Pissarides. In a seminal article published in 1979, the latest comes up with the idea that the transition out of unemployment is a trading process<sup>10</sup> between unemployed workers and firms with vacancies. This trading process needs time<sup>11</sup>. Frictions that are inherent to the labor market prevent the matching between

<sup>&</sup>lt;sup>9</sup>which is costly

<sup>&</sup>lt;sup>10</sup>An aspect which is absent in the Keynesian perspective

<sup>&</sup>lt;sup>11</sup>An aspect which is absent in the Walrasian perspective

unemployed workers and vacancies to be instantaneous.

The matching function gives the number of positions filled during a period for given levels of the stocks of unemployed workers and vacancies. U job seekers and V vacancies at the beginning of the period generate a matching flow equal to  $M = \Phi(U, V)$  during the period. Usually, the matching function is assumed to be constant-returns to scale. The matching mechanism is completed by two other mechanisms: a mechanism of existing jobs destruction and a mechanism of demographic renewal. Both mechanisms feed the stocks of jobless workers and vacancies. The labor market reaches the equilibrium when the net flows are nil.

The theoretical framework dealing with the matching process has greatly enriched the labor economics literature as it led to a better understanding of the causes and the consequences of the phenomenon of unemployment. However, it barely includes the inefficiencies and the frictions of the labor market generated by the spatial disconnection between the workplaces and the homes locations.

### 3 The global structure of the model

Our model aims to formalize the impact of the transport system on the labor market of a city consisting on many districts and in which the economic activities and the workers homes are scattered across all the districts.

The spatial dispersion of the workers homes and the firms locations influences both the job search process of unemployed workers and the matching process between them and firms with vacant jobs. In practical terms, the consequence of the spatial dispersion consists on the need for the workers to displace to reach their workplaces. These displacements entail monetary and time expenditures whose dis-utility can be higher than the utility of working. To avoid being in such a situation, workers embed displacement costs in their job search process firstly by adjusting their reservation wage so that it absorbs the displacement costs, and secondly by choosing to fill the job allowing them to enjoy the highest take-home wage<sup>12</sup> <sup>13</sup>.

The impact of the generalized displacement costs on the reservation wage is determined and analyzed using an extension of the job search model à la Stigler presented in the fifth section. Concerning the matching process, the influence of the transport system is studied using a well behaved macroeconomic matching function that gives the spatial distribution of the successful linkages between job seekers and vacancies according to their numbers in each district. This function needs to be built microeconomecally. To do so, we start by considering the spatial framework where the action takes place then we spell out an explicit job search micro scenario used for building the function.

We consider a closed city structured in N districts indexed  $i = 1...N^{14}$ , and assume time to be discrete and consisting on an infinite sequence of short periods.

The reasoning is done period per period. At the beginning of each period, each district i contains  $H_i$  inhabitants,  $U_i$  unemployed workers and  $V_i$  vacant jobs. Unemployed workers are looking for unfilled vacancies to send their applications, firms are looking for workers to fill their vacant jobs, and workers that have already a job don't plan on changing it. We assume that as long as they are jobless, workers cannot move <sup>15</sup>. For simplicity, we consider that employed

<sup>&</sup>lt;sup>12</sup>The wage minus the displacement costs

<sup>&</sup>lt;sup>13</sup>If they have many job opportunities

<sup>&</sup>lt;sup>14</sup>This numbering does not contain any notion of order and aims only to distinguish between the districts.

<sup>&</sup>lt;sup>15</sup>There are two main reasons for this: firstly, moving has a high cost that jobless can't afford, and secondly lessors rarely accept to sign rental contract with job seekers

workers don't plan to move neither, at least at the short run. In view of these assumptions, the number of unemployed workers and vacancies in each district can change from a period to an other while the number of its inhabitants remains constant.

Workers are risk-neutral, infinitely lived, future discounting, and endowed with one unit of labor that can be supplied inelastically to firms. Each worker's home and each firm's location are predetermined and unemployed workers are free to search for a job everywhere in the city. To reach their workplaces, workers have to displace by making use of transport infrastructure and services. These displacements have a time cost and a monetary cost that we group in what we call the generalized displacement cost. <sup>16</sup>.

In this context, the matching between job seekers and vacancies occurs according to the following process :

- 1. At the beginning of each period, each unemployed worker is informed of the availability of many vacant jobs compatible with his skills. The firms where these jobs are to be filled are located in different districts. The information relate to the availability of the job and the productivity (and thus the salary) of the worker if hired. For simplicity, we assume that the wages consist on a deterministic part w common to all the vacant positions, and a stochastic part  $\epsilon$  that depends on each couple firm-worker.
- 2. Based on these information, each unemployed worker decides to send one application at the most<sup>17</sup>. A worker can send no application if all the wages he is proposed minus the generalized displacement costs are lower than his reservation wage.
- 3. On the other side, employers receive a certain number of applications for each vacant job. If more than one application is received for the same vacant job, the employer hires the most productive worker which is also the one who is proposed the highest salary.
- 4. At the term of this process, the successful linkages correspond to the jobs that received at least one application. The jobs to whom no application was sent remain vacant, and are proposed in the next period. On the other hand, workers who didn't send any application or whose application wasn't chosen remain jobless and restart looking for a job in the next period.

In the matching function obtained based on this scenario, space intervenes in the stage when the unemployed workers select the jobs to apply to. Knowing that the more the generalized displacement costs to reach the job's location are high the more the utility enjoyed from exercising it is low, the job seeker applies for a job located far away from home rather than a closer one only if the dis-utility resulting from the increase in the generalized displacement costs is more than compensated by the utility obtained by the wage differential between the two jobs. Consequently, the probability that a job seeker applies for a job decreases as the generalized displacement costs to reach its location increase.

In order to have a model describing the labor market entirely, we complete the matching process by a classical random and exogenous process of jobs destruction feeding the stocks of

<sup>&</sup>lt;sup>16</sup>In anticipation, we are particularly interested in modeling how the generalized displacement costs impact the workers behavior. The matching process and the outcomes of the global labor market and those of all the local ones are simply consequences of the workers behavior. Thus we don't pay much attention to the firms behavior, particularly the circumstances under which they create vacancies.

<sup>&</sup>lt;sup>17</sup>We assume the periods to be too short and the applications to be time consuming

jobless workers and vacancies. The macroeconomic equilibrium of the labor market is reached when the number of destroyed jobs is exactly compensated by the number of successful linkages between jobless workers and vacancies.

### 4 The matching function including the transport costs

In the following sub-sections we use the results shown in Appendix Part A.

The proofs of the results shown in this section are presented in Appendix Part B.

#### 4.1 The applications

We consider a job seeker residing in a district *i* and having information about *N* job opportunities, each in a district<sup>18</sup>. The wage proposal of a job located in *j* is  $w_j = w * + \epsilon_j$ . We assume  $\epsilon_j$ to be a random term drawn from a Gumbel distribution with parameter  $\mu$ . The trips between *i* and *j* entail a generalized displacement cost noted  $\theta_{ij}$ . The take-home wage can be written as  $v_{ij} = w_{ij} - \theta_{ij} = w^* - \theta_{ij} + \varepsilon_j$ .

According to Appendix part A, the best after-displacement costs take-home wage proposal the job seeker gets can be written

$$v_i = \max_j \left( v_{ij} \right) = w^* - \Theta_i + \eta \tag{4.1}$$

where  $\Theta_i = -\mu \log \left( \sum_j e^{-\frac{\theta_{ij}}{\mu}} \right)$  and  $\eta$  is a random term drawn from a Gumbel distribution of parameter  $\mu$ .

The probability of the opportunity coming from j to be the best one is:

$$P_{i,j} = \Pr\left\{v_{ij} = \max_{k} v_{ik}\right\} = \exp\left(-e^{\frac{\Theta_i + \theta_{ij}}{\mu}}\right)$$
(4.2)

Moreover, (4.1) is also valid conditionally on the fact that the best opportunity is coming from j:

$$\Pr\left\{v_{ij} \le v \,|\, v_{ij} = \max_k v_{ik}\right\} = \exp\left(-e^{-\frac{v-w^*+\Theta_i}{\mu}}\right)$$

We can easily demonstart that the  $P_{ij}$  decreases as  $\theta_{ij}$  increase. Figure 4.1 shows how  $P_{ij}$  decreases for different values of the other parameters.

 $<sup>^{18}</sup>$ We assume for simplicity that the absence of information about job opportunities is also an information and that the wage related to the absence of information is equal to 0



Figure 4.1:  $P_{ij}$  according to  $\theta_{ij}$  for different values of the other parameters

And as in a system of communicating vessels, the probability  $P_{ij}$  increases when the general displacement costs between district i and a district  $k \neq j$  increase. Figure 4.2 shows that  $P_{ij}$  is less affected by variations of  $\theta_{ik\neq j}$  than by variations of  $\theta_{ij}$ , and that the impact of  $\theta_{ik\neq j}$  is more observable when their value is low.



Figure 4.2:  $P_{ij}$  according to  $\theta_{ik}$  for different values of the other parameters

#### 4.2 The vacant jobs

Lets consider a vacant job located in j. From each district i = 1, ..., N,  $0 \le k_i \le U_i$  job seekers have information about this vacancy. Knowing that the productivity of all the workers is such as  $w_i = w * + \epsilon_i$  where  $\epsilon_i$  is a random term drawn from a Gumbell distribution whose parameter is  $\mu$ , the following results are obtained :

- There are two conditions for the information received by an unemployed to result in an application. The first conditions is that the vacancy must be the best choice for the applicant. The second one is that the take-home wage,  $v_{ij}$  must be higher than the reservation value.
- It is worth noting that the generalized displacement costs impact the deterministic part of the best wage proposal in a negative manner (cf. 4.1). An increase of  $\theta_{ij}$ , whatever

j, leads inevitably to the decrease of the probability for the best wage proposal made to the job seekers living in i to be greater than their reservation wage. This means that the job seekers living in districts where the transport facilities are bad are less incited to participate in the labor market and are thus more likely to remain jobless. Let us temporarilly neglect the later condition. From now on, we call "potential applicant" an unemployed who is informed of the vacancy and for which the vacancy is the best choice.

- We know from 4.2 that, with probability  $P_{i,j} = \Pr\{v_{ij} = \max_k v_{ik}\} = \exp\left(-e^{\frac{\Theta_i + \theta_{ij}}{\mu}}\right)$ , the unemployed is a potential applicant and, with probability  $1 P_{ij}$ , he is not a potential applicant.
- Then, the number of potential applications to the vacancy,  $q_i$ , follows a binomial distribution:  $\Pr \{q_i | k_i\} = \frac{k_i!}{q_i!(k_i-q_i)!} P_{ij}^{q_i} (1-P_{ij})^{k_i-q_i}$
- We also know from 4.1 that, conditionnally of being a potential applicant, the take home wage is  $v_{ij} = w^* \Theta_i + \eta$ , where  $\eta$  follows a Gumbel distribution with parameter  $\mu$ . Knowing that  $v_{ij} = w_{ij} \theta_{ij}$ , we have  $w_{ij} = w^* \Theta_i + \theta_{ij} + \eta$ . Then, knowing that there are  $q_{ij}$  potential applicants, the productivity of the most productive applicant from district *i* is:

$$\max_{i} (w_{ij}) = \mu \log \left( q_i e^{(w^* - \Theta_i + \theta_{ij})/\mu} \right) + \eta$$
$$= w^* - \Theta_i + \theta_{ij} + \mu \log q_i + \eta$$
(4.3)

where  $\eta$  is a random term drown from a Gumbell distribution of parameter  $\mu$ .

#### The probability for a vacancy in j to be filled by a job seeker from i

We assume that from each district i,  $q_i$  job seekers are potential applicants on the vacancy in j.

Let us recall that the productivity of the most productive potential applicant from i is  $w^* + \Delta_{ij} + \mu \log q_i + \eta$ , where  $\Delta_{ij} = \theta_{ij} - \Theta_i$  and  $\eta_i$  is a random term drawn from a Gumbell distribution of parameter  $\mu$ . For the job seeker from i to fill the position, for every origin  $m \neq i$ , at least one of two following conditions must be satisfied :

- 1. the best applicant from m must be less productive than the best candidacy from i. This means that the inequality  $\eta_m < \Delta_{ij} \Delta_{mj} + \mu \log (q_i/q_m) + \eta_i$  is met.
- 2. the best candidacy from m is not productive enough to cover the generalized displacement costs to j. This means that the inequality  $\eta_m < \Theta_m w^* + \overline{v}_m \mu \log q_m$  is met.

From the two conditions above, we deduce that for each  $m \neq i$  the following inequality must be satisfied to ensure the vacancy in j to be filled by a worker from i:

$$\eta_m < \Delta_{ij} - \Delta_{mj} + \mu \log \left( q_i / q_m \right) + \max \left( \eta_i, -\Delta_{ij} - w^* - \mu \log q_i + \overline{v}_m + \theta_{mj} \right)$$
(4.4)

The problem here is to determine  $\max(\eta_i, -\Delta_{ij} - w^* - \mu \log q_i + \overline{v}_m + \theta_{mj}).$ 

Knowing that, if the most productive worker from *i* is candidate, we have  $\eta_i > \Theta_i - w^* + \overline{v}_i - \mu \log q_i$ , for each district *m* two scenarios are possible

1.  $\overline{v}_i + \theta_{ij} > \overline{v}_m + \theta_{mj}$ . In that case it is clear that  $\max(\eta_i, -\Delta_{ij} - w^* - \mu \log q_i + \overline{v}_m + \theta_{mj}) = \eta_i$ 

2.  $\overline{v}_i + \theta_{ij} < \overline{v}_m + \theta_{mj}$ . In that case it is impossible to determine directly  $\max(\eta_i, -\Delta_{ij} - w^* - \mu \log q_i + \overline{v}_m + \theta_{mj})$ 

To solve the problem of the second case we rank the districts according to the value of the reservation wage of their inhabitants plus the generalized costs entailed by moving to j, and define the permutations  $\kappa_{j}(i)$  and  $\iota_{j}(k)$  from  $\{1, ..., N\}$  to itself such as  $\kappa_{j}(i)$  corresponds to the ranking of district i, and  $\iota_{j}(k)$  corresponds the district ranked  $k^{th}$ .  $\kappa_{j}(i)$  and  $\iota_{j}(k)$  verify the following properties :

$$\kappa_{j}(n) < \kappa_{j}(m) \Longleftrightarrow \overline{v}_{n} + \theta_{nj} \le \overline{v}_{m} + \theta_{mj} \tag{4.5}$$

$$k = \kappa_j(m) \Longleftrightarrow m = \iota_j(k) \tag{4.6}$$

As our focus is on the situations where the most productive worker from i is an applicant, we only consider the values of  $\eta_i$  such that

$$\eta_i \ge \Theta_i - w^* + \overline{v}_i - \mu \log q_i \tag{4.7}$$

 $\operatorname{From}\left\{1\;,\;\ldots\;,\;N\right\} \text{ we can find } k = \bar{k}\left(j\right) \operatorname{such as } \left(\eta_i - \Theta_i + w^* + \mu \log q_i\right) \in \left[\bar{v}_{\iota_j(k)} + \theta_{\iota_j(k),j}, \bar{v}_{\iota_j(k+1)} + \theta_{\iota_j(k+1),j}\right].$ The inequality (4.7) implies that  $k \ge \kappa_i$  (i). Thus, additionally to the fact that (4.7) is verified, the most productive worker from i is selected only if :

- 1. For districts m such as  $\kappa_i(m) < \kappa_i(i)$ , the inequality  $\eta_m < \Delta_{ij} \Delta_{mj} + \mu \log(q_i/q_m) + \eta_i$ is verified.
- 2. For districts m such as  $\kappa_j(i) < \kappa_j(m) \le k$ , the inequality  $\eta_m < \Delta_{ij} \Delta_{mj} + \mu \log (q_i/q_m) + \mu \log (q_i/q_m)$  $\eta_i$  is verified<sup>19</sup>.
- 3. For districts m such as  $\kappa_i(m) > k$ , the inequality  $\eta_m < \Theta_m w^* + \overline{v}_m \mu \log q_m$  is verified<sup>20</sup>.

Consequently, the probability that the most productive candidate from *i* is hired conditionally to the value of  $\eta$ , and knowing that  $(\eta_i - \Theta_i + w^* + \mu \log q_i) \in [\bar{v}_{\iota_j(k)} + \theta_{\iota_j(k),j}, \bar{v}_{\iota_j(k+1)} + \theta_{\iota_j(k+1),j}]$ is :

$$\widetilde{P}_{ij}(\eta_i, q, \dots, q_I) = \exp\left(-\sum_{m, \kappa_j(m) \le \bar{k}(j)} e^{-[\Delta_{ij} - \Delta_{mj} + \mu \log(q_i/q_m) + \eta_i]/\mu} - \sum_{m, \kappa_j(m) > \bar{k}(j)} e^{-[\Theta_m - w^* + \bar{v}_m - \mu \log q_m]/\mu}\right)$$

$$= \exp\left(-e^{-[\Delta_{ij} + \eta_i]/\mu} \sum_{m, \kappa_j(m) \le \bar{k}(j)} \frac{q_m}{q_i} e^{\Delta_{mj}/\mu} - e^{w^*/\mu} \sum_{m, \kappa_j(m) > \bar{k}(j)} q_m e^{-[\Theta_m + \bar{v}_m]/\mu}\right)$$

$$= \exp\left(-e^{[S_{ij} - \Delta_{ij} - \eta_i]/\mu} - e^{(w^* - W_j)/\mu}\right)$$

(4.8)

Where

$$S_{i,j} = \mu \log \sum_{m,\kappa_j(m) \le \bar{k}(j)} \frac{q_m}{q_i} e^{\Delta_{mj}/\mu}$$
  
$$W_{j,k} = -\mu \log \sum_{m,\kappa_j(m) > \bar{k}(j)} q_m e^{-[\Theta_m + \bar{v}_m]/\mu}$$

By integrating  $\widetilde{P}_{ij}(\eta_i, q_1, \ldots, q_I)$  for  $\eta_i \ge \Theta_i - w^* + \overline{v}_i - \mu \log q_i$  we obtain  $\widetilde{P}_{ij}(q_1, \ldots, q_I)$ , the pobability that most productive candidate from i is hired which expression is :

<sup>&</sup>lt;sup>19</sup>In this case, max  $(\eta_i, -\Delta_{ij} - w^* - \mu \log q_i + \overline{v}_m + \theta_{mj}) = \eta_i$ <sup>20</sup>In this case max  $(\eta_i, -\Delta_{ij} - w^* - \mu \log q_i + \overline{v}_m + \theta_{mj}) = -\Delta_{ij} - w^* - \mu \log q_i + \overline{v}_m + \theta_{mj}$ 

$$\begin{split} \widetilde{P}_{ij}(q_{1}..q_{I}) &= \mu^{-1} \int_{\eta_{i} \ge \Theta_{i} - w^{*} + \overline{v}_{i} - \mu \log q_{i}} \exp\left(-e^{[S_{ij} - \Delta_{ij} - \eta_{i}]/\mu} - e^{(w^{*} - W_{j})/\mu}\right) e^{-\eta_{i}/\mu} \exp\left(-e^{-\eta_{i}/\mu}\right) d\eta_{i} \\ &= \mu^{-1} \exp\left(-e^{(w^{*} - W_{j})/\mu}\right) \int_{\eta_{i} \ge \Theta_{i} - w^{*} + \overline{v}_{i} - \mu \log q_{i}} e^{-\eta_{i}/\mu} \exp\left(-e^{[S_{ij} - \Delta_{ij} - \eta_{i}]/\mu} - e^{-\eta_{i}/\mu}\right) d\eta_{i} \\ &= \mu^{-1} \exp\left(-e^{(w^{*} - W_{j})/\mu}\right) \int_{\eta_{i} \ge \Theta_{i} - w^{*} + \overline{v}_{i} - \mu \log q_{i}} e^{-\eta_{i}/\mu} \exp\left(-\left(1 + e^{(S_{ij} - \Delta_{ij})/\mu}\right) e^{-\eta_{i}/\mu}\right) d\eta_{i} \\ &= \mu^{-1} \exp\left(-e^{(w^{*} - W_{j})/\mu}\right) \int_{\eta_{i} \ge \Theta_{i} - w^{*} + \overline{v}_{i} - \mu \log q_{i}} e^{-\eta_{i}/\mu} \exp\left(-e^{-(\eta_{i} - Z_{ij} + \Delta_{ij})/\mu}\right) d\eta_{i} \\ &= \mu^{-1} \exp\left(-e^{(w^{*} - W_{j})/\mu}\right) e^{(\Delta_{ij} - Z_{ij})/\mu} \int_{\eta_{i} - Z_{ij} + \Delta_{ij} \ge \Theta_{ij} - Z_{ij} - w^{*} + \overline{v}_{i} - \mu \log q_{i}} e^{-(\eta_{i} - Z_{ij} + \Delta_{ij})/\mu} \exp\left(-e^{-(\eta_{i} - Z_{ij} + \Delta_{ij})/\mu}\right) d\eta_{i} \\ &= \exp\left(-e^{(w^{*} - W_{j})/\mu}\right) e^{(\Delta_{ij} - Z_{ij})/\mu} \left[1 - \exp\left(-e^{-(\Theta_{ij} - Z_{ij} - w^{*} + \overline{v}_{i} - \mu \log q_{i})/\mu}\right)\right] \tag{4.10}$$

where  $Z_{ij} = \mu \log \left( e^{S_{ij}/\mu} + e^{\Delta_{ij}/\mu} \right)$ 

The probability that the vacant position in j is filled by a worker from i decreases as the displacement costs between i and j increase, and it increases when the displacement costs between  $k \neq i$  and j increase.

### Proof.

\*  $\widetilde{P}_{ij}(k_1, \ldots, k_I)$  consists on the sum of positive terms. An increase in  $\theta_{ij}$  leads to an increase in  $\kappa_j(i)$  and thus to a diminution of the number of terms in the sum. Consequently  $\widetilde{P}_{ij}(k_1, \ldots, k_I)$  decreases.

\* Similarly, if  $\theta_{kj}$  with  $k \neq i$  increases,  $\kappa_j(i)$  decreases leading to an increase in the number of terms in the sum and thus to the increase of  $\widetilde{P}_{ij}(k_1, \ldots, k_I)$ 

#### 

#### 4.3 The firms

For a vacant job in j, the number of workers from i having information about it follows a Poisson distribution of parameter  $\lambda_{ij} = \frac{U_i}{V_i}$ . Thus:

- The probability that  $k_i$  unemployed workers from *i* are informed of the vacancy is  $e^{-\lambda_{ij}} \lambda_{ij}^{k_i}/k_i!$
- We that the probability that, among these  $k_i$  informed workers,  $q_i$  are potential applicants is  $\Pr\{q_i \mid k_i\} = \frac{k_i!}{q_i!(k_i-q_i)!} P_{ij}^{q_i} (1-P_{ij})^{k_i-q_i}$ , where  $P_{i,j} = \exp\left(-e^{\frac{\Theta_i \theta_{ij}}{\mu}}\right)$ .
- Then, combining the Poisson law and the binomial law, the number of potential applicants from district i,  $q_i$ , follows a Poisson distribution with parameter  $\lambda_{ij}P_{ij}$
- The probability the position to receive no candidacy and thus to remain vacant is :

$$\pi_{0j}\left(\Lambda_{j}\right) = e^{-\bar{\lambda}_{j}} \sum_{(k_{1},\dots,k_{N})\geq 0} \prod_{i} \frac{\left(\lambda_{ij}P_{ij}\right)^{k_{i}}}{k_{i}!} \overline{P}_{ij}\left(k_{i}\right)$$

Where  $\Lambda_j = (\lambda_{1,j}, \ldots, \lambda_{I,j}), \Lambda_j = (\lambda_{1,j}, \ldots, \lambda_{I,j})$  and  $\overline{\lambda}_j = \sum_i \lambda_{ij} P_{ij}$ Whatever the value of  $k_i$  and for the *is*,  $\overline{P}_{ij}(k_i)$  decreases when  $\theta_{ij}$  increases. Consequently,  $\pi_{0j}(\Lambda_j)$  increases as the displacement costs towards *j* increase. • The probability the postion to be filled by a worker from i is :

$$\pi_{ij}\left(\Lambda_{j}\right) = e^{-\bar{\lambda}_{j}} \sum_{(k_{1},\dots,k_{N})\geq 0} \prod_{i} \frac{\left(\lambda_{ij}P_{ij}\right)^{k_{i}}}{k_{i}!} \widetilde{P}_{ij}\left(k_{1},\dots,k_{I}\right)$$

Similarly, the probability for the position in j to be filled by a worker from i increases when the displacement costs between i and j decrease or when the displacement between  $k \neq i$  and j increase.

#### 4.4 The matching functions

• The number of worker from i hired in j

$$M_{ij}\left(V_1,\ldots,V_N,U_1,\ldots,U_N\right)=V_j\pi_{ij}\left(\frac{U_1}{V_j},\ldots,\frac{U_N}{V_j}\right)$$

• The number of vacancies filled in j

$$M_{.,j}(V_1,...,V_N,U_1,...,U_N) = \sum_i M_{ij}(V_1,...,V_N,U_1,...,U_N) \\ = V_j \left[ 1 - \pi_{0j} \left( \frac{U_1}{V_j},...,\frac{U_N}{V_j} \right) \right]$$

• The number of workers from i hired

$$M_{i,.}(V_1,...,V_N,U_1,...,U_N) = \sum_j M_{ij}(V_1,...,V_N,U_1,...,U_N)$$

• The total number of hired people (and filled vacancies)

$$M_{.,.}(V_1,...,V_N,U_1,...,U_N) = \sum_j M_{.,j}(V_1,...,V_N,U_1,...,U_N)$$
$$= \sum_j V_j \left[ 1 - \pi_{0j} \left( \frac{U_1}{V_j},...,\frac{U_N}{V_j} \right) \right]$$

It is easy to show that all these functions are increasing for all their arguments, concave, and and homogeneous of degree one.

Also for all of them we have  $M(0, ..., 0, U_1, ..., U_N) = M(V_1, ..., V_N, 0, ..., 0) = 0.$ 

It follows from the analysis in section 4.3 that :

- 1. The matching between the job seekers living in i and the vacancies located in j becomes easier when the displacement costs between i and j decrease.
- 2. The matching between the job seekers living in i and the vacancies located in j becomes easier when the displacement costs between the other couples of districts increases.

#### 5 Reservation wage by district

As explained above, workers need to adjust their reservation wage according to the accessibility conditions of their living districts in order to avoid being in a situation in which the utility generated by the job they exercise is lower than the disutility of the generalized displacement costs to reach their workplaces. In principle, the heterogeneity of the transport infrastructures and services quality in the same city should result in an uneven distribution of the workers reservation wages depending on their living districts : workers living in accessible ones must have lower reservation wages than those living in isolated ones.

To take account of the impact of a district's accebility conditions on the reservation wage of its inhabitants, we take the basic job search à la Stigler as a strating point and add a spatial diemension to it by considering that the search occurs in the spatial framework discribed above and that the workers embed the generalized displacement costs in their job search process.

In the basic job search model à la Stigler, time is represented by an infinite sequence of discrete periods of length h, and workers are assumed risk neutral, infinitely lived, and discounting the futur at a certain constant rate. In the beginning of each period, each job seeker receives a certain number of job proposals. Based on the information he has on the wages, the worker must conduct an arbitration and decide whether to accept the best proposal he got in that period or to conduct a new job search in the following period.

The optimal search strategy consists on stopping the search when the value of searching during the next period is lower than, the value of stopping the search and accepting the current best job proposal. The best job proposals are assumed to be iid so that the information on wages remains constant over the periods. Based on these assumption, a Bellman equation can be built, and its solution for the wage rate is the worker's reservation wage.

The approach we adopt in the present paper is slightly different from the one of the basic model in the sens that the job seeker does not receive job proposals but rather information about job opportunities, and must then decide whether to send an application to one of them or to wait the information of the next period. If he sends an application, it is the firm that decides to hire him or not. The worker cannot put an end to his job search, this decision is in the hands to the firm.

Consequently, we define here the reservation wage as the minimum wage beyond which the job seeker considers sending his application.

Without loss of generality, we consider the spatial framework described in the section 3, and look at things from the perspective of a job seeker residing in district *i*. In the beginning of each period the latest is informed of the existence of one job opportunity by district at the most (he cannot be informed of job opportunities of all the districts of the city). Lets note D the set containing all the districts of the city, and  $D_{inf}$  the subset of those where a job opportunity exists and is known to the job seeker. Thus Card(D) = N and  $0 \leq Card(D_{inf}) \leq N$ . As explained above, we consider that the wage proposed by a vacant job located in j is  $w_j = w + \epsilon_j$ where  $\epsilon_j$  is a random term drawn from a Gumbel distribution of parameter  $\mu$ . We consider also that the distribution of  $\epsilon_j$  is know to the workers. The take-home wage if exercising a job located in j can be written  $v_{ij} = w_j - \theta_{ij} = w^* - \theta_{ij} + \varepsilon_j$ . Thus, the best proposal minus the displacement costs is :

$$w_i = Max_j \left( w^* - \theta_{ij} + \epsilon_{ij} \right)$$

As the  $\epsilon_{ij}$  are iid, the best wages sequence is also iid. The worker doesn't get additional

information from a period to another. If we consider h = 1, what corresponds to the actual value of a search in the next period in our case is :

$$V_i = b - c + \beta E \left( max \left( v_i , V_i \right) \right)$$

where  $V_i$  is the actual value of a search in the next period, b the value of a time unit dedicated to leisure, c the cost of job search per time unit,  $\beta$  a discount rate and  $E(max(v_i, V_i))$  is the expected present value of the next period optimal stoping decision.

If we note q(n) the probability that  $Card(D_{inf}) = n$ , and knowing that  $v_i = \mu \ln \left( \sum_{j \in D_{inf}} e^{\frac{w - \theta_{ij}}{\mu}} \right) + \eta$  where  $\eta$  is a random term drawn from a Gumbel distribution which parameter is  $\mu$  (Proof see Appendix Part A), the actual value of a search in the next period can be written

$$V_{i} = b - c + \beta \mu \sum_{n=0}^{N} q(n) \ln \left( \sum_{j \in E_{inf}, Card(D_{inf})=n} e^{\frac{w - \theta_{ij}}{\mu}} \right) + \beta \sum_{n=0}^{N} q(\eta) \int_{-\infty}^{+\infty} max \left( \eta, V_{i} - \mu \ln \left( \sum_{j \in D_{inf}, Card(E_{inf})=n} e^{\frac{w - \theta_{ij}}{\mu}} \right) \right)$$
(5.1)

*Proof.* See Appendix Part B

Since the equation (5.1) has a unique solution for  $V_i$ , the reservation wage of a worker residing in *i* is such as

 $v_i = V_i$ 

### 6 Conclusion :

So far, we succeeded in building a matching function with a spatial dimension allowing us to study the impact of the generalized displacement costs on the number of links created between the job seeker living in an area i and the firms located in area j.

This matching function is build without making any restriction on the size and the shape of the city. Thus it can be used whatever the structure of the studied city is.

We demonstrated that better transport infrastructures increase the probability that the job seeker prefer working to remaining jobless.

Also enhancing the performances of the transport system increases the number of matchings in each period.

Remaining work : finish the search model - study the different macro equilibrium of the city resulting from the partial equilibria of each district.

The version to be presented in June will contain the detailed demonstrations.

### 7 Appendix

### **A** - Prerequisites

Let  $\{\mu_1, \ldots, \mu_I\}$  be a set of random variables such as for each i,  $\mu_i = a_i + \epsilon_i$ , where  $a_i$  is deterministic and  $\epsilon_i$  is a random term drawn from a Gumbel distribution of parameter  $\mu$ . The following properties are verified :

- 1.  $Max_i \{\mu_i\} = \mu L_a + \eta$  where
  - (a)  $L_a = \log\left(\sum_i e^{\frac{a_i}{\mu}}\right)$
  - (b)  $\eta$  is a random term drawn from a Gumbel distribution of parameter  $\mu$ .
- 2. Pr { $\mu_i = Max_j(\mu_j)$ } = exp  $\left(\frac{a_i}{\mu} L_a\right)$
- 3. Conditionally to the fact that  $\mu_i = Max_j(\mu_j)$ ,  $\mu_i = \mu L_a + \eta$ , where  $\eta$  is a random term drawn from a Gumbel distribution of parameter  $\mu$ .

#### Proof of (1)

$$\begin{aligned} \Pr\left\{\max\left(\mu_{1}, \dots, \mu_{I}\right) \leq v\right\} &= \Pr\left\{\max\left(a_{1} + \varepsilon_{1}, \dots, a_{I} + \varepsilon_{I}\right) \leq v\right\} \\ &= \Pr\left(a_{1} + \varepsilon_{1} < v\right) \Pr\left(a_{2} + \varepsilon_{2} < v\right) \dots \Pr\left(a_{I} + \varepsilon_{I} < v\right) \\ &= \Pr\left(\varepsilon_{1} < v - a_{1}\right) \dots \Pr\left(\varepsilon_{I} < v - a_{I}\right) \\ &= \exp\left(-e^{-\frac{v-a_{I}}{\mu}}\right) \dots \exp\left(-e^{-\frac{v-a_{I}}{\mu}}\right) \\ &= \exp\left(-e^{-\frac{v}{\mu}}\left(e^{\frac{a_{1}}{\mu}} + \dots + e^{\frac{a_{I}}{\mu}}\right)\right) \\ &= \exp\left(-e^{L_{a} - \frac{v}{\mu}}\right) \end{aligned}$$

Thus  $\eta = \max(\mu_1, \dots, \mu_I) - \mu L_a$  is a random variable drawn from a Gumbel distribution of parameter  $\mu$ .

Consequently max  $(\mu_1, \ldots, \mu_I) = \mu L_a + \eta$ 

#### Proof of (2)

Without loss of generality we calculate  $\Pr\{\max(u_2, ..., u_I) = u_1\}$ , and put  $L_1 = \log\left(\sum_{k=2}^{I} e^{\frac{a_k}{\mu}}\right)$ 

$$\begin{aligned} \Pr\{\max(u_{2},...,u_{I}) = u_{1}\} &= & \Pr\{\max(u_{2},...,u_{I}) < u_{1} < v\} \\ &= & \Pr\{\mu L_{1} + \eta < a_{1} + e_{1} < v\} \\ &= & \Pr\{\mu L_{1} + \eta < a_{1} - \mu L_{1} \text{ tf } e_{1} < v > a_{1}\} \\ &= & \int_{\epsilon_{1}=-\infty}^{\psi-a_{1}} \int_{\epsilon_{1}=-\infty}^{\epsilon_{1}+a_{1}-\mu L_{1}} f(\epsilon_{1}) f(\eta) d\epsilon_{1} d\eta \\ &= & \int_{\epsilon_{1}=-\infty}^{\psi-a_{1}} \int_{\epsilon_{1}=-\infty}^{e_{1}-a_{1}} f(\epsilon_{1}) F(\epsilon_{1} + a_{1} - \mu L_{1}) d\epsilon_{1} \\ &= & \frac{1}{\mu} \int_{\epsilon_{1}=-\infty}^{\psi-a_{1}} e^{-\frac{\epsilon_{1}}{\mu}} \exp\left(-e^{-\frac{\epsilon_{1}}{\mu}}\left(1 + e^{L_{1}-\frac{a_{1}}{\mu}}\right)\right) d\epsilon_{1} \\ &= & \frac{1}{\mu} \int_{\epsilon_{1}=-\infty}^{\psi-a_{1}} e^{-\frac{\epsilon_{1}}{\mu}} \exp\left(-e^{-\frac{\epsilon_{1}}{\mu}}\left(1 + e^{-\frac{a_{1}}{\mu}} e^{L_{1}}\right)\right) d\epsilon_{1} \\ &= & \frac{1}{\mu} e^{\frac{a_{1}}{\mu} - L_{a}} \int_{\epsilon_{1}=-\infty}^{\psi-a_{1}} e^{-\frac{\epsilon_{1}}{\mu}} e^{\frac{a_{1}}{\mu} - e^{L_{1}}} \exp\left(-e^{-\frac{\epsilon_{1}}{\mu}}\left(1 + e^{-\frac{a_{1}}{\mu}} e^{L_{1}}\right)\right) d\epsilon_{1} \\ &= & \frac{1}{\mu} e^{\frac{a_{1}}{\mu} - L_{a}} \int_{\epsilon_{1}=-\infty}^{\psi-a_{1}} e^{-\frac{\epsilon_{1}}{\mu}} \left(e^{\frac{a_{1}}{\mu}} + e^{L_{1}}\right) \exp\left(-e^{-\frac{\epsilon_{1}}{\mu}}\left(1 + e^{-\frac{a_{1}}{\mu}} e^{L_{1}}\right)\right) d\epsilon_{1} \\ &= & \frac{1}{\mu} e^{\frac{a_{1}}{\mu} - L_{a}} \int_{\epsilon_{1}=-\infty}^{\psi-a_{1}} e^{-\frac{\epsilon_{1}}{\mu}} \left(1 + e^{-\frac{a_{1}}{\mu}} + e^{L_{1}}\right) \exp\left(-e^{-\frac{\epsilon_{1}}{\mu}}\left(1 + e^{-\frac{a_{1}}{\mu} + L_{1}\right)\right) d\epsilon_{1} \\ &= & e^{\frac{a_{1}}{\mu} - L_{a}} \exp\left(-e^{-\frac{\epsilon_{1}}{\mu}}\left(1 + e^{-\frac{a_{1}}{\mu} + L_{1}\right\right)\right) d\epsilon_{1} \\ &= & e^{\frac{a_{1}}{\mu} - L_{a}} \exp\left(-e^{-\frac{\psi}{\mu}}\left(e^{\frac{a_{1}}{\mu} + e^{\frac{a_{1}}{\mu}} e^{\frac{\Sigma_{h=2}}{\mu} a_{h} - a_{h}}\right)\right) \\ &= & e^{\frac{a_{1}}{\mu} - L_{a}} \exp\left(-e^{-\frac{\psi}{\mu}}\left(e^{\frac{a_{1}}{\mu} + e^{\frac{a_{1}}{\mu}} e^{\frac{\Sigma_{h=2}}{\mu} a_{h} - a_{h}}\right)\right) \\ &= & e^{\frac{a_{1}}{\mu} - L_{a}} \exp\left(-e^{-\frac{\psi}{\mu}}\left(e^{\frac{a_{1}}{\mu} + e^{\frac{a_{1}}{\mu}} e^{\frac{\Sigma_{h=2}}{\mu} a_{h} - a_{h}}\right)\right) \right) \\ &= & e^{\frac{a_{1}}{\mu} - L_{a}} \exp\left(-e^{-\frac{\psi}{\mu}}\left(e^{\frac{\omega}{\mu} + e^{\frac{\omega}{\mu}} e^{-\frac{\omega}{\mu}}\right)\right) \\ &= & e^{\frac{a_{1}}{\mu} - L_{a}} \exp\left(-e^{-\frac{\psi}{\mu}}\left(e^{\frac{\omega}{\mu} + e^{\frac{\omega}{\mu}} e^{-\frac{\omega}{\mu}}\right)\right) \right) \end{aligned}$$

 $\Pr\left\{\max\left(u_{2},...,u_{n}\right) < u_{1}\right\} = \Pr\left\{\max\left(u_{2},...,u_{n}\right) < u_{1} < +\infty\right\} = e^{\frac{a_{1}}{\mu} - L_{a}} = \frac{e^{\frac{\mu}{\mu}}}{\sum_{k=1}^{n-1} e^{\frac{a_{k}}{\mu}}}$ 

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