

Can we Identify the Fed's Preferences?*

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Abstract

A pre-test of Ramsey optimal policy versus time-consistent policy rejects time-consistent policy and (optimal) simple rule for the U.S. Fed during 1960 to 2006, assuming the reference new-Keynesian Phillips curve transmission mechanism with auto-correlated cost-push shock. The number of reduced form parameters is larger with Ramsey optimal policy than with time-consistent policy although the number of structural parameters, including central bank preferences, is the same. The new-Keynesian Phillips curve model is under-identified with Ramsey optimal policy (one identifying equation missing) and hence under-identified for time-consistent policy (three identifying equations missing). Estimating a structural VAR for Ramsey optimal policy during Volcker-Greenspan period, the new-Keynesian Phillips curve slope parameter and the Fed's preferences (weight of the volatility of the output gap) are not statistically different from zero at the 5% level.

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1 Introduction

Can we *pre-test* if the Fed follows Ramsey optimal policy or a time-consistent policy (Cohen and Michel (1988))? Are central bank preferences facing the same *identification* problem as simple Taylor rule parameters? Cochrane (2011) found that the simple Taylor rule parameter describing the response of the interest rate to inflation is not identified in new-Keynesian models including forward-looking inflation and a non-observable autoregressive shock. Finally, can we *jointly test* on US data the new-Keynesian Phillips curve monetary policy transmission mechanism *and* a representation of the rule of the policy with passed the pre-test?

There are several estimations of Fed's preferences assuming inflation is a predetermined variable or a forward-looking variable, assuming Ramsey optimal policy or time-consistent policy, assuming Phillips curve or new-Keynesian Phillips curve as a monetary policy transmission mechanism: e.g. Cechetti and Ehrmann (2002), Ozlale (2003),

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Castelnuovo and Surico (2004), Castelnuovo (2006), Juillard et al. (2006), Adjemian and Devulder (2011), Leveuge and Lucotte (2014), Paez-Farrell (2015)...

Söderlind's (1999) is a commonly used maximum likelihood estimation of the Hamiltonian system of Ramsey optimal policy and of time-consistent policy. The functional form of the Hamiltonian system which is estimated does not take into account the transversality conditions seeking the stable path of the Hamiltonian. *Firstly this method has the same probability to fit the optimal path of the Hamiltonian system than the probability to select a given point on a continuous line, which is equal to zero. Secondly, identification issue are overlooked in this estimation.* This paper carefully deals with both issues.

We pre-test and test Ramsey optimal policy versus time-consistent policy with the reference new-Keynesian Phillips curve including an auto-regressive cost-push shock (Gali (2015, chapter 5)).

Firstly, this paper proposes a *pre-test* of the two spanning conditions of Ramsey optimal policy versus time-consistent policy and (optimal) simple rules (Kollmann (2002) and (2008) is a precursor for optimal simple rules simulations in models including the new-Keynesian Phillips curve). The principle is to test the number of linearly independent observed variables with stable dynamics predicted by Ramsey policy or by time-consistent policy for any DSGE model. Ramsey optimal policy has *richer dynamics* than time-consistent policy. It *includes* policy maker's marginal condition with respect to forward-looking variables such as inflation. Time-consistent policy *excludes* these equations. Ramsey optimal policy rules respond to a number of variables equal to the number of predetermined *and* of forward-looking variables. Time-consistent policy rules respond to a number of variable equal to the number of predetermined variables only. For US data from 1960 to 2006, time-consistent policy and (optimal) simple rule are *strongly rejected* by pre-test with respect to Ramsey optimal policy.

Secondly, Fed's preferences and monetary policy transmission mechanism are *tested* for the model which passed the pre-test. The number of reduced form parameters is larger with Ramsey optimal policy than with time-consistent policy although the number of structural parameters, including central bank preferences, is the same. Hence, at least one of the two equilibria is over-identified or under-identified. Because of the lower number of non-collinear variables in time-consistent policy than in Ramsey optimal policy, three identifying equations are missing in time-consistent policy. In the case of Ramsey optimal policy, Fed's preferences and monetary policy transmission channel structural parameters are identified when the Fed's discount factor is exogenously given, because only one identifying equation is missing.

In the test of Ramsey optimal policy, the two key parameter estimates are not statistically different from zero. The first parameter is the slope of the new-Keynesian Phillips curve (parameter κ) which models the monetary policy transmission effect of the policy instrument on inflation. This result is perfectly in line with hundreds of estimates of the parameter κ in limited-information single-equation of the new-Keynesian Phillips curve in Mavroeidis, Plagbord-Möller and Stock's (2014) *Journal of Economic Literature* survey. In this paper, the evidence is obtained estimating a full-information structural vector auto-regressive (VAR) of Ramsey optimal policy, including two equations: the new-Keynesian Phillips curve monetary policy transmission mechanism and a representation of Ramsey optimal policy rule.

The second parameter is the Fed's preference of the cost of changing the policy instrument. If the monetary policy transmission effect is zero, even if the central bank is

willing to stabilize inflation at any cost, it is not able to achieve this policy. There is nothing counterfactual in these estimates.

Unconstrained VAR parameter estimates are close to the values of reduced form parameters computed from structural parameters estimates of Ramsey optimal policy. The inflation equation of the VAR has identical estimates in both cases. In the policy instrument equation of VAR, its persistence shifts from an unconstrained estimate equal to 0.9 to a Ramsey optimal policy estimate close to a unit root. Ramsey optimal policy predicts *too much persistence* of the policy instrument. *Ad hoc* additional assumptions such as inflation indexation and consumption habits in the new-Keynesian Phillips curve would push towards even more persistence for Ramsey optimal policy, that is, an even more sizable misspecification.

Section 2 presents Ramsey optimal policy and time-consistent policy estimation method. Section 3 presents the pre-test theory and its implementation on US data during 1960-2006. Section 4 tests Ramsey optimal policy. Section 5 evaluates the robustness to misspecification of Ramsey optimal policy versus time-consistent policy. The last section concludes.

2 The Monetary Policy Problem: The Case of an Efficient Steady State

Gali's (2015, chapter 5) reference model for Ramsey optimal policy considers the case of an efficient steady state. The welfare losses experienced by the representative household are, up to a second-order approximation, proportional to:

$$v(\pi_0, u_0) = \max_{\{x_t, \pi_t\}} - \frac{1}{2} E_0 \left\{ \sum_{t=0}^{+\infty} \beta^t (\pi_t^2 + \alpha_x x_t^2) \right\} \quad (1)$$

where x_t represents the welfare-relevant output gap, i.e. the deviation between (log) output and its efficient level. π_t denotes the rate of inflation between periods $t - 1$ and t . u_t denotes a cost-push shock. β denotes the discount factor. E_t denotes the expectation operator. $v(\pi_0, u_0)$ denotes the optimal value function. Coefficient $\alpha_x > 0$ represents the weight of the fluctuations of the marginal cost of the firm (measured by the output gap) relative to inflation in the loss function. Coefficient $\alpha_x > 0$ is the relative cost of the changing the policy instrument with respect to the costs of fluctuations of the policy target, which is inflation. It is given by $\alpha_x = \frac{\kappa}{\eta}$ where κ is the coefficient on the marginal cost of the firm x_t in the New Keynesian Phillips curve, and η is the elasticity of substitution between goods. More generally, and stepping beyond the welfare-theoretic justification for (1), one can interpret α_x as the weight attached by the central bank to deviations of output from its efficient level (relative to price stability) in its own loss function, which does not necessarily have to coincide with the household's. A structural equation relating inflation and the welfare-relevant output gap can be derived leading to the new-Keynesian Phillips curve:

$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa x_t + u_t \text{ where } \kappa > 0, 0 < \beta < 1 \quad (2)$$

The central bank minimizes (1) subject to the sequence of constraints given by (2). The cost push shock u_t includes an exogenous auto-regressive component:

$$u_t = \rho u_{t-1} + \varepsilon_{u,t} \text{ where } 0 < \rho < 1 \text{ and } \varepsilon_{u,t} \text{ i.i.d. normal } N(0, \sigma_u^2) \quad (3)$$

where ρ denotes the auto-correlation parameter and ε_t is identically and independently distributed (i.i.d.) according to a normal distribution with constant variance σ_u^2 .

2.1 Ramsey Optimal Policy

2.1.1 Solution using Lagrange Multipliers

A policy maker with a mandate for a new policy regime revised on date $t = 0$ (corresponding to a structural break in econometrics) commits to Ramsey optimal policy from the current date *until* a given known date T where the optimal policy is optimized again. The duration T of commitment ranges from a minimal duration of *two weeks* between official meetings of the boards of governors for the European Central Bank up to *ten to twenty years* of a stable monetary policy regime. Ramsey optimal policy can be solved directly using Bellman's equation, substituting the law of motion of the economy into the policy-maker's loss function without Lagrange multipliers. With the Lagrange intermediate computations, the Lagrangian of Ramsey optimal policy includes a sequence of Lagrange multipliers γ_{t+1} .

$$\mathcal{L} = -E_0 \sum_{t=0}^{t=T} \beta^t \left[\frac{1}{2} (\pi_t^2 + \alpha_x x_t^2) + \gamma_{t+1} (\pi_t - \kappa x_t - \beta \pi_{t+1}) \right] \quad (4)$$

The law of iterated expectations has been used to eliminate the condition expectations that appeared in each constraint. Because of the certainty equivalence principle for determining optimal policy in the linear quadratic regulator including additive normal random shocks (Simon (1956)), the expectations of random variables u_t are set to zero and do not appear in the Lagrangian.

The program includes given initial u_0 and final boundary conditions for the predetermined forcing variable $\lim_{t \rightarrow +\infty} \beta^t u_t = 0$. It also includes optimal initial and final boundary values of the forward-looking variable inflation. These transversality conditions minimize the optimal value of the central bank's loss function at the initial and the final date:

$$\frac{\partial v(\pi_t, u_t)}{\partial \pi_t} = 0 = \beta^t \gamma_t \text{ predetermined for } t = \{0, T\} \Leftrightarrow \pi_t = \pi_t^* \text{ for } t = \{0, T\} \quad (5)$$

$$\lim_{T \rightarrow +\infty} \frac{\partial v(\pi_T, u_T)}{\partial \pi_T} = 0 = \lim_{T \rightarrow +\infty} \beta^T \gamma_T \Leftrightarrow \lim_{t \rightarrow +\infty} \pi_T = \lim_{t \rightarrow +\infty} \pi_T^* \text{ if } T \rightarrow +\infty \quad (6)$$

We follow Gali (2015) and we consider the limit case where the revision for a new policy regime happens in the infinite horizon. Differentiating the Lagrangian with respect to the policy instrument (output gap x_t) and to the policy target (inflation π_t) yields the first order optimality conditions:

$$\frac{\partial \mathcal{L}}{\partial x_t} = 0 \Rightarrow \alpha_x x_t - \kappa \gamma_{t+1} = 0 \quad (7)$$

$$\frac{\partial \mathcal{L}}{\partial \pi_t} = 0 \Rightarrow \pi_t + \gamma_{t+1} - \gamma_t = 0 \quad (8)$$

$$\gamma_0 = 0 \Rightarrow x_{-1} = -\frac{\kappa}{\alpha_x} \gamma_0 = 0 \text{ and } \pi_0 = -\gamma_1 = -\frac{\kappa}{\alpha_x} x_0 \quad (9)$$

that must hold for $t = 1, 2, \dots$ where $\gamma_0 = 0$, because the inflation Euler equation corresponding to period 0 is not an effective constraint for the central bank choosing its optimal plan in period 0. The former commitment to the value of the policy instrument of the previous period x_{-1} is not an effective constraint. The policy instrument is predetermined at the value zero $x_{-1} = 0$ at the period preceding the commitment.

Combining the two optimality conditions to eliminate the Lagrange multipliers yields the optimal initial anchor of forward inflation π_0 on the predetermined policy instrument x_0 :

$$\pi_0 = -\frac{\alpha_x}{\kappa} x_0 \quad (10)$$

and the central bank's Euler equation for the periods following period 0, for $t = 1, 2, 3, \dots$

$$x_t = x_{t-1} - \frac{\kappa}{\alpha_x} \pi_t. \quad (11)$$

The central bank's Euler equation links recursively the future or current value of central bank's policy instrument x_t to its current or past value x_{t-1} , because of the central bank's relative cost of changing her policy instrument is strictly positive $\alpha_x > 0$. This non-stationary Euler equation adds an unstable eigenvalue in the central bank's Hamiltonian system including three laws of motion of one forward variable (inflation π_t) and of two predetermined variables (u_t, x_t) or (u_t, γ_t).

Ljungqvist and Sargent (2012, chapter 19) seek the stationary equilibrium process using the augmented discounted linear quadratic regulator (ADLQR) solution of the Hamiltonian system (Anderson, Hansen, McGrattan and Sargent (1996)) as an intermediate step. Using the method of undetermined coefficients, this solution seeks optimal negative-feedback rule parameters $\mathbf{F}_R = (F_{\pi,R}, F_{u,R})$ function of structural parameters $(\alpha_x, \beta, \kappa, \rho)$ satisfying the infinite horizon transversality conditions. The policy instrument should be exactly correlated with private sectors variables:

$$x_t = F_{\pi,R}(\alpha_x, \beta, \kappa) \pi_t + F_{u,R}(\alpha_x, \beta, \kappa, \rho) u_t. \quad (12)$$

Ljungqvist and Sargent (2012, chapter 19) ADLQR intermediate step basis vectors (π_t, u_t) of the stable subspace or Ljungqvist and Sargent (2012, chapter 19) final step basis vectors (γ_t, u_t) and Galí's (2015, chapter 5) basis vectors (x_t, u_t) include the non-observable predetermined cost-push shock u_t in their VAR(1) within the Hamiltonian system (H). How to derive one representation from the other is described in the appendix.

2.1.2 Issues of Söderlind's (1999) Estimation Method.

Söderlind (1999) estimation method of Ramsey optimal policy faces two problems.

Problem 1: Estimating the wrong model.

Söderlind (1999) estimates the Hamiltonian system outside its stable subspace. In our case, it omits the stable subspace optimal rule and restriction: $x_t = F_{\pi,R}\pi_t + F_{u,R}u_t$. It estimates instead the policy maker’s first order equation. In Söderlind’s method, the central bank’s preferences parameter appears in a simple explicit linear form in the Hamiltonian system of equations. It is easy to compute the likelihood of this vector-auto-regressive (VAR) model.

In the general case, as soon as there are at least two endogenous controllable variables, the solution for $(F_{\pi,R}, F_{u,R})$ is an implicit function of central bank preferences and monetary policy transmission parameters, given by Riccati and Sylvester equations. This is the reason why the stable subspace optimal policy rule constraint is not handled in Söderlind’s estimation method.

It does not make sense to estimate optimal policy using the saddle-point equilibrium Hamiltonian system omitting its stable subspace optimal rule constraint. The saddle-point equilibrium Hamiltonian system is a *fictitious intermediate computational* step to solve optimal policy. One can solve linear quadratic optimal control with Bellman’s equation to find optimal negative-feedback rule parameters $(F_{\pi,R}, F_{u,R})$ directly, without Lagrange multipliers. The probability that Söderlind’s methods estimates the unique path of optimal policy within the stable subspace is *zero*.

Problem 2: Identification issue.

Estimating a VAR with lagged dependent variable and non-observable auto-regressive leads to a classic identification problem (Griliches (1967), Feve, Matheron, Poilly (2007)), where two sets of parameters are observationally equivalent for the auto-correlation parameter of the cost-push shock (see appendix).

2.1.3 Solving Issues of Söderlind’s (1999) Estimation Method.

In the case of a single controllable variable with *explicit* solutions for $(F_{\pi,R}, F_{u,R})$, it is feasible to solve the issues raised by Söderlind’s (1999) estimation method. This new estimation method give insights and hope for further research with *implicit* solutions for \mathbf{F} based on matrix Riccati and Sylvester equations when there is at least two controllable variables. Firstly, the stable subspace optimal rule constraint is complicated. But, secondly, it helps to remove a fundamental limited-information Griliches (1967) identification issue for policy rules.

Firstly, we substitute the non-stationary Euler equation by the stationary optimal rule including endogenous $(F_{\pi,R}, F_{u,R})$.

Secondly, we do a *full-information estimation* of the optimal policy rule and its transmission mechanism, including all the equations of optimal policy for observable variables (inflation and the policy instrument). We use the *stable subspace optimal rule constraint* to eliminate the auto-regressive shock in the VAR(1) in the inflation equation and *substitute it by the policy instrument*. In this VAR(1), the auto-correlation parameter of the shock appears *inside* the matrix of the auto-correlation of the observable variables (inflation and the policy instrument), but *not* in the residuals of this VAR, which are white noise. The auto-correlation parameter of the shock appears into the residuals estimating of the inflation equation, with lagged inflation and auto-correlated cost-push shock, (leading to Griliches identification problem) only because it is a *limited-information* estimation, estimating only the inflation (transmission mechanism) equation of the model without estimating jointly the optimal policy rule equation. This full-information method

holds with an *auto-correlated* cost-push shock (with its parameter estimated): it does not assume this auto-correlation is zero to obtain white noise in the full information VAR.

We suggest using the basis vectors (π_t, x_t) of the stable subspace for the VAR(1) representation within the Hamiltonian system, using the mathematical equivalence of systems of equations for $t = 1, 2, 3, \dots$:

$$(H) \begin{cases} \begin{pmatrix} \pi_{t+1} \\ u_{t+1} \end{pmatrix} = (\mathbf{A} + \mathbf{B}\mathbf{F}_R) \begin{pmatrix} \pi_t \\ u_t \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varepsilon_t \\ x_t = F_{\pi,R}\pi_t + F_{u,R}u_t \\ \pi_0 = -\frac{\alpha_x}{\kappa}x_0 \text{ and } u_0 \text{ given} \end{cases} \\ \Leftrightarrow \begin{cases} \begin{pmatrix} \pi_{t+1} \\ x_{t+1} \end{pmatrix} = \mathbf{M}^{-1}(\mathbf{A} + \mathbf{B}\mathbf{F})\mathbf{M} \begin{pmatrix} \pi_t \\ x_t \end{pmatrix} + \mathbf{M}^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varepsilon_t \\ u_t = \frac{1}{F_{u,R}}x_t - \frac{F_{\pi,R}}{F_{u,R}}\pi_t \\ \pi_0 = -\frac{\alpha_x}{\kappa}x_0 \text{ and } u_0 \text{ given} \end{cases} \quad (13)$$

with:

$$\mathbf{A} + \mathbf{B}\mathbf{F}_R = \begin{pmatrix} \frac{1}{\beta} - \frac{\kappa}{\beta}F_{\pi,R} & -\frac{1}{\beta} - \frac{\kappa}{\beta}F_{u,R} \\ 0 & \rho \end{pmatrix} \\ \begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} \pi_t \\ u_t \end{pmatrix} \text{ with } \mathbf{M}^{-1} = \begin{pmatrix} 1 & 0 \\ F_{u,R} & F_{\pi,R} \end{pmatrix}$$

$F_{u,R}$ is eliminated using $-\frac{1}{\beta} - \frac{\kappa}{\beta}F_{u,R} = (1 - \rho)\lambda_R \frac{F_{u,R}}{F_{\pi,R}}$ and $F_{\pi,R} = \frac{\lambda_R}{1 - \lambda_R} \frac{\kappa}{\alpha_x}$ (see appendix 1):

$$\mathbf{M}^{-1}(\mathbf{A} + \mathbf{B}\mathbf{F})\mathbf{M} = \begin{pmatrix} \rho\lambda_R & (1 - \rho)\lambda_R \frac{1}{F_{\pi,R}} \\ \rho(\lambda_R - 1)F_{\pi,R} & \rho + (1 - \rho)\lambda_R \end{pmatrix} = \begin{pmatrix} \rho\lambda_R & (1 - \rho)(1 - \lambda_R) \frac{\alpha_x}{\kappa} \\ -\rho\lambda_R \frac{\kappa}{\alpha_x} & \rho + (1 - \rho)\lambda_R \end{pmatrix}$$

with:

$$\lambda_R \begin{pmatrix} \beta, \alpha_x, \kappa \\ -, +, - \end{pmatrix} = \frac{1 - \kappa F_{\pi}}{\beta} = \frac{1}{2} \left(1 + \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} \right) - \sqrt{\frac{1}{4} \left(1 + \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} \right)^2 - \frac{1}{\beta}} = \delta$$

where the two invariant stable eigenvalues of the stable subspace are λ_R denoted δ by Gali (2015) and ρ (appendix 2). The other representations of the VAR(1) including the non-observable cost-push shocks u_t amounts to estimate the VAR(1) as a partial adjustment inflation or output gap equation with serially correlated cost-push shocks u_t . These equations face a classic problem of identification and multiple equilibria, because the auto-correlation of the dependent variable and of the disturbances are competing to model persistence (Griliches (1967), Blinder (1986), McManus et al. (1994), Fève, Matheron Poilly (2007), appendix).

We eliminate the non-observable serially correlated cost-push shock u_t with a change of basis vectors (π_t, x_t) including observable variables. Hence, we are able to fit a structural VAR(1) with the assumption of *white noise shocks instead of serially correlated shocks* (Sims (1980)). Structural parameters are estimated with feasible generalized non-linear

least squares for a system of equations. Theory-based constraints on the four reduced form parameters of the matrix $\mathbf{M}^{-1}(\mathbf{A} + \mathbf{BF})\mathbf{M}$ imply that only three structural parameters can be identified: $\rho, \lambda_R, F_{\pi,R}$ or $\rho, \lambda_R, \frac{\alpha_x}{\kappa}$ or $\rho, \alpha(\beta), \kappa(\beta)$ for a given value of the discount factor β :

$$\kappa(\beta) = \frac{1 - \lambda_R \beta}{F_{\pi,R}} \Rightarrow \alpha_x(\beta) = \left(\frac{\lambda_R}{1 - \lambda_R} \right) \frac{1}{F_{\pi,R}} \kappa(\beta). \quad (14)$$

If initial values of inflation and of the policy instrument (in deviation from their equilibrium values) were perfectly measured at the date of commitment, the ratio $\frac{\alpha_x}{\kappa}$ would be over-identified by the optimal initial anchor of forward inflation on the predetermined policy instrument equation:

$$\frac{\alpha_x}{\kappa} = \frac{-\pi_0}{x_0}. \quad (15)$$

The semi-reduced form cost-push shock rule parameter $F_{u,R}$ requires an identification restriction, for example, setting a value for β (see appendix 2):

$$F_{u,R}(\beta) = \frac{-1}{1 - \beta \rho \lambda_R} F_{\pi,R} < 0. \quad (16)$$

The standard error σ_u of cost-push shock is computed using the standard error of residuals $\sigma_{\varepsilon,x}$ of the output gap rule equation in the VAR(1). It requires an identification restriction, because it depends on $F_{u,R}$:

$$\sigma_u(\beta) = \frac{\sigma_{\varepsilon,x}}{F_{u,R}(\beta)}. \quad (17)$$

The standard error of the measurement of the inflation equation σ_π (which is theoretically predicted to be zero) and its covariance with the cost push shock $\sigma_{x\pi} = F_{u,R} \sigma_{xu}$ are also available.

One identifying equation is missing in order to identify the remaining four structural parameters ($\alpha_x, \kappa, \beta, \sigma_u$) and the negative feedback rule parameter $F_{u,R}$. We set an identification restriction on the discount factor to a given value: $\beta = 0.99$ or $\beta = 1$ in the estimations. One of the reason why an identification restriction is required for Ramsey optimal policy is that the AR(1) cost-push shock u_t is not observable. The usual practice of DSGE modelers is to include a number of AR(1) processes equal to the number forward variables, i.e. all prices and flows of quantities variables in their model. The larger the number of non-observable AR(1) processes, the more likely identifying structural parameters of Ramsey optimal policy (including central bank preferences) would require additional identification restrictions.

2.2 Time-Consistent Policy

Gali (2015, chapter 5) considers the case of a particular time-consistent policy (Cohen and Michel (1988), Oudiz and Sachs (1985)) when the policy maker's knows perfectly the parameters of the policy transmission mechanism. The central bank minimizes its loss function subject to the new-Keynesian Phillips curve *and* subject to two additional constraints. These constraints forces the marginal value of the loss function with respect to inflation (the policy maker's Lagrange multiplier on inflation) to stick to the value zero at all periods. Hence, this rule does not change if the policy maker optimizes at the

initial date or at any future date.

These constraints *assume* that both the private sector and the central bank commit for ever to *restricted* policy rules where their policy instrument reacts only to the contemporary *predetermined variable* u_t at all periods t , with a *perfect correlation*. These time-consistent rules are determined by time-invariant rule parameters N_{TC} and $F_{u,TC}$ to be optimally chosen for all periods, assuming common and complete knowledge of structural parameters including preferences of both agents:

$$\pi_t = N_{TC}u_t \text{ and } x_t = F_{u,TC}u_t \quad (18)$$

Indeed, the central bank policy time-consistent rule has a representation where its policy instruments responds only to current inflation, after substitution of private sector time-consistent rule:

$$x_t = F_{u,TC}u_t = F_{\pi,TC}\pi_t \text{ with } F_{\pi,TC} = \frac{F_{u,TC}}{N_{TC}} \quad (19)$$

The Central Bank *commits for ever* to a *restricted time-consistent rule* where the policy instrument responds only to current inflation or only to the current non-observable cost-push shock *with a perfect correlation*. Time-consistent policy is the *opposite* of time-inconsistent discretion, which is the ability of not sticking to any policy rules over time because new problems arise that could not be anticipated. It is a recent mistake since the 2000's to refer to *time-inconsistent discretion* for this solution instead of the initial name of *time-consistent policy* given by researchers who invented them in the 80's.

In order to have policy rule parameters to be identified, the reduced form representations of the rules of the optimal policy reacts to a number of variables equal to the number of predetermined variables. In Ramsey optimal policy, this number is equal to two. It is equal to one with time-consistent policy. Ramsey optimal policy allows a less-restricted, more-flexible reduced form representations of its policy rules, where the policy instruments responds to its lagged value *in addition* to current inflation or current cost-push shock. In practice, this rule has more room for flexibility and adjustment due to misspecification than rules of time-consistent policy.

We derive the solution of time-consistent monetary policy facing the new-Keynesian Phillips curve according to Cohen and Michel (1988) and Oudiz and Sachs (1985). Their solution *slightly differs* from Gali (2015) who additionally assumes that the Central Bank does not know the rules at future dates (see appendix). Substituting the private sector's inflation rule and the policy rule in the loss function:

$$\max_{\{\pi, x_t\}} -\frac{1}{2}E_0 \sum_{t=0}^{+\infty} \beta^t (\pi_t^2 + \alpha_x x_t^2) = \max_{\{F_{u,TC}, N_{TC}\}} -\frac{1}{2} (N_{TC}^2 + \alpha_x F_{u,TC}^2) \frac{u_0^2}{1 - \beta\rho^2}$$

The central bank first order condition is:

$$0 = N_{u,TC} \frac{\partial N_{u,TC}}{\partial F_{u,TC}} + \alpha_x F_{u,TC}$$

$$F_{\pi,TC} = \frac{F_{u,TC}}{N_{TC}} = -\frac{1}{\alpha_x} \frac{\partial N_{u,TC}}{\partial F_{u,TC}}$$

Substituting the private sector's inflation rule and the policy rule in the inflation law of motion leads to the following relation between N_{TC} on date t , $N_{TC,t+1}$ and $F_{u,TC}$:

$$\begin{aligned}\pi_t &= \beta E_t [\pi_{t+1}] + \kappa x_t + u_t \Rightarrow \\ N_{TC} u_t &= \beta N_{TC,t+1} \rho u_t + \kappa F_{u,TC} u_t + u_t \\ N_{TC} &= \beta \rho N_{TC,t+1} + \kappa F_{u,TC} + 1\end{aligned}$$

In the reference Oudiz and Sachs' (1985) dynamic Nash equilibrium, the central bank foresees that $N_{TC,t+1} = N_{TC}$ in its optimization (see appendix):

$$N_{TC} = \frac{\kappa F_{u,TC} + 1}{1 - \beta \rho} = \frac{\kappa F_{\pi,TC} N_{TC} + 1}{1 - \beta \rho} \Rightarrow \frac{\partial N_{u,TC}}{\partial F_{u,TC}} = \frac{\kappa}{1 - \beta \rho}$$

The endogenous rule parameters are increasing function of the central bank cost of changing the policy instrument α_x . They are bounded by limit values of $\alpha_x \in]0, +\infty[$:

$$\begin{aligned}0 < \frac{\pi_{t,TC}}{u_t} = N_{TC}(\alpha_x) &= \frac{\alpha_x (1 - \beta \rho)}{\alpha_x (1 - \beta \rho)^2 + \kappa^2} < N = \frac{1}{1 - \beta \rho} \\ -\frac{1}{\kappa} < \frac{x_{t,TC}}{u_t} = F_{u,TC}(\alpha_x) &= \frac{-\kappa}{\alpha_x (1 - \beta \rho)^2 + \kappa^2} < 0 \\ -\infty < \frac{x_{t,TC}}{\pi_{t,TC}} = F_{\pi,TC}(\alpha_x) &= \frac{F_{u,TC}}{N_{TC}} = \frac{-\kappa}{\alpha_x (1 - \beta \rho)} < \frac{-\kappa}{\alpha_x} < 0\end{aligned}$$

For an infinite cost of changing the policy instrument $\alpha_x \rightarrow +\infty$, we label this equilibrium as "laissez-faire" because two policy rule parameters are both equal to zero $F_{\pi,TC} = 0 = F_{u,TC}$. The policy instrument x_t is set to zero at all dates: it is eliminated in the model. It corresponds to the maximal initial response of inflation (in absolute values) to cost-push shock $Nu_t = \frac{1}{1-\beta\rho}u_t$ for time-consistent policy.

For the limit case of a zero cost of changing the policy instrument ($\alpha_x \rightarrow 0$), the policy instrument (output gap) has its largest response to cost-push shock $x_0 = -\frac{1}{\kappa}u_0$ so that the policy target (inflation) does not respond to the cost-push shock (N_{TC} is zero).

The policy instrument (the output gap) x_t is exactly *negatively* correlated ($F_{\pi,TC} < 0$) with the policy target (inflation) π_t . When increasing the central bank's preferences (α_x) for the relative cost of changing the output gap from zero to infinity, the strictly negative rule parameter $F_{\pi,TC}$ increases from minus infinity to zero. There is one stable eigenvalue and one unstable eigenvalue:

$$0 < \rho < 1 < \frac{1}{\beta} \leq \lambda_{TC} = \frac{1 - \kappa F_{\pi,TC}}{\beta} < +\infty. \quad (20)$$

The welfare loss of time-consistent policy v_{TC} as a proportion of the limit maximal value of the welfare loss with the largest volatility of inflation (laissez-faire) v_{LF} turns to be equal to the ratio of inflation under time-consistent policy to inflation under laissez-faire. It increases from zero to one when the cost of changing the policy instrument increases from zero to infinity:

$$0 < \frac{v_{TC}}{v_{LF}} = \frac{N_{TC}^2 + \alpha_x F_{u,TC}^2}{N^2} = \frac{\alpha_x (1 - \beta \rho)^2}{\alpha_x (1 - \beta \rho)^2 + \kappa^2} = \frac{N_{TC}}{N} = \frac{\pi_{t,TC}}{\pi_{t,LF}} < 1$$

3 Pre-test of Ramsey versus Time-Consistent Policy and Optimal Simple Rule

3.1 A Bifurcation of the Economy Dynamical System

This theoretical section adds *new analytical results* with respect to Gali (2015, chapter 5). It compares Taylor rule parameter responding to inflation for Ramsey versus time-consistent policy (only presented for time-consistent policy in Gali (2015)). They have opposite signs. They correspond respectively to negative-feedback versus positive-feedback rule. This section compares the inflation eigenvalue for Ramsey versus time-consistent policy. It is only presented for Ramsey policy in Gali (2015). We provide their functional form depending on structural parameters which is not done in Gali (2015) but useful for a correct estimation. Ramsey optimal policy eigenvalue (λ_R) is stable. Time-consistent policy eigenvalue (λ_{TC}) is unstable.

Shifting from Ramsey policy to time-consistent policy corresponds to a *bifurcation* of the dynamic system of the economy. This section shows that optimal simple rule corresponds to reduced form of time-consistent policy. These major differences are overlooked in Gali (2015). They provide the key *idea of our pre-test* of the maximal number of variables predicted to evolve in a stable VAR for Ramsey policy versus for time-consistent policy.

Simple rule assume that there is no policy-maker loss function nor welfare function and that the policy instruments are forward variables. For simple rule, when inflation and the policy instrument are forward variables, only the cost-push auto-regressive shock is a predetermined variable with an exogenous stable eigenvalue ρ . Blanchard and Kahn's (1980) determinacy condition imply that the controllable eigenvalue, indexed by S for "simple rule": $\lambda_S = \frac{1-\kappa F_{\pi,S}}{\beta}$ should be unstable ($|\lambda_S| > 1$). This implies constraints on the values of the inflation rule parameter $F_{\pi,S} = \frac{1-\beta\lambda}{\kappa}$. Because of the simple rule stable subspace is of dimension one, omitting the identifying restriction $F_{u,S} = 0$ would imply that both rule parameters $F_{\pi,S}$ and $F_{u,S}$ are not identified.

Proposition 1: *In Gali's (2015) model, an optimal simple rule minimizing the central bank loss function is a reduced form of time-consistent policy.*

Proof: For a given monetary policy transmission mechanism $(\beta, \kappa, \rho, \sigma_u)$, a simple rule with a *strictly negative* inflation parameter $F_{\pi,S}$, forcing an unstable eigenvalue $\lambda_S \in]\frac{1}{\beta}, +\infty[$ by positive feedback, is the reduced form of time-consistent policy with a unique central bank preference parameter α_x given by:

$$F_{\pi,S} = F_{\pi,TC} = -\frac{\kappa}{\alpha_x} \frac{1}{1-\beta\rho} < 0 \implies \alpha_x = -\frac{\kappa}{F_{\pi,S}} \frac{1}{1-\beta\rho} \quad (21)$$

The remaining cases of simple rules with *positive* rule parameter $F_{\pi,S} \in [0, \frac{1-\beta}{\kappa}[\cup]\frac{1+\beta}{\kappa}, +\infty[$ forcing an unstable eigenvalue $\lambda_S \in [1, \frac{1}{\beta}[\cup]-\infty, -1[$ by positive feedback do not minimize a central bank loss function in time-consistent policy. For $F_{\pi,S} \in [0, \frac{1-\beta}{\kappa}[$, these simple rules imply a jump of inflation larger than in laissez-faire ($N_S > N$). For $F_{\pi,S} \in]\frac{1+\beta}{\kappa}, +\infty[$, these simple rules imply a jump of inflation with an opposite sign with respect to laissez-faire ($N_S < 0 < N$). Those simple rule solution are never optimal simple rule nor reduced form of time-consistent policy. When the policy instrument is a forward variable, simple rule parameter $F_{\pi,S}$ is never the reduced form rule parameter

of the Ramsey optimal policy inflation rule parameter $F_{\pi,R}$ forcing a stable eigenvalue $\lambda_S \in]0, \beta[$ by negative feedback, where the policy instrument is predetermined. Kollmann (2002 and 2008) is a precursor for computing optimal simple rules with new-Keynesian Phillips curve models. Q.E.D.

Sargent and Ljungqvist's (2012) LQR intermediate step allows a direct comparison between the reduced form inflation rule parameters $F_{\pi,TC}$ of time-consistent policy (respectively $F_{\pi,R}$ of Ramsey optimal policy), which is *affine negative function* of the eigenvalue λ_{TC} (respectively λ_R):

$$F_{\pi,TC} = -\frac{1}{1-\beta\rho} \frac{\kappa}{\alpha_x} \text{ and } \lambda_{TC} = \frac{1}{\beta} - \frac{\kappa}{\beta} F_{\pi,TC} = \frac{1}{\beta} + \frac{1}{\beta} \frac{1}{1-\beta\rho} \frac{\kappa^2}{\alpha_x} \quad (22)$$

$$F_{\pi,R} = \frac{1}{\kappa} - \frac{\beta}{\kappa} \lambda_R \text{ and } \lambda_R = \frac{1}{2} \left(1 + \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} \right) - \sqrt{\frac{1}{4} \left(1 + \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} \right)^2 - \frac{1}{\beta}} \quad (23)$$

Figure 1 plots the eigenvalue λ_{TC} of time-consistent policy (and respectively the eigenvalue λ_R of Ramsey optimal policy) as non-linear decreasing (respectively increasing) function of the relative cost of changing the policy instrument α_x for the estimated parameters $\rho = 0.995$, $\kappa = 0.340$ for a given $\beta = 0.99$ of the Ramsey optimal policy model during Volcker-Greenspan's Fed starting 1979q3-2006q2 (see estimation section, with estimated $\lambda_R = 0.856$ and $\alpha_x = 4.552$). For a minimal cost of changing the policy instrument, the eigenvalue λ_R tends to zero for Ramsey optimal policy and λ_{TC} tends to infinity for time-consistent policy. For an infinite cost of changing the policy instrument: the eigenvalue λ_R tends to one for Ramsey optimal policy and λ_{TC} tends to $1/\beta > 1$ for time-consistent policy.

Figure 2 plots inflation rule parameter $F_{\pi,TC}$ of time-consistent policy (and respectively $F_{\pi,R}$ of Ramsey optimal policy) as non-linear decreasing (respectively increasing) function of the relative cost of changing the policy instrument α_x for the same estimated parameters than for figure 1 (with estimated $F_{\pi,R} = 0.447$ and $\alpha_x = 4.552$). For a minimal cost of changing the policy instrument, the inflation rule parameter $F_{\pi,R}$ tends to $1/\kappa$ and $F_{\pi,TC}$ tends to minus infinity for time-consistent policy. For an infinite cost of changing the policy instrument, the inflation rule parameter $F_{\pi,R}$ tends to $(1-\beta)/\kappa$ for Ramsey optimal policy and $F_{\pi,TC}$ tends to zero for time-consistent policy.

Figures 1 and 2: Eigenvalues λ and inflation rule parameter F_π function of α_x for Ramsey optimal policy (solid line) and time-consistent policy (dash line)

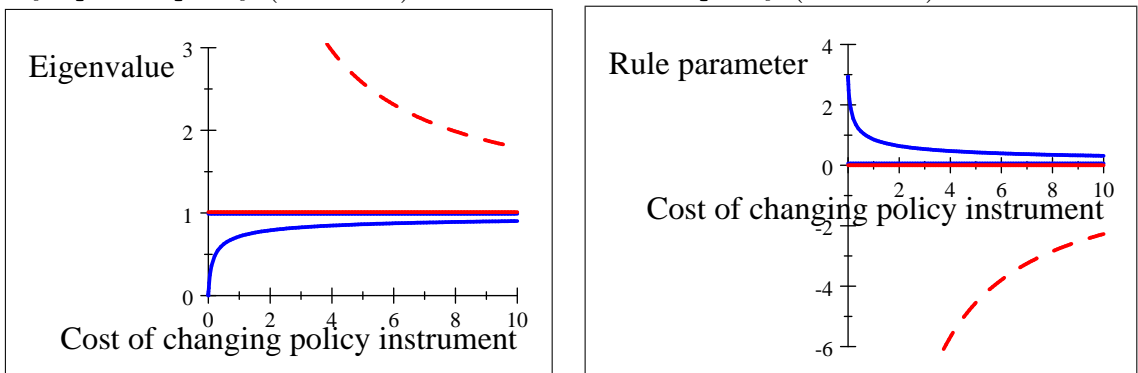


Table 1 summarizes the opposite properties of Ramsey optimal policy versus time-consistent policy and simple rule when inflation is a forward variable.

Table 1: Ramsey optimal policy, time-consistent policy and simple rule

Policy	Predetermined inflation π_t	Forward inflation π_t
Ramsey	2 predetermined: u_t, π_t stable subspace dim=2 1 forward: x_t $\alpha_x \in]0, +\infty[$ $F_{\pi,R} \in]\frac{1-\beta}{\kappa}, \frac{1}{\kappa}[$ $\lambda_R \in]0, 1[$ negative feedback	2 predetermined: u_t, x_t or γ_t stable subspace dim=2 1 forward: π_t $\alpha_x \in]0, +\infty[$ $F_{\pi,R} \in]\frac{1-\beta}{\kappa}, \frac{1}{\kappa}[$ $\lambda_R \in]0, 1[$ negative feedback
Time-Consistent	2 predetermined: u_t, π_t stable subspace dim=2 1 forward: x_t $\alpha_x \in]0, +\infty[$ $F_{\pi} \in]\frac{1-\beta}{\kappa}, \frac{1}{\kappa}[$ $\lambda \in]0, 1[$ negative feedback	1 predetermined: u_t stable subspace dim=1 2 forward: π_t, x_t $\alpha_x \in]0, +\infty[$ $F_{\pi,TC} \in]-\infty, 0[, F_{u,TC} = 0$ $\lambda_{TC} \in]\frac{1}{\beta}, +\infty[$ positive feedback
Simple rule	2 predetermined: u_t, π_t stable subspace dim=2 1 forward: x_t $\lambda \in]-1, 1[$ $F_{\pi} \in]\frac{1-\beta}{\kappa}, \frac{1+\beta}{\kappa}[$ negative feedback .	1 predetermined: u_t stable subspace dim=1 2 forward: π_t, x_t $\lambda_S \in]\frac{1}{\beta}, +\infty[\cup [1, \frac{1}{\beta}[\cup]-\infty, -1[$ $F_{\pi,S} \in]-\infty, 0[\cup [0, \frac{1-\beta}{\kappa}[\cup]\frac{1+\beta}{\kappa}, +\infty[$ positive feedback $F_{u,S} = 0$

Ramsey optimal policy: limit cases.

For very small or very large values of α_x , time-consistent policy and Ramsey optimal policy paths and hence central bank losses are identical. *Optimizing at later periods leads to a negligible deviation from the optimal path chosen at the initial period.* When $\alpha_x \rightarrow 0$, inflation π_t tends to zero at all dates for any initial value of the cost-push shock u_0 for Ramsey optimal policy and time-consistent policy. For Ramsey optimal policy, the policy rules are $F_{\pi,R} = \frac{1-\beta}{\kappa}$ and $F_{u,R} = \frac{-F_{\pi,R}}{1-\beta\rho^2}$. For time-consistent policy, the policy rule parameter $F_{\pi,TC} \rightarrow -\infty$ and $F_{u,TC} = 0$. When $\alpha_x \rightarrow +\infty$, π_t tends to laissez-faire inflation $\frac{1}{1-\beta\rho}u_t$ at all dates for any initial value of the cost-push shock u_0 for both Ramsey and time-consistent policy. For Ramsey optimal policy, the policy rules are $F_{\pi,R} = \frac{1}{\kappa} = -F_{u,R}$. For time-consistent policy, the policy rule parameters are both equal to zero (laissez-faire equilibrium) $F_{\pi,TC} = F_{u,TC} = 0$.

Although the equilibrium paths of time-consistent policy and Ramsey optimal policy are the same and evolve in a subspace of dimension one, the distinct policy rule parameters for time-consistent policy versus Ramsey optimal policy reflects that there exists a two-dimension stable subspace where Ramsey optimal policy path is surrounded by stable out-of-equilibrium paths which does not exist for time-consistent policy and for laissez-faire. This matters for robustness to specification when the central bank does not know exactly the parameters of the monetary policy transmission mechanism.

Timeless-perspective assumes that the inflation jump π_{-20} occurred for example 20 periods before, even if the cost-push shock u_0 occurs now. This is equivalent to consider as the current inflation jump is $\pi_0 = \pi_{20}$, the value of inflation 20 periods ahead. Hence, inflation jump is very small and very close to the long run value of inflation. Timeless-perspective is equivalent to a time-consistent policy with a very low cost α_x of changing the policy rate and with a maximal volatility of the policy instrument. Timeless-

perspective simulation raises two issues. Firstly, it is inconsistent with optimization. The *ad hoc* near-zero jump of inflation $\pi_0 = \pi_{20}$, is optimally consistent with a large volatility of the policy instrument related to near-zero cost of changing the policy instrument ($\alpha_x \rightarrow 0$). Timeless-perspective is usually assumed *without a large volatility of the policy instrument*, corresponding to a *non-negligible* cost of changing the policy instrument α_x . Second, it assumes away tests of real world structural breaks of new commitment. Timeless-perspective simulations explains Volcker's structural break in 1979-1982 by Martin chairman of the Fed during 1951-1970.

When the finite time-horizon for the end of the commitment is short (for example, *the central bank re-optimize every two periods (two-quarters or every two-weeks)*), Ramsey optimal policy is close to a time-consistent path. Finally, in the case where the forward-looking variables are not controllable with stable dynamics, Ramsey optimal policy and time-consistent policy are identical.

3.2 Null Hypothesis of the Pre-Test

Time-consistent policy is described by a permanent anchor of inflation on the output gap and by two AR(1) processes of inflation and of the output gap. Variables, such as inflation ($\pi_t + \pi^*$) are not computed as deviations of equilibrium (already denoted π_t). Estimates of equilibrium values (π^*, x^*) are then sample mean values found in the estimates of intercepts. The reduced form time-consistent policy rule to be tested (which corresponds to a permanent anchor of inflation on the output gap) allows to estimate the reduced form time-consistent policy rule parameter $F_{\pi,TC}$:

$$\begin{aligned} x_t + x^* &= F_{\pi,TC} (\pi_t + \pi^*) + (x^* - F_{\pi,TC}\pi^*) + \varepsilon_{x\pi,t} \text{ with} \\ \varepsilon_{x\pi,t} &= 0 \text{ for all dates, } R^2 = 1 \text{ and } F_{\pi,TC} < 0 \end{aligned} \quad (24)$$

The simple correlation between the output gap and inflation provides another estimate of the time-consistent policy rule parameter $F_{\pi,TC} = r_{x\pi}\sigma_{\varepsilon,x}/\sigma_{\varepsilon,\pi} < 0$. It is equal to the one found using the ratio of standard errors of residuals of the AR(1) estimations for inflation and for the output gap : $F_{\pi,TC} = -\sigma_{\varepsilon,x}/\sigma_{\varepsilon,\pi}$ only if the following condition is satisfied: $r_{x\pi} = -1$. Testing time-consistent policy against Ramsey optimal policy amounts to test the perfect negative correlation between the output gap and inflation: $r_{x\pi} = -1$. Because of test of a simple correlation exactly equal to -1 cannot be performed, we can perform a one-sided test of a composite null hypothesis of a simple correlation very close to minus one (subscript TC is for time-consistent policy):

$$H_{0,TC} : r_{x\pi} < -0.99 \quad (25)$$

However, this non-perfect correlation may be due to measurement errors. Hence, the key test of time-consistent policy versus Ramsey optimal policy is a test of the auto-correlation of residuals of the output gap policy rule function of inflation. In time-consistent policy, errors should not be auto-correlated. If they are auto-correlated, this alternative hypothesis suggests that at least the lagged policy instrument is missing in the regression of the policy rule, which is exactly a reduced form of the *Ramsey optimal policy rule* (which also depends on inflation):

$$H_{0,TC} : \rho_{\varepsilon,x\pi} = 0 \text{ for } \varepsilon_{x\pi,t} = \rho_{\varepsilon,x\pi}\varepsilon_{x\pi,t} + \eta_t \text{ with } \eta_t \text{ i.i.d.} \quad (26)$$

Additionally, the two AR(1) process for inflation and the output gap are:

$$\pi_t + \pi^* = \rho(\pi_{t-1} + \pi^*) + (1 - \rho)\pi^* + N_{TC}\varepsilon_{u,t} \text{ with } \varepsilon_{u,t} \text{ i.i.d.} \quad (27)$$

$$x_t + x^* = \rho(x_{t-1} + x^*) + (1 - \rho)x^* + F_{\pi,TC}N_{TC}\varepsilon_{u,t} \text{ with } \varepsilon_{u,t} \text{ i.i.d.} \quad (28)$$

If the two previous tests did not reject the null hypothesis, we can perform a third test that the auto-correlation coefficients are identical for inflation and for the output gap:

$$H_{0,TC} : \rho_\pi = \rho_x \quad (29)$$

If this hypothesis is not rejected, the AR(1) estimates identify the auto-correlation parameter of the non-observable cost-push shock: ρ . The ratio of the standard errors of residuals of each AR(1) estimations of inflation and output gap provides another estimate of $F_{\pi,TC}$, (if $r_{x\pi} = 1$) with a negative sign restriction predicted by theory, and grounded by positive feedback:

$$F_{\pi,TC} = -\sigma_{\varepsilon,x}/\sigma_{\varepsilon,\pi} \quad (30)$$

The variance $\sigma_{\varepsilon,\pi}^2$ of perturbations of the inflation AR(1) process is:

$$\sigma_{\varepsilon,\pi}^2 = N_{TC}^2 \sigma_{\varepsilon,u}^2 \Rightarrow N_{TC}^2 = \frac{\sigma_{\varepsilon,\pi}^2}{\sigma_{\varepsilon,u}^2} \quad (31)$$

Unfortunately, the cross equations covariance $\sigma_{\varepsilon,\pi x}$ between the residuals of both AR(1) process of inflation and of the output gap does not allow to identify either the private sector parameter N_{TC} anchoring inflation on the cost-push shock or the variance of the cost-push shock $\sigma_{\varepsilon,u}^2$. The simple correlation between the two residuals is predicted to be exactly negatively correlated ($r_{\varepsilon,\pi x} = -1$):

$$\sigma_{\varepsilon,\pi x} = -\frac{\sigma_{\varepsilon,x} \sigma_{\varepsilon,\pi}^2}{\sigma_{\varepsilon,\pi} \sigma_{\varepsilon,u}^2} \sigma_{\varepsilon,u}^2 = -\sigma_{\varepsilon,x} \sigma_{\varepsilon,\pi} < 0. \quad (32)$$

Finally, only one structural parameter related to the cost-push shock ρ and one reduced form parameter $F_{\pi,TC}$ are identified. However, there remain four structural parameters. For exogenous central bank preferences α_x , these parameters are $\kappa, \beta, \alpha_x, \sigma_{\varepsilon,u}^2$. For a welfare loss function, as $\alpha_x = \frac{\kappa}{\eta}$, these parameters are $\kappa, \beta, \eta, \sigma_{\varepsilon,u}^2$ where η is the elasticity of substitution between goods. It is not possible to identify at least one of these four parameters separately, because the identified parameter $F_{\pi,TC}$ does not depend only on one of these four structural parameters:

$$F_{\pi,TC} = \frac{-1}{1 - \beta\rho} \frac{\kappa}{\alpha_x} < 0. \quad (33)$$

Three identifying equations are missing in the case of time-consistent policy instead of one identifying equation in the case of Ramsey optimal policy. In the case of endogenous central bank preferences (welfare loss function case,) it is not possible to identify η the elasticity of substitution between goods. In the case of exogenous central bank preferences, it is not possible to disentangle exogenous central bank preferences α_x from the monetary transmission mechanism parameter κ (the identical discount rate β is identical to central banks and to the private sector). We cannot disentangle whether an estimated impulse response function is obtained by a large cost α_x of changing the policy instrument

and a large marginal effect κ of the policy instrument on the policy target or by a low cost α_x with a large response of the instrument in the policy rule and a low marginal effect κ of the policy instrument on the policy target.

This is unfortunate because the main value added of the estimation of optimal time-consistent policy with respect to the estimation of a reduced form positive feedback simple-rule parameter $F_{\pi,TC}$ is to estimate central bank preferences α_x .

Finally, the tests of reduced form parameters of bivariate VAR(1) of time-consistent policy versus Ramsey optimal policy are not feasible. The exact multicollinearity (exact correlation) between regressors (current output gap and current inflation) imply a bivariate VAR(1) with infinite coefficients with denominator including the term $1 - r_{x\pi}^2$ equal to zero:

$$\begin{pmatrix} x_{t+1} \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} +\infty & -\infty \\ -\infty & +\infty \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} N_{TC} \\ F_{\pi,TC}N_{TC} \end{pmatrix} \varepsilon_t \quad (34)$$

The time-consistent policy equilibrium predicts that out-of-equilibrium behavior corresponds to a non-stationary bivariate VAR including one unstable eigenvalue λ_{TC} and one stable eigenvalue ρ , which cannot be estimated. By contrast, the stationary structural VAR(1) of output gap and inflation with Ramsey optimal policy allows to identify a larger number of structural parameters.

$$\begin{pmatrix} x_{t+1} \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} F_{u,R} \\ 0 \end{pmatrix} \varepsilon_t$$

Two additional reduced form parameters (b, c) are available, because the stable subspace of the VAR process is of dimension two with Ramsey optimal policy instead of dimension one with time-consistent policy.

3.3 Pre-test of Fed's Ramsey versus Time-consistent policy

The annualized quarter-on-quarter rate of inflation and the congressional budget office (CBO) measure of the output gap are taken from Mavroeidis' (2010) online appendix (detailed information at the end of this paper's appendix). The pre-Volcker sample covers the period 1960q1 to 1979q2 and the Volcker-Greenspan sample runs until 2006q2. The period of Paul Volcker's tenure is 1979q3 to 1987q2. The period of Alan Greenspan's tenure is 1987q3 to 2006q1.

According to Gali (2015, chapter 5), a structural break is related either to the beginning of a permanent anchor of forward inflation on output gap in the case of time-consistent policy or to an initial anchor (jump) of forward inflation on output gap in the case of Ramsey optimal policy. Clarida, Gali and Gertler (2000) and Mavroeidis (2010) consider the beginning of Paul Volcker's mandate 1979q3 as a structural break. Givens (2012) considers 1982q1 as a structural break, after 1981 fall of inflation and before the 1982 recession. Matthes (2015) estimation of the private sectors beliefs regarding central bank regimes also points to 1982q1 as a structural break. Table 2 presents summary statistics before and after the 1979q3 and 1982q1 structural breaks.

Table 2: Summary statistics of inflation and output gap for 1979q3 and 1982q1 breaks.

Break	obs.	var.	mean	min	max
before79	78	π_t	4.39 (2.71)	0.59	11.79
before79	78	x_t	0.47 (2.59)	-4.97	6.10
before82	88	π_t	4.86 (2.90)	0.59	11.79
before82	88	x_t	0.20 (2.59)	-4.97	6.10
after79	108	π_t	3.18 (2.03)	0.64	10.93
after79	108	x_t	-1.11 (2.07)	-7.95	3.01
after82	98	π_t	2.64 (1.08)	0.64	5.61
after82	98	x_t	-1.03 (2.12)	-7.95	3.01

Standard deviations in parentheses.

The mean of inflation and of output gap are lower during Volcker-Greenspan than before Volcker. Excluding the period 1979q3 to 1981q4, in particular the sharp disinflation which occurred during 1981 (figure 3), the standard error of inflation decreases by half from 2.03 to 1.08 .

Table 3: Pre-tests of inflation and output gap permanent anchor correlation for 1979q3 and 1982q1 breaks.

Break	obs	$r_{x\pi}$	Low 95% r	p	$R_{x\pi}^2$	$F_{\pi,TC}$	c	$\rho_{\varepsilon,x\pi}$
before79	78	-0.13	-0.21	< 0.001	0.02	-0.13 (0.11)	1.03 (0.56)	0.92 (0.04)
before82	108	-0.30	-0.42	< 0.001	0.09	-0.30 (0.09)	-0.14 (0.35)	0.91 (0.04)
after79	88	-0.24	-0.31	< 0.001	0.06	-0.22 (0.09)	1.25 (0.53)	0.92 (0.05)
after82	98	-0.40	-0.53	< 0.001	0.16	-0.78 (0.18)	1.03 (0.52)	0.89 (0.05)

The pre-tests of the null hypothesis of a quasi perfect negative correlation $H_0 : r_{x\pi} < -0.99$ between observed inflation and observed output gap are rejected. The test uses Fisher's Z transformation using the procedure corr with the software SAS. The threshold of the composite null hypothesis -0.99 is far away from the 95% single tail confidence interval, where the lowest 95% confidence limit reported in table 3 is at most equal to -0.53 for the period after 1981q4 (figure 4). The opposite null hypothesis $H_0 : r(x_t, \pi_t) = 0$ is not rejected before 1979q3. time-consistent policy predicts a perfect correlation for the anchor of inflation expectations with the output gap. If the time-consistent policy equilibrium occurred before 1979q3, we do not reject the null hypothesis $H_0 : r(E_{t-1}(\pi_t), \pi_t) = 0$ that the rational expectations of inflation are orthogonal to observed inflation.

The pre-tests of the null hypothesis of the auto-correlation of residuals $H_0 : \rho_{\varepsilon,x\pi} = 0$ are strongly rejected, with a point estimate at least equal to 0.89 (figure 5). These tests gives a hint of model misspecification. They suggest an omitted lagged policy instrument in the policy rule. When it is included in Ramsey optimal policy rule, the R^2 increases from 16% (table 3, last line) to 93% (table 6, last line) beginning in 1982q1.

Figure 3. Inflation and output gap anchor correlation (insert).

Table 4 investigates the auto-correlation and unit roots of inflation and output gap.

Table 4: Auto-correlation and AR(1) statistics for 1979q3 and 1982q1 breaks.

Break	obs.	var.	r	R^2	ρ	c	σ_ε	ρ_ε	DF	PP
before79	78	π_t	0.86	0.74	0.88 (0.06)	0.62 (0.30)	1.38	-0.22 (0.12)	0.55	0.41
before79	78	x_t	0.93	0.86	0.93 (0.04)	0.03 (0.11)	0.99	0.26 (0.11)	0.20	0.33
before82	88	π_t	0.88	0.78	0.88 (0.05)	0.63 (0.28)	1.35	-0.19 (0.11)	0.34	0.23
before82	88	x_t	0.92	0.85	0.93 (0.04)	-0.03 (0.11)	1.03	0.23 (0.11)	0.27	0.37
after79	108	π_t	0.89	0.79	0.85 (0.04)	0.42 (0.16)	0.93	-0.27 (0.09)	0.04	0.01
after79	108	x_t	0.94	0.88	0.94 (0.03)	-0.07 (0.08)	0.70	0.34 (0.09)	0.11	0.25
after82	98	π_t	0.64	0.41	0.59 (0.07)	1.06 (0.21)	0.83	-0.20 (0.10)	0.00	0.00
after82	98	x_t	0.96	0.92	0.95 (0.03)	-0.02 (0.07)	0.60	0.35 (0.09)	0.07	0.35

The output gap and inflation are highly auto-correlated (respectively 0.93 and 0.86), except when inflation excludes the 1981 disinflation for the period after 1981q4. For the period 1982q1 to 2006q2, the inflation auto-correlation coefficient falls in the 95% confidence interval 0.6 ± 0.14 and it is statistically different from the output gap auto-correlation coefficient in the 95% confidence interval 0.95 ± 0.06 (figures 4 and 5). As the time-consistent policy equilibrium predicts that the auto-correlation of the output gap and of inflation should be the same, this is an additional test against time-consistent policy, which holds for the period 1982q1 to 2006q2.

There is a negative auto-correlation of residuals ρ_ε for inflation and a (statistically significant at the 5% level) positive auto-correlation of residuals for the output gap. The column DF reports the p-value of the Dickey-Fuller test of unit root with one lag without trend. The column PP reports the p-value of the Phillips-Perron test of unit root, which takes into account auto-correlation, with one lag without trend. The null hypothesis of a unit root is rejected for inflation after 1979q2 and after 1981q4.

4 A New Test of Fed's Ramsey Optimal Policy

Table 5 presents estimates of structural parameters, with small changes for estimates with two given values for the discount factor $\beta = 1$ or $\beta = 0.99$ for the Volcker-Greenspan period. Estimates are found using three structural VAR estimations for $(\rho, \lambda_R, F_{\pi,R})$, $(\rho, \lambda_R, \frac{\alpha_x}{\kappa})$ and $(\rho, \kappa(\beta), \alpha_x(\beta))$.

The goodness of fit indicators (coefficients of determination R^2) of the Ramsey optimal policy structural VAR(1) are far larger than for time-consistent policy rules. However, the cost-push shock faces is extremely persistent, close to a unit root, with ρ estimate close to one. The new-Keynesian Phillips curve parameter κ is not significantly different from zero at the 5% level, as in many estimates of Mavroeidis, Plagbord-Möller, Stock (2014) limited-information single-equation estimations. The Fed's preference parameter α_x is not significantly different from zero at the 5% level, as well as the reduced form rule parameter $F_{\pi,R}$. However, the ratio $\frac{\alpha_x}{\kappa}$ is statistically significant when starting the estimation in 1979q3. We check another property which has never been done in previous estimations concerning Ramsey optimal initial anchor of inflation. Ramsey optimal initial anchor $\frac{-\pi_0}{x_0} = -5$ is not in the confidence interval of $\frac{\alpha_x}{\kappa}$ for the period after 1979, but it is within the confidence interval of $\frac{\alpha_x}{\kappa}$ for the period after 1982.

Table 5: Tests of Ramsey optimal policy structural parameters for given

discount factor β (excluding periods with conjugate complex roots).

Break	ρ	λ_R	$F_{\pi,R}$	$\frac{\alpha_x}{\kappa}$	$\frac{-\pi_0}{x_0}$	β	$\kappa(\beta)$	$\alpha_x(\beta)$	$F_{u,R}(\beta)$	$\sigma_u(\beta)$
after79	0.995* (0.024)	0.857* (0.054)	0.447 (0.292)	13.375* (6.627)	-5	1	0.321 (0.303)	4.296 (5.447)	-3.027	0.229
after79	0.995* (0.024)	0.857* (0.054)	0.447 (0.292)	13.375* (6.627)	-5	0.99	0.340 (0.314)	4.552 (5.703)	-2.861	0.242
after82	1.011* (0.015)	0.560* (0.079)	0.189 (0.121)	6.729 (3.932)	0.55	1	2.325 (1.710)	15.64 (19.22)	-0.436	1.562
after82	1.011* (0.015)	0.560* (0.079)	0.189 (0.121)	6.729 (3.932)	0.55	0.99	2.354 (1.726)	15.84 (19.44)	-0.431	1.583

We compare two observationally equivalent representations of the reduced form of the policy rule of Ramsey optimal policy for the period after 1979q3, with the rule parameter $F_{u,R}(\beta)$ identified with the identification restriction $\beta = 0.99$. The first one includes the reduced form parameters of the structural VAR(1) of observable variables. The second one is the ADLQR intermediate representation including the non-observable cost-push shock:

$$x_t = 0.995x_{t-1} - 0.064\pi_{t-1} - 2.861\varepsilon_{u,t} \text{ and } x_t = 0.447\pi_t - 2.861u_t \text{ for } \beta = 0.99 \quad (35)$$

Although both representations look different, in particular with the opposite sign of the policy rule parameter related to current or lagged inflation, they are *observationally equivalent within the Hamiltonian system of equations taking into account the stable subspace constraint*.

By contrast, the reduced form policy rule for time-consistent policy would be for the period after 1979q3:

$$x_t = -0.22\pi_t \quad (36)$$

The policy instrument responds to two variables for Ramsey stable subspace of dimension two. The policy instrument responds to one variable in the time-consistent policy stable subspace of dimension one.

Another check never done in the existing literature is to compare the reduced form parameters of Ramsey optimal policy with parameters of an unconstrained VAR (table 6). Goodness of fit R^2 are very close for the structural Ramsey optimal policy VAR and the unconstrained VAR. The inflation equation of the VAR does not change. The second equation of the VAR(1) is a representation of the optimal policy rule. *Ramsey optimal policy over-estimates the persistence of the output gap: its VAR auto-correlation parameter shifts from 0.93 to 1*. There is also a small decline of the effect of lagged inflation on the output gap and with a fall of the constant.

The increase of R^2 of the VAR(1) including cross-correlations with respect to the R^2 obtained with the AR(1) process, denoted ΔR^2 , is at most of 1% for each equations. This is reflected in the impulse response functions which are not statistically significant for cross-correlation even in the most favorable case of after 1981q4 with a parameter -0.17 for the output gap policy rule. In the unconstrained VAR, the policy target (inflation) does not depend on the policy instrument (output gap) in Gali's (2015) Ramsey optimal policy subject to the new-Keynesian Phillips curve according to the t -test after 1979q2 and after 1981q4. As seen in table 8, there is no Granger causality of output gap on inflation in the reduced form VAR(1).

Hand in hand with a small contribution of cross-correlations is so small, the auto-correlation coefficients for each equations of the unconstrained VAR(1) are nearly identical

to the ones of AR(1) equations of inflation and of the output gap.

The remaining auto-correlation of residuals of the unconstrained VAR(1) is not negligible. The p-value of the Lagrange multiplier test of auto-correlation of order 1 of residuals of both equations are for each period in chronological order: 0.02, 0.04, 0.00 and 0.00. One does not reject the auto-correlation of residuals at the 5% level, with very low p-value for the Volcker-Greenspan period. This auto-correlation of residuals test suggests a misspecification of Ramsey optimal policy subject to the new-Keynesian Phillips curve during the Volcker-Greenspan period. Lags of order two and/or including omitted regressors may improve the specification of the unconstrained VAR(1). This suggests a related structural VAR of Ramsey optimal policy subject to *another* transmission mechanism than the new-Keynesian Phillips curve.

Table 6: Inflation and output gap structural VAR(1) reduced form versus unconstrained VAR (U)

Break	obs.	var.	S/U	π_{t-1}	x_{t-1}	c	ρ_ε	R^2	ΔR^2	λ
after79	108	π_t	S	0.85 0.85	0.009 0.001	0.428 (0.16)		0.793		0.857* (0.054)
			U	0.85 (0.04)	0.00 (0.04)	0.43 (0.16)	-0.27 (0.09)	0.79	0.00	0.85
after79	108	x_t	S	-0.064	0.999	0.198 (0.121)		0.888		0.995* (0.024)
			U	-0.084 (0.03)	0.917 (0.03)	0.17 (0.11)	0.29 (0.09)	0.89	0.01	0.92
after82	98	π_t	S	0.56	-0.03	1.086 (0.21)		0.416		0.560* (0.079)
			U	0.56 (0.07)	-0.04 (0.04)	1.09 (0.21)	-0.17 (0.10)	0.42	0.01	0.54
after82	98	x_t	S	-0.084	1.005	0.259 (0.14)		0.919		1.011* (0.015)
			U	-0.17 (0.05)	0.91 (0.03)	0.38 (0.14)	0.30 (0.09)	0.92	0.01	0.933

For the period before Volcker-Greenspan, table 7 presents the unconstrained VAR estimations. There is a fall of the auto-correlation of inflation from 0.857 starting 1979q3 to 0.560 starting 1982q1, which is a major difference of correlations before versus during Volcker-Greenspan. Before Volcker-Greenspan, as the auto-correlation of inflation and of the output gap are close to the same 0.9 values, this opened a possibility for a VAR with complex roots. The imaginary components of the eigenvalues are small. But they contradict the assumption of an exogenous real auto-correlation coefficient for the cost-push shock before 1979q2 and before 1981q4. Hence, before Volcker-Greenspan, one cannot identify structural parameters of Ramsey optimal policy. The estimations does not converge for the structural VAR(1) using non-linear estimation before Volcker-Greenspan period.

Table 7: Inflation and output gap unconstrained VAR(1) before 1979q3 and 1982q1 breaks (periods with conjugate roots).

Break	obs.	var.	π_{t-1}	x_{t-1}	c	ρ_ε	R^2	dR^2	$\lambda, \lambda $
before79	78	π_t	0.89 (0.06)	0.09 (0.06)	0.54 (0.30)	-0.26 (0.11)	0.75	0.01	0.90± 0.08i, 0.91
before79	78	x_t	-0.09 (0.04)	0.91 (0.04)	0.41 (0.21)	0.21 (0.11)	0.87	0.01	0.90± 0.08i, 0.91
before82	88	π_t	0.89 (0.05)	0.06 (0.06)	0.55 (0.29)	-0.22 (0.11)	0.79	0.01	0.90± 0.07i, 0.90
before82	88	x_t	-0.08 (0.04)	0.91 (0.04)	0.39 (0.21)	0.17 (0.11)	0.85	0.01	0.90± 0.07i, 0.90

We finally investigate Granger causality in the unconstrained VAR (table 8). There are larger correlation from lagged inflation to output gap than from lagged output gap to inflation, although those correlation coefficient are far beyond auto-correlation coefficients except after 1981q1 for the auto-correlation of inflation. They are related to the partial correlations of the unconstrained VAR(1). We report the p-values of Granger causality Wald test that the cross-variables coefficient is zero in the two columns GCW in table 8, for comparison with simple cross-correlations. Even though the simple correlation of inflation with lagged output gap increased for periods more and more recent in table 5, their partial effect in the VAR including lagged inflation is small enough to reject Granger causality of the output gap to inflation, in particular for the Volcker-Greenspan period. By contrast, Granger causality of inflation to the output gap is not rejected at the 5% level.

Table 8: Inflation and output gap cross-correlogram for 1979q3 and 1982q1 breaks and Granger Causality.

Break	obs.	GCW	$r(\pi_{t-1}, x_t)$	$r_{\pi_t x_t}$	$r(\pi_{t+1}, x_t)$	GCW
before79	78	0.03	-0.21	-0.13	-0.03	0.15
before82	88	0.02	-0.31	-0.24	-0.15	0.29
after79	108	0.01	-0.33	-0.30	-0.24	0.99
after82	98	0.00	-0.47	-0.40	-0.33	0.34

5 Robustness to Misspecification: Ramsey vs Time-Consistent

There is a lot of *uncertainty on the estimates* of the slope κ of the new-Keynesian Phillips curve in this paper and in limited-information single-equation estimations, which is often found *not to be statistically different from zero* in Mavroeidis *et al.* (2014). There are also *measurement errors* of the output gap and of inflation. First, gross domestic product (GDP) implicit price deflator may differ from various measures of core inflation which is the target of central banks. Subtracting the changes in the quality of goods in the increase of prices is not done with a perfect accuracy. National accountants may change the scope of GDP over time, which also changes the price deflator. For example, they included investment in intangible assets after 2006. The current measure of output does not correspond to the final measure after revisions by national accountants up to three years after. Output gap can be computed using different methods leading to distinct values, in particular when there is uncertainty on structural breaks on the trend of productivity growth, as in 1973 and in 2007.

Time-consistent policy and optimal simple rules assume a perfect knowledge of the monetary policy transmission parameters by the central bank. This raises the question of their robustness to misspecification. Figure 7 (respectively figure 8) represents impulse response functions of output gap and inflation after a small cost-push shock $u_0 = 1\%$ for time-consistent policy (respectively Ramsey optimal policy) using Ramsey optimal policy estimates of Volcker-Greenspan 1979q3-2006q2 period ($\alpha_x = 4.55$, $\beta = 0.99$, $\kappa = 0.340$ and $\rho = 0.995$). Figures 7 and 8 plots two out-of-equilibrium impulse responses for an evil-agent $x_0 \pm 0.1\%$ taking into account a small measurement error of the initial date output gap. In the case of time-consistent policy for a measurement error of the initial output gap $x_0 - 0.1\%$, the evil-agent out-of-equilibrium time-consistent policy path leads to depression in a year coupled to hyperinflation in two years (figure 7). For a

measurement error of the initial output gap $x_0 + 0.1\%$, the evil-agent out-of-equilibrium time-consistent policy path leads to boom in four quarters coupled with deflation in five quarters. On figure 8, evil-agent out-of-equilibrium Ramsey optimal policy inflation path are converging with the optimal path in three years, with near-zero inflation in five years. Because the policy instrument linearly responds to the near-unit-root cost-push shock, the convergence towards equilibrium of out-of-equilibrium and equilibrium paths is very long. These figures shows that for plausible evil-agent measurement errors of the inflation anchor, Ramsey optimal policy is a pre-condition for robust-control optimal policy (Hansen and Sargent (2008)), unless the behavior of the evil-agent is strictly restricted to fool the central bank in the smaller stable subspace of time-consistent policy (Giordani and Söderlind (2004)).

6 Conclusion

This paper proposes new pre-tests and test of Ramsey optimal policy versus time-consistent policy and (optimal) simple rules. It applied this new estimation methods using the reference new-Keynesian Phillips curve with an auto-regressive cost-push shock as the transmission mechanism of monetary policy.

The pre-test rejects time-consistent policy on US data for the period 1960-2006. The number of identification restrictions required for time-consistent policy is three instead of one for Ramsey optimal policy. The test of Ramsey optimal policy does not find statistical significance of the slope of the new-Keynesian Phillips curve and hence of the Fed's preference parameter.

This paper suggests further research estimating Ramsey optimal policy and time-consistent policy using *alternative* transmission mechanism than the new-Keynesian Phillips curve with an auto-regressive cost-push shock. Because of the very weak partial correlation between lagged output gap and inflation, adding many other equations while keeping the new-Keynesian Phillips curve with an auto-regressive cost-push shock into the monetary policy transmission mechanism does not appear to be promising to fit US data since the 60's.

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6.1 Appendix 1: Augmented Discounted Linear Quadratic Regulator

The new-Keynesian Phillips curve can be written as a function of the Lagrange multiplier:

$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa \frac{\kappa}{\alpha_x} \gamma_{t+1} + u_{\pi,t} \text{ where } \kappa > 0, 0 < \beta < 1$$

It can be written:

$$E_t [\pi_{t+1}] + \frac{\kappa^2}{\beta \alpha_x} \gamma_{t+1} = \frac{1}{\beta} \pi_t - \frac{1}{\beta} u_{\pi,t} \text{ where } \kappa > 0, 0 < \beta < 1$$

The solution of the Hamiltonian system are based on the demonstrations of the augmented discounted linear quadratic regulator in Anderson, Hansen, McGrattan and Sargent [1996]:

$$\mathbf{L}^a \begin{pmatrix} \pi_{t+1} \\ \gamma_{t+1} \\ u_{t+1} \end{pmatrix} = \mathbf{N}^a \begin{pmatrix} \pi_t \\ \gamma_t \\ u_t \end{pmatrix}$$

where

$$\mathbf{L}^a = \begin{pmatrix} 1 & \frac{\kappa^2}{\beta\alpha_x} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \mathbf{N}^a = \begin{pmatrix} \frac{1}{\beta} & 0 & \frac{-1}{\beta} \\ -1 & 1 & 0 \\ 0 & 0 & \rho \end{pmatrix}$$

As \mathbf{L}^a is non singular:

$$(\mathbf{L}^a)^{-1} \mathbf{N}^a = \mathbf{M}^a = \begin{pmatrix} \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} & -\frac{\kappa^2}{\beta\alpha_x} & -\frac{1}{\beta} \\ -1 & 1 & 0 \\ 0 & 0 & \rho \end{pmatrix} = \begin{pmatrix} \frac{1}{\beta a} - 1 & 1 + \frac{1}{\beta} - \frac{1}{\beta a} & -\frac{1}{\beta} \\ -1 & 1 & 0 \\ 0 & 0 & \rho \end{pmatrix}$$

where Gali (2015) denotes $a = a(\beta, \kappa, \alpha_x) = \frac{\alpha_x}{\alpha_x(1+\beta)+\kappa^2} = \frac{1}{1+\beta+\frac{\kappa^2}{\alpha_x}}$. The characteristic polynomial of matrix \mathbf{M}^a :

$$(X - \rho) \left(X^2 - \frac{1}{\beta a} X + \frac{1}{\beta} \right) = 0$$

Matrix \mathbf{M}^a has two stable roots with bounded discounted quadratic loss function (below $\sqrt{\frac{1}{\beta}}$): ρ and $\lambda_R = \frac{1-\sqrt{1-4\beta a^2}}{2\beta a}$ (λ_R is denoted δ in Gali (2015)) and one unstable root $\lambda_U = \frac{1+\sqrt{1-4\beta a^2}}{2\beta a}$ because the determinant of the matrix \mathbf{M}^a is $\rho\lambda_R\lambda_U = \rho\sqrt{\frac{1}{\beta}}\sqrt{\frac{1}{\beta}}$ and $\lambda_R < \sqrt{\frac{1}{\beta}}$ imply $\lambda_U = \frac{1+\sqrt{1-4\beta a^2}}{2\beta a} = \frac{1}{\beta\lambda_R} > \sqrt{\frac{1}{\beta}}$.

$$\lambda_R(\beta, \kappa, \alpha_x) = \frac{1}{2} \left(1 + \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} - \sqrt{\left(1 + \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} \right)^2 - \frac{4}{\beta}} \right)$$

$$\frac{\partial\lambda_R}{\partial\alpha_x} > 0, \lim_{\alpha_x \rightarrow 0} \lambda_R = 0 \text{ and } \lim_{\alpha_x \rightarrow +\infty} \lambda_R = 1 < \frac{1}{\sqrt{\beta}}$$

Identification of $\frac{\kappa}{\alpha_x}$: The ratio $\frac{\kappa}{\alpha_x}$ is identified using the following two equalities defining the inflation rule parameter $F_{\pi,R}$, which are found for the characteristic polynomial equal to zero:

$$F_{\pi,R} = \frac{1 - \beta\lambda_R}{\kappa} = \left(\frac{\lambda_R}{1 - \lambda_R} \right) \frac{\kappa}{\alpha_x} \Rightarrow$$

$$0 = \beta\lambda_R - \left(1 + \beta + \frac{\kappa^2}{\alpha_x} \right) \lambda_R + 1$$

Positive sign restriction of $F_{\pi,R}$: The eigenvalue λ_R is a linear decreasing function of the inflation rule parameter $F_{\pi,R}$. It varies between zero (for the relative cost of changing the interest rate tending to zero: $\alpha_x \rightarrow 0$) and the inverse $\frac{1}{\beta}$ of the laissez-faire eigenvalue $\frac{1}{\beta}$ (for the relative cost of changing the interest rate tending to infinity: $\alpha_x \rightarrow +\infty$). This sets boundaries restrictions of the inflation rule parameter $F_{\pi,R}$, which is strictly positive (see appendix):

$$F_{\pi,R} = \frac{1}{\kappa} - \frac{\beta}{\kappa} \lambda_R = \left(\frac{\lambda_R}{1 - \lambda_R} \right) \frac{\kappa}{\alpha_x} \in \left] \frac{1 - \beta^2}{\kappa}, \frac{1}{\kappa} \right[. \quad (37)$$

Ricatti equation solution: P_π is the slope of eigenvectors of the stable eigenvalue λ_R of the matrix \mathbf{H} of the Hamiltonian system when $u_0 = 0 = u_t$

$$\begin{aligned} \begin{pmatrix} \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} & -\frac{\kappa^2}{\beta\alpha_x} \\ -1 & 1 \end{pmatrix} &= \begin{pmatrix} 1 & 1 \\ P_\pi & P_U \end{pmatrix} \begin{pmatrix} \lambda_R & 0 \\ 0 & \lambda_U \end{pmatrix} \begin{pmatrix} 1 & 1 \\ P_\pi & P_U \end{pmatrix}^{-1} \\ \begin{pmatrix} \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} & -\frac{\kappa^2}{\beta\alpha_x} \\ -1 & 1 \end{pmatrix} &= \begin{pmatrix} 1 & 1 \\ \frac{1}{1-\lambda_R} & \frac{1}{1-\frac{1}{\beta\lambda_R}} \end{pmatrix} \begin{pmatrix} \lambda_R & 0 \\ 0 & \frac{1}{\beta\lambda_R} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ \frac{1}{1-\lambda_R} & \frac{1}{1-\frac{1}{\beta\lambda_R}} \end{pmatrix}^{-1} \end{aligned}$$

The stable eigenvalue λ_R is the stable solution of the characteristic polynomial of the hamiltonian matrix \mathbf{H} :

$$\lambda_R = \frac{1}{2} \left(\frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} + 1 - \sqrt{\left(\frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} + 1 \right)^2 - \frac{4}{\beta}} \right)$$

The slope P_π of eigenvectors of the stable eigenvalue λ_R is given by:

$$\begin{aligned} P_\pi &= \frac{\lambda_R - a_{11}}{a_{12}} = \frac{a_{21}}{\lambda_R - a_{22}} = \frac{1}{1 - \lambda_R} \in [1, +\infty[\text{ where } \begin{pmatrix} a_{11} = \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} & a_{12} = -\frac{\kappa^2}{\beta\alpha_x} \neq 0 \\ a_{21} = -1 & a_{22} = 1 \end{pmatrix} \text{ or} \\ P_\pi &= \frac{1}{2} \frac{\beta\alpha_x}{\kappa^2} \left(\frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} - 1 + \sqrt{\left(\frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} + 1 \right)^2 - \frac{4}{\beta}} \right) \text{ or} \\ P_\pi &= \frac{1}{2} \left(\frac{\alpha_x}{\kappa^2} + 1 - \frac{\beta\alpha_x}{\kappa^2} + \sqrt{\left(\frac{\alpha_x}{\kappa^2} + 1 - \frac{\beta\alpha_x}{\kappa^2} \right)^2 + 4\beta \frac{\alpha_x}{\kappa^2}} \right) \end{aligned}$$

P_π is also the positive solution of a scalar Ricatti equation (demonstration using undetermined coefficients in proposition 1):

$$\begin{aligned} \frac{P_\pi - 1}{P_\pi} &= \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} - \frac{\kappa^2}{\beta\alpha_x} P_\pi \\ 0 &= -\frac{\kappa^2}{\beta\alpha_x} P_\pi^2 + \left(\frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} - 1 \right) P_\pi + 1 \\ 0 &= P_\pi^2 - \left(\frac{\alpha_x}{\kappa^2} + 1 - \frac{\beta\alpha_x}{\kappa^2} \right) P_\pi - \frac{\beta\alpha_x}{\kappa^2} \end{aligned}$$

Proposition 1: Rule parameters F_u and P_u of the cost-push shock u_t satisfy:

$$\frac{P_u}{P_\pi} = \frac{-\lambda_R}{1 - \lambda_R \rho \beta} = \frac{-\frac{1}{\beta}}{\frac{1}{\beta \lambda_R} - \rho} \quad \text{and} \quad \frac{F_u}{F_\pi} = -1 + \beta \rho \frac{P_u}{P_\pi} = \frac{-1}{1 - \lambda_R \rho \beta} = \frac{1}{\lambda_R} \frac{P_u}{P_\pi} \quad (38)$$

$$P_u = \frac{-\lambda_R}{1 - \lambda_R \rho \beta} \frac{1}{1 - \lambda_R} \quad \text{and} \quad F_u = \frac{-1}{1 - \lambda_R \rho \beta} \left(\frac{1}{\kappa} - \frac{\beta}{\kappa} \lambda_R \right) = \frac{-1}{1 - \lambda_R \rho \beta} \left(\frac{\lambda_R}{1 - \lambda_R} \right) \frac{\kappa}{\alpha_x} = P_u \frac{\kappa}{\alpha_x} \quad (39)$$

and the optimal initial anchor of inflation on the cost-push shock is:

$$\pi_0 \begin{pmatrix} \lambda_R, \rho \\ +, - \end{pmatrix} = \frac{-P_u}{P_\pi} u_0 = \frac{\frac{1}{\beta}}{\frac{1}{\beta \lambda_R} - \rho} u_0 = \frac{-\alpha_x}{\kappa} x_0 \quad \text{with} \quad \rho < 1 < \frac{1}{\beta} < \frac{1}{\beta \lambda_R} = \lambda_U \quad (40)$$

Demonstration: It uses the method of undetermined coefficients of Anderson, Hansen, McGrattan and Sargent's (1996), section 5, on Galí's (2015) Ramsey optimal policy. Using the infinite horizon transversality conditions, the solution is the one that stabilizes the state-costate vector for any initialization of inflation π_0 and of the exogenous shock u_0 in a stable subspace of dimension two within a space of dimension three (π_t, γ_t, u_t) of the Hamiltonian system. We seek a characterization of the Lagrange multiplier γ_t of the form:

$$\gamma_t = P_\pi \pi_t + P_u u_t.$$

To deduce the control law associated with matrix (P_π, P_u) , we substitute it into the Hamiltonian system:

$$\mathbf{L}^a \begin{pmatrix} \pi_{t+1} \\ P_\pi \pi_{t+1} + P_u u_{t+1} \\ u_{t+1} \end{pmatrix} = \mathbf{N}^a \begin{pmatrix} \pi_t \\ P_\pi \pi_t + P_u u_t \\ u_t \end{pmatrix}$$

If we write the three equations in this system separately,

$$\begin{aligned} \left(1 + \frac{\kappa^2}{\beta \alpha_x} P_\pi \right) \pi_{t+1} + \frac{\kappa^2}{\beta \alpha_x} P_u u_{t+1} &= \frac{1}{\beta} \pi_t - \frac{1}{\beta} u_t \\ P_\pi \pi_{t+1} + P_u u_{t+1} &= (P_\pi - 1) \pi_t + P_u u_t \\ u_{t+1} &= \rho u_t \end{aligned}$$

Substitute the last equation into the first and solve for π_{t+1} :

$$\pi_{t+1} = \left(1 + \frac{\kappa^2}{\beta \alpha_x} P_\pi \right)^{-1} \left(\frac{1}{\beta} \pi_t + \left(-\frac{1}{\beta} - \frac{\kappa^2}{\beta \alpha_x} P_u \rho \right) u_t \right)$$

It is straightforward to verify that:

$$\frac{1}{1 + \frac{\kappa^2}{\beta \alpha_x} P_\pi} = 1 - \frac{\frac{\kappa^2}{\beta \alpha_x} P_\pi}{1 + \frac{\kappa^2}{\beta \alpha_x} P_\pi} = 1 - \frac{\frac{\kappa^2}{\beta} P_\pi}{\alpha_x + \frac{\kappa^2}{\beta} P_\pi}$$

The policy instrument evolves in the stable subspace of the Hamiltonian. We seek a

characterization of the policy rule of the form:

$$x_t = F_\pi \pi_t + F_u u_t.$$

The evolution equation of inflation can be rewritten with a feedback rule as:

$$\pi_{t+1} = \left(\frac{1}{\beta} - \frac{\kappa}{\beta} F_\pi \right) \pi_t + \left(-\frac{1}{\beta} - \frac{\kappa}{\beta} F_u \right) u_t$$

where F_π is given by:

$$F_\pi = \frac{\frac{\kappa}{\beta} P_\pi}{\alpha_x + \frac{\kappa^2}{\beta} P_\pi} = \frac{P_\pi}{\beta + \frac{\kappa^2}{\alpha_x} P_\pi} \frac{\kappa}{\alpha_x} = \left(\frac{\lambda_R}{1 - \lambda_R} \right) \frac{\kappa}{\alpha_x} \quad (41)$$

where F_u is given by (demonstration (1) below):

$$\frac{F_u}{F_\pi} = -1 + \beta \rho \frac{P_u}{P_\pi}$$

where $\frac{P_u}{P_\pi}$ is given by (demonstration (2) below):

$$\frac{P_u}{P_\pi} = \frac{-\lambda_R}{1 - \lambda_R \rho \beta}$$

so that F_u is given by:

$$\frac{F_u}{F_\pi} = -1 - \frac{\beta \rho \lambda_R}{1 - \beta \rho \lambda_R} = \frac{-1}{1 - \lambda_R \rho \beta} = \frac{1}{\lambda_R} \frac{P_u}{P_\pi}$$

Demonstration (1) is:

$$\begin{aligned} & \left(1 - \frac{\frac{\kappa^2}{\beta} P_\pi}{\alpha_x + \frac{\kappa^2}{\beta} P_\pi} \right) \left(-\frac{1}{\beta} - \frac{\kappa^2}{\beta \alpha_x} P_u \rho \right) \\ &= -\frac{1}{\beta} - \frac{\kappa^2}{\beta \alpha_x} P_u \rho - \frac{\frac{\kappa^2}{\alpha_x \beta} P_\pi}{1 + \frac{\kappa^2}{\alpha_x \beta} P_\pi} \left(-\frac{1}{\beta} - \frac{\kappa^2}{\beta \alpha_x} P_u \rho \right) \\ &= -\frac{1}{\beta} + \frac{-\frac{\kappa^2}{\beta \alpha_x} P_u \rho + \frac{\kappa^2}{\alpha_x \beta} P_\pi \frac{1}{\beta}}{1 + \frac{\kappa^2}{\alpha_x \beta} P_\pi} = -\frac{1}{\beta} - \frac{\kappa}{\beta} \frac{\frac{\kappa}{\alpha_x} P_u \rho - \frac{\kappa}{\alpha_x \beta} P_\pi}{1 + \frac{\kappa^2}{\alpha_x \beta} P_\pi} \Rightarrow \\ F_u &= \frac{-\frac{\kappa}{\alpha_x \beta} P_\pi + \frac{\kappa}{\alpha_x} P_u \rho}{1 + \frac{\kappa^2}{\alpha_x \beta} P_\pi} = \frac{\frac{\kappa}{\alpha_x \beta} P_\pi}{1 + \frac{\kappa^2}{\alpha_x \beta} P_\pi} \left(-1 + \beta \rho \frac{P_u}{P_\pi} \right) \\ \frac{F_u}{F_\pi} &= -1 + \beta \rho \frac{P_u}{P_\pi}. \end{aligned}$$

For demonstration (2), substitute the auto-regressive equation of the forcing variable u_t into the law of motion of the Lagrange multiplier remaining in stable subspace and solve for $P_\pi \pi_{t+1}$:

$$\begin{aligned} P_\pi \pi_{t+1} + P_u u_{t+1} &= (P_\pi - 1) \pi_t + P_u u_t \\ P_\pi \pi_{t+1} &= (P_\pi - 1) \pi_t + (P_u - \rho P_u) u_t \end{aligned}$$

The coefficient on u_t is $P_u - \rho P_u$. To obtain an alternative formula for this coefficient, premultiply the evolution equation for inflation including the feedback rule by $\frac{1}{\beta}P_\pi$:

$$\frac{1}{\beta}P_\pi\pi_{t+1} = \frac{1}{\beta}P_\pi\left(\frac{1}{\beta} - \frac{\kappa}{\beta}F_\pi\right)\pi_t + \frac{1}{\beta}P_\pi\left(-\frac{1}{\beta} - \frac{\kappa}{\beta}F_u\right)u_t$$

Using both formulas of the feedback rule, we rewrite the coefficient on u_t . First:

$$\begin{aligned} & \left(\frac{1}{\beta} - \frac{\kappa}{\beta}F_\pi\right)\left(P_\pi\frac{-1}{\beta} + P_u\rho\right) \\ &= \frac{1}{\beta}P_\pi\frac{-1}{\beta} + \frac{1}{\beta}P_u\rho - \frac{\kappa}{\beta}\frac{\frac{\kappa}{\alpha_x\beta}P_\pi}{1 + \frac{\kappa^2}{\alpha_x\beta}P_\pi}\left(P_\pi\frac{-1}{\beta} + P_u\rho\right) \\ &= \frac{1}{\beta}P_\pi\left(\frac{-1}{\beta} - \frac{\kappa}{\beta}\frac{\frac{\kappa}{\alpha_x\beta}(-P_\pi + P_u\beta\rho)}{1 + \frac{\kappa^2}{\alpha_x\beta}P_\pi}\right) + \frac{1}{\beta}P_u\rho \end{aligned}$$

Hence:

$$\frac{1}{\beta}P_\pi\left(-\frac{1}{\beta} - \frac{\kappa}{\beta}F_u\right) = \left(\frac{1 - \kappa F_\pi}{\beta}\right)\left(P_\pi\frac{-1}{\beta} + P_u\rho\right) - \frac{1}{\beta}P_u\rho$$

That is:

$$-\frac{1}{\beta} - \frac{\kappa}{\beta}F_u = \lambda_R\frac{\beta}{P_\pi}\left(P_\pi\frac{-1}{\beta} + P_u\rho\right) - \frac{\beta}{P_\pi}\frac{P_u}{\beta}\rho$$

That is:

$$\begin{aligned} -\frac{1}{\beta} - \frac{\kappa}{\beta}F_u &= \lambda_R\left(-1 + \beta\rho\frac{P_u}{P_\pi}\right) - \frac{P_u}{P_\pi}\rho = \lambda_R\frac{F_{u,R}}{F_{\pi,R}} - \lambda_R\frac{F_{u,R}}{F_{\pi,R}}\rho \\ -\frac{1}{\beta} - \frac{\kappa}{\beta}F_u &= (1 - \rho)\lambda_R\frac{F_{u,R}}{F_{\pi,R}} \end{aligned}$$

Equating coefficients on u_t in the two equations results in a scalar Sylvester equation:

$$\begin{aligned} P_u - P_u\rho &= \left(\frac{1 - \kappa F_\pi}{\beta}\right)(-P_\pi + P_u\beta\rho) - P_u\rho \\ P_u &= \lambda_R(-P_\pi + P_u\beta\rho) \\ P_u &= \frac{-\lambda_R P_\pi}{1 - \lambda_R\rho\beta} \implies \frac{P_u}{P_\pi} = \frac{-\lambda_R}{1 - \beta\rho\lambda_R} \end{aligned}$$

Hence:

$$\frac{F_{u,R}}{F_{\pi,R}} = -1 + \beta\rho\frac{P_u}{P_\pi} = -1 + \beta\rho\left(\frac{-\lambda_R}{1 - \lambda_R\rho\beta}\right) = \frac{-1}{1 - \beta\rho\lambda_R}$$

Q.E.D.

6.2 Appendix 2: A representation of the optimal policy rule function of the non-observable AR(1) cost-push shock.

Gali (2015) stationary equilibrium process for the output gap and the cost-push shock, using basis vectors (u_t, x_t) :

$$u_t = \rho u_{t-1} + \varepsilon_{u,t} \quad (42)$$

$$x_t = \lambda_R x_{t-1} - \frac{\lambda_R}{1 - \beta\rho\lambda_R} \frac{\kappa}{\alpha_x} u_t \quad (43)$$

corresponds to a change of basis vectors (u_t, x_t) of the ADLQR representation:

$$\begin{pmatrix} u_t \\ x_t \end{pmatrix} = \mathbf{N}^{-1} \begin{pmatrix} u_t \\ \pi_t \end{pmatrix} \text{ with } \mathbf{N}^{-1} = \begin{pmatrix} 1 & 0 \\ F_{u,R} & F_{\pi,R} \end{pmatrix}$$

implying Gali (2015) observationally and mathematically equivalent third representation of the VAR(1) of Ramsey optimal policy:

$$\begin{cases} \begin{pmatrix} u_{t+1} \\ \pi_{t+1} \end{pmatrix} = (\mathbf{A} + \mathbf{B}\mathbf{F}_C) \begin{pmatrix} u_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \varepsilon_t \\ x_t = F_{\pi,R}\pi_t + F_{u,R}u_t \\ \pi_0 = -\frac{\alpha_x}{\kappa}x_0 \text{ and } u_0 \text{ given} \end{cases} \\ \Leftrightarrow \begin{cases} \begin{pmatrix} u_{t+1} \\ x_{t+1} \end{pmatrix} = \mathbf{N}^{-1}(\mathbf{A} + \mathbf{B}\mathbf{F})\mathbf{N} \begin{pmatrix} u_t \\ x_t \end{pmatrix} + \mathbf{N}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \varepsilon_t \\ \pi_t = \frac{1}{F_{\pi,R}}x_t - \frac{F_{u,R}}{F_{\pi,R}}\pi_t \\ \pi_0 = -\frac{\alpha_x}{\kappa}x_0 \text{ and } u_0 \text{ given} \end{cases}$$

with Gali (2015) representation of the Ramsey optimal policy rule as the second line of the VAR(1). The output gap rule depends on its lagged value and on the lagged value of the cost-push shock u_t :

$$\mathbf{N}^{-1}(\mathbf{A} + \mathbf{B}\mathbf{F})\mathbf{N} = \begin{pmatrix} \rho & 0 \\ -\frac{\lambda_R}{1 - \beta\rho\lambda_R} \frac{\kappa}{\alpha_x} \rho & \lambda_R \end{pmatrix}$$

for $t = 1, 2, 3, \dots$ where the two stable eigenvalues of the stable subspace ρ and λ_R are invariant to changes of basis vectors. This is obtained with intermediate computations:

$$\begin{pmatrix} 1 & 0 \\ AF_{\pi,R} & F_{\pi,R} \end{pmatrix} \begin{pmatrix} \rho & 0 \\ (1 - \rho)A\lambda_R & \lambda_R \end{pmatrix} \begin{pmatrix} 1 & 0 \\ AF_{\pi,R} & F_{\pi,R} \end{pmatrix}^{-1} = \begin{pmatrix} \rho & 0 \\ (1 - \lambda_R)AF_{\pi,R}\rho & \lambda_R \end{pmatrix}$$

where:

$$(1 - \lambda_R)AF_{\pi,R} = (1 - \lambda_R) \frac{-1}{1 - \beta\rho\lambda_R} \left(\frac{\lambda_R}{1 - \lambda_R} \right) \frac{\kappa}{\alpha_x}$$

6.3 Appendix 3: Identification issue for reduced form including a non-observable AR(1) shock.

Because the auto-correlation of the policy instrument x_t and the auto-correlation of the cost-push shock are competing to explain the persistence of the policy instrument x_t , this partial adjustment model with serially correlated shocks has a problem of identification and multiple equilibria (Griliches (1967), Blinder (1986), McManus et al. (1994), Fève, Matheron Poilly (2007)). This VAR(1) can be written as:

$$x_t = \lambda_R x_{t-1} + \eta_t \text{ and } \eta_t = \rho \eta_{t-1} + \varepsilon_{\eta,t}$$

where $\eta_t = -\frac{\kappa}{\alpha_x} \frac{\lambda_R}{(1-\lambda_R \rho \beta)} u_t$. It is an AR(2) model of the policy instrument rule:

$$\begin{aligned} x_t &= \lambda_R x_{t-1} + \rho (x_{t-1} - \lambda_R x_{t-2}) + \varepsilon_{\eta,t} \\ x_t &= b_1 x_{t-1} + b_2 x_{t-2} + \varepsilon_{\eta,t} \text{ with } b_1 = \lambda_R + \rho \text{ and } b_2 = -\lambda_R \rho. \end{aligned}$$

The structural parameter ρ and the semi-structural parameter λ_R are functions of reduced form parameters b_1 and b_2 solutions of:

$$X^2 - b_1 X - b_2 = 0$$

which are given by:

$$\lambda_R = \frac{b_1 \pm \sqrt{b_1^2 + 4b_2}}{2} \text{ and } \rho = \beta b - \lambda_R$$

where $\Delta = b_1^2 + 4b_2 = (\rho - \lambda_R)^2$. If $\Delta \neq 0$ and $\rho \neq \lambda_R$, two sets of values for λ_R and ρ are observationally equivalent. The first solution is such that $\lambda_R > \rho$ and the second solution is such that $\lambda_R < \rho$. The larger Δ , the larger the identification issue, because it increases the gap between a more inertial monetary policy with lower correlation of monetary policy shocks and a less inertial monetary policy, that we cannot distinguish. The ADLQR representation and Galí (2015) representation of the stationary solution of the VAR(1) of optimal policy are not useful to identify parameters, because they include the cost-push shock u_t which is not observable.

The reduced form estimated variance σ_η provides another equation with a theoretical positive sign restriction $\frac{\kappa}{\alpha_x} \frac{\lambda_R}{(1-\lambda_R \rho \beta)} > 0$ for five unknowns structural parameters $(\alpha_x, \kappa, \rho, \beta, \sigma_u)$:

$$\frac{\kappa}{\alpha_x} \frac{\lambda_R}{(1-\lambda_R \rho \beta)} \sigma_u = \sigma_\eta$$

6.4 Appendix 4: Oudiz and Sachs (2015) Time Consistent Discretionary policy

Substituting the private sector's inflation rule (8) and policy rule (9) in the inflation law of motion (1) and comparing it with the forcing variable law of motion (2) leads to the following relation between N_{TC} on date t , $N_{TC,t+1}$ and $F_{u,TC}$:

$$\begin{aligned}
\pi_t &= \beta E_t [\pi_{t+1}] + \kappa x_t + u_t \Rightarrow \\
N_{TC} u_t &= \beta N_{TC,t+1} \rho u_t + \kappa F_{u,TC} u_t + u_t \\
N_{TC} &= \beta \rho N_{TC,t+1} + \kappa F_{u,TC} + 1
\end{aligned}$$

A myopic central bank does not notice that $N_{TC,t+1} = N_{TC}$ (Gali (2015)) in its optimization:

$$\begin{aligned}
N_{TC,Gali} &= \beta \rho N_{TC,t+1} + \kappa F_{u,TC} + 1 \Rightarrow \frac{\partial N_{TC,Gali}}{\partial F_{u,TC}} = \kappa \\
F_{\pi,TC} &= \frac{F_{u,TC}}{N_{TC}} = -\frac{\kappa}{\alpha_x} < 0
\end{aligned}$$

This first order condition of the central bank optimization is substituted into the new-Keynesian Phillips curve equation, where, only at this stage, players of the game discover that it is assumed $N_{TC,t+1} = N_{TC,t} = N_{TC}$. Gali's (2015) solutions are:

$$\begin{aligned}
F_{u,TC,Gali} &= -\frac{\kappa}{\kappa^2 + \alpha_x (1 - \beta \rho)} = -\frac{\kappa}{\alpha_x} N_{TC} \\
N_{TC,Gali} &= \frac{\alpha_x}{\kappa^2 + \alpha_x (1 - \beta \rho)}
\end{aligned}$$

In time-consistent equilibrium (Oudiz and Sachs (1985)), the central bank does fore-see that $N_{TC,t+1} = N_{TC}$ in its optimization, with the following solutions, that we consider for the remaining part of the paper:

$$\begin{aligned}
N_{TC} &= \frac{\kappa F_{u,TC} + 1}{1 - \beta \rho} = \frac{\kappa F_{\pi,TC} N_{TC} + 1}{1 - \beta \rho} \Rightarrow \frac{\partial N_{TC}}{\partial F_{u,TC}} = \frac{\kappa}{1 - \beta \rho} \\
F_{\pi,TC} &= \frac{F_{u,TC}}{N_{TC}} = -\frac{\kappa}{\alpha_x} \frac{1}{1 - \beta \rho} = -\frac{\kappa}{\alpha_x} N_{TC} < 0 \\
F_{u,TC} &= -\frac{\kappa}{\kappa^2 + \alpha_x (1 - \beta \rho)^2} \\
N_{TC} &= \frac{\alpha_x (1 - \beta \rho)}{\kappa^2 + \alpha_x (1 - \beta \rho)^2}
\end{aligned}$$

In Oudiz and Sachs' (1985) general solution, this is the condition after substitutions of the private sector's rule (matrix N_{TC}) and the policy maker's rule (matrix $F_{u,TC}$) for *both* dates t and $t + 1$ into the law of motion of the private sector dynamics:

$$\begin{aligned}
N_{TC,t} &= J - K F_{u,TC} \\
J &= (A_{22} + N_{TC,t+1} A_{12})^{-1} (N_{TC,t+1} A_{11} + A_{21}) \\
K &= (A_{22} + N_{TC,t+1} A_{12})^{-1} (N_{TC,t+1} B_1 + B_2)
\end{aligned}$$

with general notations and equalities with Gali's (2015) transmission mechanism:

$$\begin{pmatrix} u_{t+1} \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} A_{11} = \rho & A_{12} = 0 \\ A_{21} = -\frac{1}{\beta} & A_{22} = \frac{1}{\beta} \end{pmatrix} \begin{pmatrix} u_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} B_1 = 0 \\ B_2 = -\frac{\kappa}{\beta} \end{pmatrix} x_t$$

In Oudiz and Sachs (1985), $N_{TC,t+1} = N_{TC,t}$ at all dates, whereas Gali (2015) assumes myopia (or $N_{TC,t+1} = 0$) for the policy maker. This assumption changes the initial jump of inflation, impulse response functions of inflation and the output gap and welfare. It does not change the identification problem of discretion raised in this paper, because the stable subspace of discretion have the same dimension (one) using the reference Oudiz and Sachs (1985) discretion equilibrium or Gali (2015) and Clarida, Gali, Gertler (1999) myopia assumption.

6.5 Appendix 5: Definition of data variables

Mavroeidis data are running from 1960-Q1 to 2006-Q2.

Inflation is annualized quarter-on-quarter rate of inflation, $400 * \text{LN}(\text{GDPDEF}/\text{GDPDEF}(-1))$ with GDPDEF: Gross Domestic Product Implicit Price Deflator, 2000=100, Seasonally Adjusted. Released in August 2006. Source: U.S. Department of Commerce, Bureau of Economic Analysis.

GAPCBO is the output gap measure: $100 * \text{LN}(\text{GDPC1}/\text{GDPPOT})$ with GDPC1: Real Gross Domestic Product, Billions of Chained 2000 Dollars, Seasonally Adjusted Annual Rate, Released in August 2006. Source: U.S. Department of Commerce, Bureau of Economic Analysis and GDPPOT: Real Potential Gross Domestic Product, Billions of Chained 2000 Dollars. Source: U.S. Congress, Congressional Budget Office.

Figure 3: Time series of inflation and output gap and Volcker's 1979q3 and 1982q1

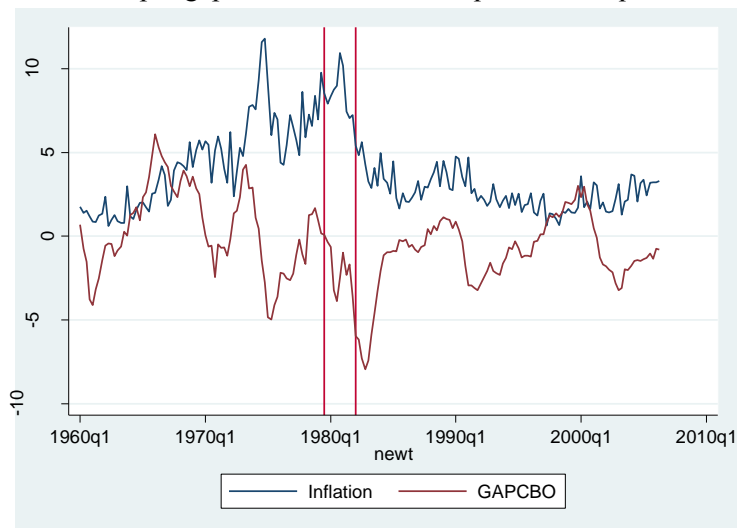


Figure 4: Time-consistent null hypothesis: perfect negative correlation (all dots should be on the regression line) is rejected ($R^2=16\% < 100\%$).

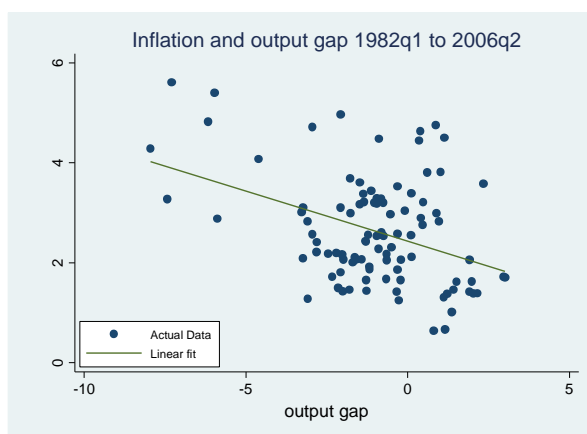


Figure 5: Time-consistent null hypothesis: zero serial correlation of residuals (horizontal regression line) is rejected ($\rho=0.89$).

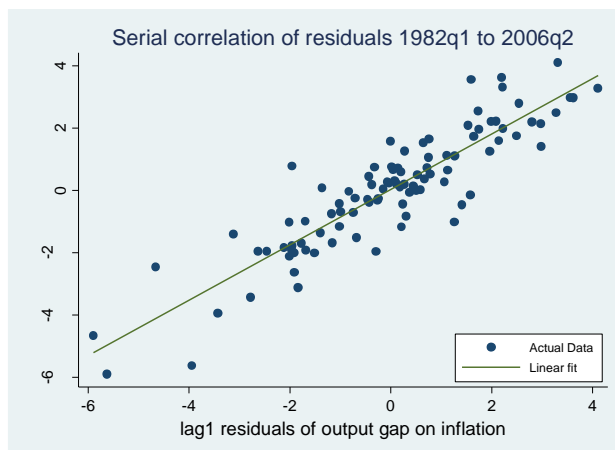
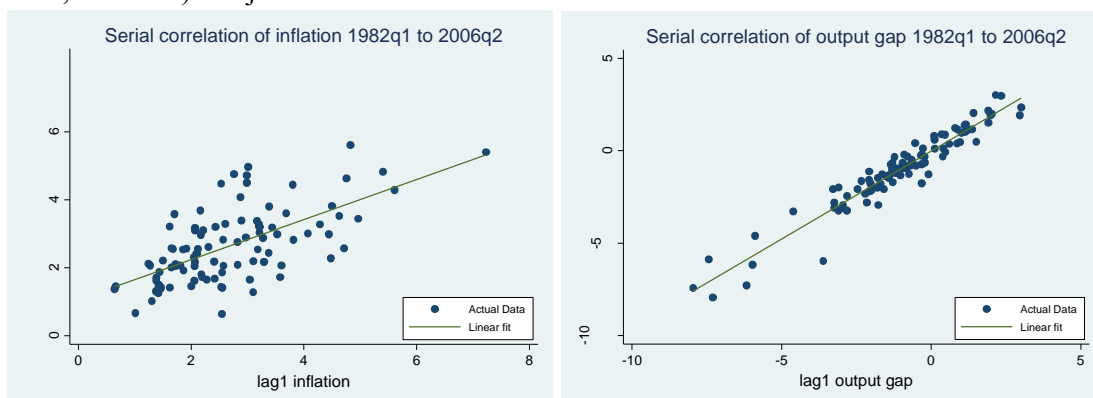


Figure 6 and 7: Time-consistent null hypothesis: identical slopes (auto-correlation) of inflation ($\rho=0.59$, $R^2=41\%$) and of output gap ($\rho=0.95$, $R^2=92\%$) is rejected.



Robustness to misspecification.

Impulse responses of expected (positive) inflation π_t and expected (negative) output gap x_t during two years for time-consistent and during six years for Ramsey optimal policy after a +1% extremely persistent autoregressive ($\rho=0.995$) shock u_t (horizontal axis) using Volcker-Greenspan (1979q3-2006q2) Ramsey estimates, with the cost of changing the policy instrument $\alpha_x=4.55$, $\beta=0.99$, $\kappa=0.34$. Figures also represent evil-agent out-of-equilibrium the impulse responses for initial output gap anchors with small likely measurement errors: $x_0 \pm 0.1\%$.

Figure 8. **Time-consistent policy:** inflation $F_{\pi,D}=-5$, shock: $F_u=0$. Initial inflation $\pi_0=0.6\%$ Initial output gap $x_0=-2.9\%$ +/- 0.1% measurement error.

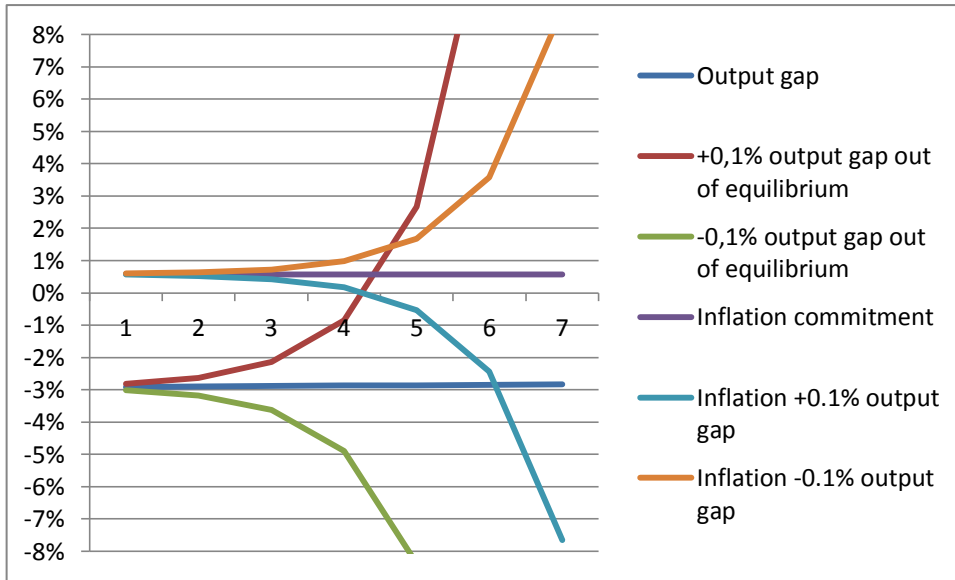


Figure 9. **Ramsey optimal policy.** Rule: inflation $F_{\pi}=-0.45$, shock: $F_u=-2.86$. Initial inflation $\pi_0=5.5\%$. Initial output gap $x_0=-0.4\%$ +/- 0.1%

