

# Endogenous growth with endogenous liquidity\*

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## Abstract

Innovation policies associate more and more programs targeted to financial constraints in addition to incentives fostering R&D spendings. To provide an encompassing framework, this paper develops a schumpeterian growth model with cash flow risk and asymmetric information between managers and investors. While being consistent with evidence on R&D spendings and internal funds at the firm level, and relying on micro-funded financial constraints, the tractability of the model allows for experiments shedding light on growth and liquidity for various industrial contexts. In particular, alleviating hindrances to venture capital seems far more efficient in spurring growth than improving corporate governance or reducing costs of liquidation. In addition, innovation features of an industry, such as entry costs, cash flow volatility or mean returns, can have opposite effects on its overall liquidity; and a positive relation between growth and liquidity occurs only for specific sets of these dimensions.

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\*This document reflects only its author's view, and not the position of the Insee in particular.

## Introduction

Innovation is at the heart of growth, and both the dynamics of firms and their innovation capacity matters. Public policies have been concentrating on fiscal incentives focused on R&D spendings. Yet, innovation also crucially depends on financial constraints. One could wonder whether alleviating these constraints is secondary or dominates direct subsidies in fostering innovation and growth. In France, global public spendings for innovation policy amount to around 0.2 percent of GDP, with three quarters devoted to R&D cost reduction and one quarter to help young firm financially (France Stratégie, 2016).

To provide an encompassing framework, this paper develops a model of schumpeterian growth with financial constraints arising from asymmetric information about cash flows between managers and investors. While including microfunded channels, its tractability allows for a large set of stylized experiments with respect to real and financial structural innovation features of different industries, and implications in terms of growth and liquidity.

More precisely, the model consists in the following elements. Firms are made up by a set of product lines. Successful R&D efforts create a new product which randomly overtakes and destroys an existing one thanks to a marginal productivity improvement. Relatedly, each firm also loses one product line with a probability equal to the creative destruction rate of the economy. Then, production is run on each product line with a linear technology with respect to labor. Cash flows are derived from the sales of this production to a representative consumer with standard preferences for diversity. Cash flows also bear firm level idiosyncratic risks, which can be related to operational unexpected outcomes. Part of these risky cash flows can be diverted by the firm manager for personal benefit, which engenders a specific manager compensation policy by investors. A firm disappears if the manager's limited liability is reached. New firms enter with a single product and initial internal funds optimized by investors.<sup>1</sup> The distribution of firms then evolves along two dimensions: their numbers of products, and their internal funds. Endogenous wage and creative destruction rate adjust in equilibrium.

Under this framework, optimal financial and investment policies at the firm level have the following properties. First, internal funds are accumulated over time to insulate the firm from liquidation losses. Then, each time innovation is successful, the ratio between internal

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<sup>1</sup>In this paper, the terms "internal funds", "liquidity", and "cash holdings" will be used interchangeably.

funds and the number of product lines is maintained. Each time competition implies the destruction of a product line, internal funds per product line are also kept at the same level. With respect to the manager's compensation, there exists a threshold of internal funds below which it is set at zero and above which it corresponds to the difference between the current internal funds and the threshold. Mostly, optimal R&D expenditures rise with internal funds, consistently with large empirical evidence.<sup>2</sup> They also reach values close to first best when internal funds approach the threshold.

Provided firm optimal policies, quantitative experiments are run along various industrial structural characteristics. First, various financial contexts are considered: when information asymmetry is more intense, when liquidation losses are higher, or when initial internal funds injection are hindered. If the model were not microfunded, disentangling these various channels would be infeasible, while they indeed have differing implications. In the first two experiments, initial internal funds adjust and alleviate liquidation risks. In a third experiment, if they can not, firm development and R&D are altered to a large extent, and growth affected by more than 10 percent. So, according to this model, alleviating hindrances to venture capital seems far more efficient in spurring growth than improving corporate governance or reducing costs of liquidation.<sup>3</sup>

Second, various innovation contexts are also addressed: with respect to R&D costs, for incumbents and entrants, and with respect to cash flow mean and volatility. These experiments can be thought as comparisons between industries, or between equilibria of a single industry with or without stylized targeted public intervention. These experiments are common for schumpeterian growth models, but have additional implications from a financial point of view here. Notably, the different technological dimensions generate diverse dynamics for firm financial management and the aggregate internal funds intensity of an industry. In particular, higher entry costs and higher cash flow volatility provoke a stringent reliance on internal funds to alleviate increased liquidation risks. On the contrary, a higher cash flow mean reduces the stock of internal funds required to insulate from liquidation risk. In the end, growth and aggregate liquidity crucially depend on the mix of the many innovation

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<sup>2</sup>See Bates, Kahle, and Stulz (2009), He and Wintoki (2016) or Pinkowitz, Stulz, and Williamson (2016) notably. The model is also consistent with Brown, Fazzari, and Petersen (2009) who find significant effects of cash flow and external equity for young, but not mature, firms.

<sup>3</sup>Brown, Martinsson, and Petersen (2012) for instance find on a large sample of European firms a major role for external equity in financing R&D.

features of a specific industry; and a positive relation only occurs in particular situations.

This paper is first related to the literature on schumpeterian growth. In recent developments, specific market or institutional designs are discussed for their impact on growth, and are confronted to firm-level data.<sup>4</sup> Yet, works following this approach, which can be already complex from a theoretical and econometric point of view, generally do not address corporate financial constraints at the firm level (Acemoglu, Akcigit, Bloom, and Kerr, 2014, for instance develop such a framework to analyze diverse designs of R&D subsidies). The corresponding literature achieves the replication of many stylized facts in terms of firm dynamics and reallocation of resources, as for instance in Klette and Kortum (2004), but not in terms of internal funds outcomes.

This paper is also related to dynamic agency models. They provided strong contributions to analyze corporate financial management under specific moral hazard or information asymmetry designs, notably with continuous-time simplifying methods (Sannikov, 2013). Yet, works relying on dynamic contracts are often partial equilibrium models or general equilibrium approaches generally preferring *ad hoc* financial constraints (Chen, 2014, and Falato, Kadyrzhanova, and Sim, 2013). In the context of endogenous growth, the intensity of the destructive creation in an economy may affect the usual determinants of firm optimal financial management.

Finally, the closest paper to this approach is Malamud and Zucchi (2016), which differs in that their corporate financial frictions arise from an exogenous external cost of funding (as in Bolton, Chen, and Wang, 2011), and that entry, exit and innovation rely on single product firms (following Acemoglu and Cao, 2010). Here, product line innovation *à la* Klette and Kortum (2004) is preferred for its empirical importance, such as depicted in the Community Innovation Surveys<sup>5</sup>, and because of a growing attention to innovative entrepreneurship and industry dynamics in public policy<sup>6</sup>. In addition, dynamic contracts *à la* DeMarzo, Fishman, He, and Wang (2012) allows to keep track of endogenous financial constraints, and

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<sup>4</sup>In particular, product creation and competition is at the core of several analyses of innovation policy though their impact on productivity gains attributable to incumbents or entrants (see Aghion, Akcigit, and Howitt, 2014, for a review).

<sup>5</sup>In France between 2010 and 2012, the share of firms making product innovation is 24 percent, and notably 55 percent for firms with more than 250 employees (Besnard, 2014). These proportions are similar to those for production, marketing or management innovation.

<sup>6</sup>In France in 2015, 1.5 billions of euros are devoted to these objectives, which corresponds to 16.4 percent of total public innovation spendings and to a rise in the diversity of programs (France Stratégie, 2016).

in particular of endogenous internal funds injection when firms are created, while venture capital market functioning is a major dimension of innovation policy debates.<sup>7</sup>

The rest of the paper is organized as follows. Section 1 describes the model. Section 2 derives the equilibrium outcomes and properties. Finally, Section 3 presents simulations with alternative scenarios.

## 1 Model

### 1.1 Production and innovation technology

The model is in continuous time. The current period is denoted by  $t$  and  $dt$  is a small time increment. A general good is produced using combined intermediary goods following:

$$\ln Y_t = \int_0^1 \ln Y_{jt} dj,$$

where  $Y_{jt}$  is the quantity produced by intermediate  $j$ . There is a fixed measure  $l_s$  of individuals who can work in three different activities: as production worker ( $l_0$ ), as R&D scientists in incumbent firms ( $h_i$ ), or as R&D scientists in entrants ( $h_e$ ).

Intermediates are produced monopolistically by the innovator who innovated last within that product line  $j$ , according to the following linear technology:

$$Y_{jt} = A_{jt} l_{0jt},$$

where  $A_{jt}$  is the product line-specific labor productivity and  $l_{0jt}$  is the labor employed for production. This implies that the marginal cost of production in  $j$  is simply  $W_t/A_{jt}$ , where  $W_t$  is the wage rate in the economy at time  $t$ . Each time an innovation is made, productivity increases by a factor  $\gamma > 1$ . Each firm is made up by  $n_{jt}$  product lines.

The final good producer spends the same amount on each variety  $j$ . As a result, the final good production function generates a unit-elastic demand with respect to each variety, with price  $p_{jt}$ :  $Y_{jt} = Y_t/p_{jt}$ . Combined with the fact that firms in a single-product line compete *à la* Bertrand, this implies that a monopolist with marginal cost  $W_t/A_{jt}$  will follow limit pricing by setting its price equal to the marginal cost of the previous innovator:  $p_{jt} = W_t/(A_{jt}/\gamma)$ . The resulting equilibrium quantity and profit in product line  $j$  are:

$$Y_{jt} = \frac{A_{jt} Y_t}{\gamma W_t} \text{ and } \Omega_{jt} = \mu Y_t, \text{ where } \mu \equiv \frac{\gamma - 1}{\gamma}, \quad (1)$$

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<sup>7</sup>In France, there is a general lack of equity, especially for innovative small and medium sized firms, while investments "have to become more "schumpeterian", and relatedly provided funds less guaranteed" (Villeroy de Galhau, 2015).

where  $\Omega_{jt}$  corresponds to profits after optimization of production labor. From now on, values corresponding to product lines will be considered as ratios over GDP, and notably wages:  $w_t = W_t/Y_t$ , and subscript  $j$  will be omitted. Optimal production labor so is:  $l_{0t} = (\gamma w_t)^{-1}$ .

Assume that after sales, the product line is subject to a cash flow shock. Denote by  $d\mu_t$  profits per GDP unit after its realization and for a small interval  $dt$ . The shocks are assumed to affect all product lines of a firm similarly. They are normally distributed with variance  $\sigma$ , such that:

$$d\mu_t \sim \mathcal{N}(\mu dt, \sigma dt). \quad (2)$$

Each firm is made up by  $n_t$  product lines where  $n_t$  is a Poisson process subject to innovation and creative destruction. The firm can invest in R&D in order to obtain another product line with a cost  $H(x, n; \theta)$ , where  $x$  is the per product innovation intensity and  $\theta$  an innovation capacity parameter. The function  $H$  is smooth, convex, and homogeneous of degree one in the number of products  $n$ . In practice, the function will have the following form:

$$H(x, n; \theta) = n\theta^{-\alpha/(1-\alpha)}x^{1/(1-\alpha)}. \quad (3)$$

When a firm is successful in its current R&D investment, it innovates over a random product line  $j' \in [0, 1]$ . Then, as previously mentioned, the productivity in line  $j'$  increases from  $A_{j'}$  to  $\gamma A_{j'}$ . The firm becomes the new monopoly producer in line  $j'$  and increases the number of its production lines to  $n + 1$ . At the same time, each of its  $n$  current product lines is subject to the creative destruction  $\tau$  by entrants and other incumbents, which will be endogenized in equilibrium. Between  $t$  and  $t + dt$ , a product of the firm is destroyed with a probability  $\tau dt$ . A firm that loses all of its product lines exits the economy.

Aggregate cumulated profits over a period  $dt$  are denoted by  $d\Pi_t$ , and are proportional to the product line number:

$$d\Pi_t = n_t[d\mu_t - w_t h(x_t) dt],$$

where  $h(x_t) \equiv \theta^{-\alpha/(1-\alpha)}x_t^{1/(1-\alpha)}$  is the per product line R&D cost, that is the number of R&D scientists paid at the general wage rate  $w_t$ .

## 1.2 Asymmetric information between investors and managers

The manager takes an action  $a_t \in [0, 1]$ . The declared cash flow is only:

$$d\hat{\mu}_t = a_t \mu dt + \sigma dS_t,$$

where  $dS_t$  is a standard normalized Brownian motion corresponding to (2). For each dollar that the manager conceals, the manager can consume  $\lambda \in [0, 1]$  and thus enjoys for  $a_t = 0$  for instance a private benefit  $b_t = \lambda(d\mu_t - d\hat{\mu}_t)$ . Investors have unlimited wealth, are risk-neutral and their discount rate is lower than the managers' one:  $r < \rho$ . The contract with the manager can be terminated at any time, in which case the firm is liquidated and investors recover a value  $l \cdot n_t$ , where  $l > 0$ .<sup>8</sup>

To maximize the value of the firm, investors offer a contract that specifies the firm's investment policy  $x_t$ , the manager's cumulative compensation  $U_t$ , and a termination time  $T$ , all of which depend on the history of the manager's performance, which is given by the profit process  $\Pi_t$ . The agent's limited liability requires the compensation process  $U_t$  to be non-decreasing. Let  $\Phi = (x_t, U_t, T)$  represent the contract. Then, the expected value for the manager is<sup>9</sup>:

$$W(\Phi) = \max_{a_s} \mathbb{E} \left\{ \int_0^T e^{-\rho s} [dU_s + b_s(a_s) ds] \right\}.$$

At the time the contract is initiated, the firm has a single product line. Given an initial payoff  $W_0$  for the manager, the investor's optimization problem is:

$$P_1(W_0) = \max_{\Phi} \mathbb{E} \left\{ \int_0^T e^{-rs} d\Pi_s + e^{-rT} l \cdot n_T - \int_0^T e^{-rs} dU_s \right\},$$

under the constraints that  $\Phi$  is incentive compatible and that  $W(\Phi) = W_0$ .

Entrants produce one unit of innovation by hiring  $\psi$  scientists. When a new entrant is successful, it innovates over a random product line by improving its productivity by  $\gamma > 1$  as incumbents. It then starts out as a single-product firm. The free-entry condition equates the total value of a new entry with the cost of innovation such that:

$$P_1(W_0) + W_0 = w\psi. \tag{4}$$

<sup>8</sup>When there is no agency problem, the intertemporal value of a product line is given by:  $q^{FB} = \max_x [\mu - h(x)] / (r + \delta - x)$ . Parameters are restricted in order to insure the definition of this value:  $\mu < h(r + \delta)$ .

<sup>9</sup>In the rest of the paper, the terms "cash" will also be used for the variable  $W_t$ . To explicit this interpretation, an implementation of the optimal contract with cash is depicted in DeMarzo et al. (2012) in Section IV. For a survey on corporate liquidity management and the role of cash, see Almeida, Campello, Cunha, and Weisbach (2014).

This equation implies that the wage cost is determined by benefits and costs in the arbitrage of entrants. Initial cash is a promised compensation value.<sup>10</sup>

$W_0$  is chosen such that it maximizes the value for investors when starting the firm, thus assuming negotiation power is fully on the side of investors. This model extension is suggested but not directly addressed by DeMarzo et al. (2012) in their partial equilibrium context. There also exist hindrances to initial liquidity injection.<sup>11</sup> It notably takes into account the possibility for a venture capital market to be undersized and not prone to radical uncertainty. Only a part  $1 - \zeta$  of optimal initial cash can be provided to the firm such that:

$$W_0^* = (1 - \zeta) \times \arg \max_{W_0 > 0} P_1(W_0). \quad (5)$$

Finally, the liquidation  $l$  is endogenized. As many industry types will be considered, this hypothesis allows to remain neutral with respect to this financial friction parameter. Notably, if  $l$  were exogenous, the distance between the liquidation value and the first best intertemporal value of the product line might change with respect to  $\mu$  for instance. Here again, the suggestion by DeMarzo et al. (2012) is adopted, and adapted to the current context:

$$l = (1 - \kappa) \times P_1(W_0^*). \quad (6)$$

In any industry, the remaining value of a firm is a fixed proportion of the initial product line value when starting the firm. According to the liquidation equation (6), liquidation is always inefficient, that is  $q^{FB} > l$ , as  $q^{FB} > P_1(W_0^*)$  in any circumstance.

## 2 Equilibrium

### 2.1 Optimal firm policy

To maintain incentive compatibility, compensation must be sufficiently sensitive to the firm's incremental profits. Let  $dJ_t^+$  and  $dJ_t^-$  denote the compensated Poisson processes such that:  $dJ_t^+ = dn_t^+ - x_t dt$  and  $dJ_t^- = dn_t^- - \tau dt$ , where  $n_t = n_t^+ - n_t^-$ , and  $n_t^+$  (resp.  $n_t^-$ ) is the cumulated number of created (destroyed) product lines. Adjusting profits by their

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<sup>10</sup>The initial cash holdings are not necessarily at the threshold value which is different from Malamud and Zucchi (2016). Thus, the distance between initial cash and the threshold value can participate to cash reserve dynamics at the firm level but also at the industry one.

<sup>11</sup>This exogenous shock could be related to risks and information asymmetries not included in the model, and in particular with respect to product line creation uncertainty. See for example Hugonnier, Malamud, and Morellec (2014) or He (2012) for such kinds of approaches.



mean and using the martingale representation theorem, this sensitivity for any incentive compatible contract is:

$$dW_t = \rho W_t - dU_t + \beta_t n_t \sigma dS_t + \beta_t^+ dJ_t^+ + \beta_t^- dJ_t^-, \quad (7)$$

which is derived from the extended Ito representation theorem in Oksendal and Sulem (2011, p.33) for processes including Brownian motions and jumps with time depending intensities.<sup>12</sup>

There is no agency problem with respect to product line noise, such that  $\beta_t^+$  and  $\beta_t^-$  are not determined at this stage. Yet, as in DeMarzo et al. (2012), the incentive compatibility requires that  $\beta = \lambda$ , such that:

$$\beta = \lambda. \quad (8)$$

Intuitively, cash dynamics are set in order to align the agent incentives to the investor's interests. When the firm is profitable according to positive cash flow shocks, the promised value of expected compensation is higher. In addition, cash accumulation in good times builds protection against termination risks in bad times.

Because investors can always compensate the agent with cash, it will cost investors at most one euro to increase  $W$  by one euro. Therefore,  $P'_n > -1$ , which implies that the total value of the firm  $P_n(W) + W$  is weakly increasing in  $W$ . Because there is a benefit in deferring the agent's compensation, the optimal contract will set cash compensation  $dU$  to zero when  $W$  is small, so that  $W$  will rise as quickly as possible. However, because the agent has a higher discount rate, there is a cost in deferring the agent's compensation. This trade-off implies that there is a compensation level  $\bar{W}_n$  such that it is optimal to pay the agent with cash if  $W_t > \bar{W}_n$  and to defer compensation otherwise. Thus:

$$dU_t = \max\{W_t - \bar{W}_n, 0\}.$$

Applying an extended version of Ito's lemma for diffusion and jump processes (Oksendal and Sulem, 2011, p.6), the Hamilton-Jacobi-Bellman equation is:

$$\begin{aligned} rP_n(W) = \max_{x, \beta_t^+, \beta_t^-} & n[\pi - wh(x)] + (\rho W - \beta_t^+ nx + \beta_t^- n\tau)P'_n(W) + \frac{1}{2}(\lambda\sigma n)^2 P''_n(W) \\ & + nx[P_{n+1}(W + \beta_t^+) - P_n(W)] + n\tau[P_{n-1}(W - \beta_t^-) - P_n(W)], \end{aligned} \quad (9)$$

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<sup>12</sup>This equation is similar the extension developed by DeMarzo et al. (2012) including persistent profitability shocks in addition to cash flow ones. Martingale representation is also used for a technology including jumps by He (2012).

where  $rP_n(W)$  has been substituted for  $\frac{dP_n(W)}{dt}$  and the time subscript  $t$  is omitted. This equation is very similar to Brémaud (1981, chap. VII, P. 203) addressing dynamic programming for intensity control, without Brownian motions.

**Proposition.** *Let  $\omega = W/n$  and  $p = P_n/n$ . The solution of the firm problem in (9) can be restated in per product line terms:*

$$rp(\omega) = \max_x \pi - wh(x) + [\rho - (x - \tau)]\omega p'(\omega) + \frac{1}{2}(\lambda\sigma)^2 p''(\omega) + (x - \tau)p(\omega), \quad (10)$$

and defines a unique function  $p(\omega)$  for the given conditions  $p(0) = l$  and  $p'(\bar{\omega}) = -1$ . The behavior of  $p$  is shown in Figure 1 (left).

*Proof.* Let us verify that  $P_n = np$  is a solution to (9). With the per product line variable  $p_n = P_n/n$  and  $\omega = W/n$ , the right hand side term of this equation becomes:

$$\begin{aligned} \max_{x, \beta_t^+, \beta_t^-} & [\pi - wh(x)] + (\rho\omega - \beta_t^+ x + \beta_t^- \tau)p'(\omega) + \frac{1}{2}(\lambda\sigma)^2 p''(\omega) \\ & + x \left[ (n+1)p \left( \frac{n\omega + \beta_t^+}{n+1} \right) - np(\omega) \right] + \tau \left[ (n-1)p \left( \frac{n\omega - \beta_t^-}{n-1} \right) - np(\omega) \right]. \end{aligned} \quad (11)$$

The first order condition with respect to  $\beta_t^+$  is:  $p'(\omega) = p' \left( \frac{n\omega + \beta_t^+}{n+1} \right)$ . Following DeMarzo et al. (2012), the function  $p'$  associated with (9) is strictly monotone for  $\omega < \bar{\omega}$  so that  $\omega = \frac{n\omega + \beta_t^+}{n+1}$  and  $\beta_t^+ = \omega$ . Similarly,  $\beta_t^- = \omega$ . Then  $x \left[ (n+1)p \left( \frac{n\omega + \beta_t^+}{n+1} \right) - np(\omega) \right]$  simplifies to  $xp(\omega)$ , another term  $-\tau p(\omega)$  also appears, and  $(\rho\omega - \beta_t^+ x + \beta_t^- \tau)p'(\omega)$  becomes  $[\rho - (x - \tau)]\omega p'(\omega)$ . So, (11) corresponds to the right hand side of (10). Thus, as  $p$  is solution to (10), it is also a solution to (9).  $\square$

This proposition has properties which differ from the approach with *ad hoc* financial constraints by Malamud and Zucchi (2016). First, this result brings notably a micro-foundation for cash policy when the firm has only one remaining product line. Malamud and Zucchi (2016) assume that shareholders receive a lumpy liquidation dividend equal to the firm's remaining cash holdings. Here, the same fact is obtained from optimal policy. Second, the optimal cash supplement necessary when innovation is successful is not the cash threshold as in Malamud and Zucchi (2016). Here, optimal additional cash is the current per product line level. Yet, if the firm already is at the cash threshold, then the same conclusion is reached.

Third, there is no optimality condition for the total production size of the firm as in Malamud and Zucchi (2016). Actually, this is the outcome of relying on product innovation where the size of the firm is associated to  $n$ , which is an outcome of the optimization program. While there is not the rich dynamics of Malamud and Zucchi (2016) in that respect here, the model has tractability thanks to this stylized approach. The volatility of cash flows is also not affected here by optimal production size, which allows to analyze its effects independently. Fourth, the effects of financial constraints can be characterized independently from firm size, and beyond the neighborhood of the cash threshold, contrary to Malamud and Zucchi (2016).

**Corollary.** *The first order condition with respect to  $x$  is*

$$h'(x) = p(\omega) - \omega p'(\omega), \quad (12)$$

*which replicates the empirical fact that cash holdings and R&D expenditures are positively correlated – for that positive relationship, see Figure 1 (right).*

This equation states that the marginal value of investing equals the current per unit value of the firm to investors,  $p(\omega)$ , plus the marginal effect of decreasing the agent’s per unit payoff  $\omega$  as the firm grows.<sup>13</sup>

The shape of this curve is consistent with the microeconomic literature relating R&D and cash at the firm level. R&D expenditures have been positively related to cash holdings for U.S. listed corporations during the last three decades (Bates et al., 2009)<sup>14</sup>, and seems independent from tax issues.<sup>15</sup>

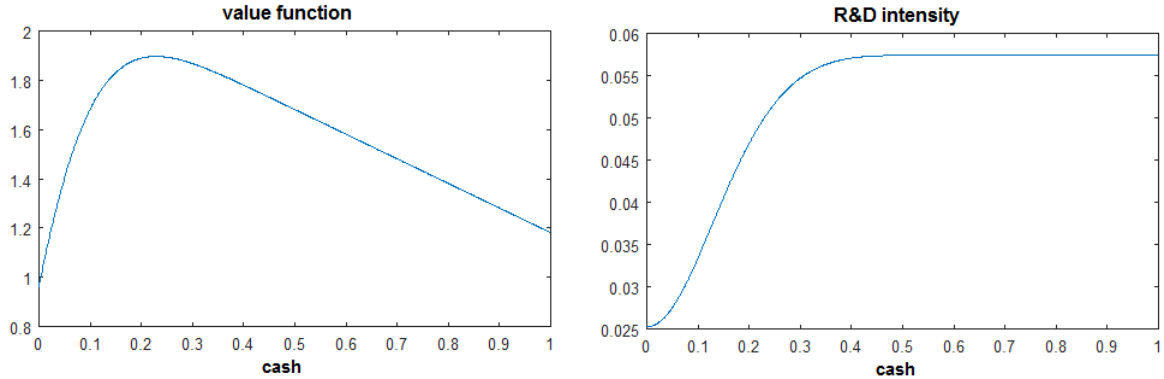
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<sup>13</sup>So, the approach by DeMarzo et al. (2012) can be adapted to a context where continuous capital accumulation is replaced by irregular product line creation. More precisely, the firm problem can be restated in a per product line form; the per product line value function with respect to the per product line cash in this model and in DeMarzo et al. (2012) are identical, and the same investment and cash policies are derived. The intuition behind this property is that, while the Poisson processes used here for innovation and creative destruction complicates analytics in the first place, they add two additional controls to determine optimal cash hoarding policy when product lines are created or destroyed. First order conditions with respect to these controls restore homogeneity.

<sup>14</sup>In particular, the relation observed in levels is also true for variations, meaning that the relation is not only driven by cross-sectional patterns among firms but also by within individual firm dynamics.

<sup>15</sup>This joint dynamics might reflect tax issues for multinationals, which can retain cash earned abroad because earning repatriation would imply substantial tax payments. Yet, the relation between R&D and cash hoarding is not concentrated in these specific multinationals: Bates et al. (2009) show that “there is no evidence that cash holdings increase more for firms with foreign pretax income [and in] particular, while the average cash ratio of firms without foreign taxable income increases from 14.3% in 1990 to 25.3% in 2006, the cash ratio of firms with foreign taxable income is 10.8% in 1990 and increases to 20.2% in 2006”. And cash hoarding at the global level is also not driven only by large firms as “the average cash ratio has a significantly positive time trend for all size quintiles.”

Figure 1 – Value function and R&D expenditures in equilibrium depending on cash



Note : Graphics are derived with standard calibration depicted in simulations below, which remains close to those used in DeMarzo et al. (2012) and appearing in the model *à la* Klette and Kortum (2004) used by Acemoglu et al. (2014). The innovation efficiency parameter is set at  $\theta = 0.60$ .

## 2.2 Aggregate equilibrium

In equilibrium, the distribution  $dM(n, \omega)$  of firms across product line numbers  $n$  and cash levels  $\omega = W/n$  is fixed. The rate of destructive creation  $\tau$  is the sum of the total innovation effort by incumbents  $z_i$  and entrants  $z_e$  :  $\tau = z_i + z_e$ .  $z_e$  corresponds to the mass of entrants which is directly linked to the distribution  $dM(n, \omega)$ . There is also a mass of firms which are liquidated,  $\nu$ , but which does not intervene in the creative destruction account. As every single firm terminates at some date, the stability is the distribution is insured by permanent flows of entrants. Assume the continuous entry of a unit mass of firms for each period. Relying on the optimal investment and cash policies stated above, this generates after a high number of periods a distribution of firms  $d\tilde{M}(n, \omega)$  and a total number of products  $N > 1$ . When normalizing the distribution so that  $dM(n, \omega) = d\tilde{M}(n, \omega)/N$ , one obtains the mass of entrants  $z_e = 1/N$ .<sup>16</sup> Then, the innovation effort by incumbents is:

$$z_i = \int_{n=1}^{+\infty} \int_{\omega=0}^{+\infty} nx(n, \omega) dM(n, \omega), \quad (13)$$

Here the growth rate has the same simple form as in the case without corporate financial frictions, that is:

$$g = \tau \ln \gamma.$$

The equilibrium outcomes for  $g$  and the mean level of cash  $C$  are not clear-cut and

<sup>16</sup>The total number of products is normalized to one as in Klette and Kortum (2004).

interact. When there is no agency problem,  $z_i$  and  $z_e$  have closed form solutions as the value function reduces to a proportional function of the product line number, and the innovation intensity of incumbents is homogeneous. Here, the destructive creation rate is lower as the investment rate is below the first best one for many firms. Inversely, growth and destructive creation are here equivalent and  $\tau$  affects the optimal financial policy of firms.

The remaining markets are settled as follows. Labor is in fixed supply  $l_s$  and split between production,  $(\gamma w_s)^{-1}$ , and R&D from entrants  $h_e = \psi z_e$ , and incumbents  $h_i$ , for which the expression implies summing over all products and cash levels as in (13). Cash and equity markets clear within a representative agent perspective including both investors and managers. All equity is own by investors while all cash is the property of managers. Yet, the sum of equity and cash corresponds to the total value of the firm. From a representative agent perspective, all the firm value is own by this agent, and cash corresponds to inside transfers.<sup>17</sup>

### 3 Quantitative analysis

#### 3.1 Setup and baseline

In this section, all the elements of the quantitative exercises are presented, with first the calibrations in the baseline case, then the precise algorithm and then an illustration of the firm distribution at equilibrium.

Calibration is the following (Table 1). Values are mainly taken from DeMarzo et al. (2012). The interest rate is exogenously set at  $r = 0.06$ . The discount rate of the agent follows  $\rho = r + 0.04$ . Mean cash flows are  $\mu = 0.20$ , which equivalently corresponds to an innovation step of  $\ln \gamma = 0.223$ . The agency problem parameter is  $\lambda = 0.20$  and the cash flow risk is  $\sigma = 0.13$ . Further, the liquidation value loss of a product line is set at 25 percent:  $\kappa = 0.25$ . And the initial distance to optimal equity verifies:  $\zeta = 0.25$ .

With respect to the general equilibrium, the measure of labor supply is  $l_s = 0.078$ . The fixed cost of entrants is  $\psi = 0.15$ . Finally, the innovation intensity with respect to the knowledge stock is  $\alpha = 0.5$ , which is close to 0.637 used in Acemoglu et al. (2014) and allows for a quadratic R&D cost function as in DeMarzo et al. (2012).

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<sup>17</sup>The Euler equation usually gives a link between  $r$  and  $g$  in presence of risk aversion. As investors are risk neutral, this is not the case here.

With respect to the computation procedure, the time interval is  $dt = 0.25$  to correspond to a quarter, and the number of periods is  $Q = 1000$ . The maximum number of products is  $N = 15$  (Figure 2). Finally, cash values  $\omega$  are between 0 and 1 on a grid with 1,000 points.<sup>18</sup>

Table 1 – Calibration

<i>param.</i>	<i>description</i>	<i>value</i>
$l_s$	measure of workers	0.078
$r$	interest rate of investors	0.060
$\rho$	interest rate of managers	0.064
$\lambda$	agency problem stringency	0.20
$\kappa$	liquidation losses share	0.25
$\zeta$	initial distance to optimal equity	0.25
$\mu$	product line mean profit	0.20
$\sigma$	cash flow shocks volatility	0.13
$\gamma$	innovation step	0.223
$\theta$	incumbent innovation capacity	0.40
$\psi$	entrants innovation capacity	0.15
$\alpha$	innovation intensity w.r.t. knowledge stock	0.50

The algorithm is made up by the following steps<sup>19</sup>:

- An initial value for the destructive creation rate  $\tau$  is set.
- Partial equilibrium loop:
  - Arbitrary initial values are given to  $w$  and  $l$ .
  - Value function loop: An initial value is chosen for the cash threshold  $\bar{\omega}$ . Then, the value function is calculated following (10) between 0 and  $\bar{\omega}$ . Finally, a new value for  $\bar{\omega}$  is chosen if  $p'(\bar{\omega})$  is above or below -1 (see Proposition) until convergence.
  - New values are derived for  $w$  using the free entry condition (4) and for the liquidation value  $l$  using (6) and compared to the previous ones until convergence.
  - Optimal initial cash  $\omega_0$  is also derived in this loop using (5) and optimal R&D spendings  $x$  using (12).
- General equilibrium loop<sup>20</sup>:
  - The distribution starts with a unit mass of entrants and then, at each period, a new distribution is calculated.<sup>21</sup>

<sup>18</sup>The maximum value 1 could be higher but is never reached in all the considered simulations.

<sup>19</sup>All loops have a convergence criteria at  $10^{-2}$ .

<sup>20</sup>The procedure by Atkeson and Burstein (2010) is followed as a core setup for firm dynamics and is adapted to the current context, notably with cash flow shocks.

<sup>21</sup>Note here that there are two ways for firms to exit. On the one hand, they can have only a single

- Random shocks  $\sigma dS_t$  are applied to every firm and the corresponding cash dynamics is derived with (7) and (8) using optimal decision parameters from the Proposition.
- The new number of products for each firm is derived using optimal R&D investment for each couple  $(\omega, n)$  with the corresponding probability of success.
- A new unit mass of entrants is introduced and the loop is run  $Q$  times.<sup>22</sup>
- Aggregates are normalized and innovation efforts by incumbents and entrants are summed up and compared to the chosen initial value of the destructive creation rate. New simulations are run until convergence for  $\tau$  and clearing of the labor market.<sup>23</sup>

As the outcome of this algorithm, one can notice a distribution of firms similar to those traditionally observed with schumpeterian models, but with a slight difference with respect to the cash dimension. In Figure 2, Panel (a) shows the overall distribution over cash and the number of products. An important mass of firms achieves the cash threshold, explaining the most distinctive part of the distribution for cash levels next to it. A second part of the distribution can be distinguished on the contrary around the entry point. The shape around it directly reflects cash flow shocks. In the end, abstracting from cash in Panel (b) where densities are calculated for each product number, the distribution follows a classic pattern (see for instance Aghion et al., 2014, Figure 1.4 p. 538).

### 3.2 Experiments related to financial parameters

Financial frictions can be exacerbated by different factors. The need to avoid inefficient liquidation is reinforced in three situations: when the agency problem is more intense, when liquidation losses are higher, or when initial cash injections are hindered. In three experiments, a single parameter corresponding to one of these financial frictions is adjusted. If the model were not microfunded, it would not be possible to disentangle the effects of these various channels. Yet, there implications are not similar. Generally, optimal financial policy, through cash accumulation and initial cash injection, allows to compensate higher risks to liquidate the firm in more constraining contexts. However, if initial cash injection can

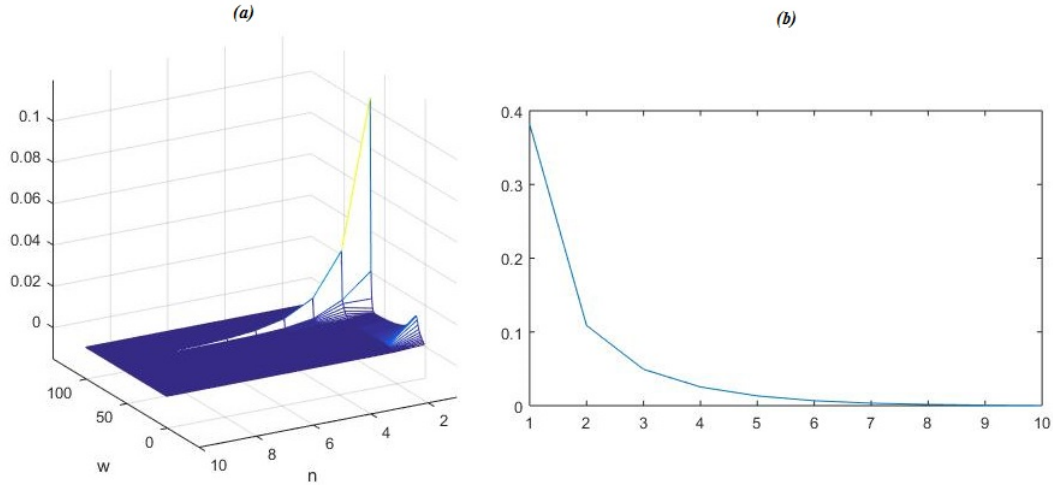
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remaining product line and experience a product line destruction. On the other hand, whatever their number of product lines, they are liquidated if they are running out of cash (equivalently, if they reach the manager's limited liability constraint). This second source of exit is not due to creative destruction.

<sup>22</sup>The chosen value for  $Q$  allows for stability of aggregates and the firm distribution.

<sup>23</sup>If the simulated  $\tau_{\text{out}}$  is above the initial chosen value  $\tau_{\text{in}}$ , total labor demand is also too high, and new simulations are run with a lower  $\tau_{\text{in}}$ .

Figure 2 – Example of firm distribution



Note: On panel (a), the overall distribution is represented on the  $(\omega, n)$  grid. The number of points for cash values is 1000. On panel (b), the masses of firm are calculated for each level of product number  $n$ .

not reach the optimal level, firm development and R&D and growth can be altered to a large extent. So, according to this model, developing venture capital markets appears far more efficient than improving corporate governance, to reduce asymmetric information, or liquidation settlements.

When the agency problem is stronger, cash accumulation depends on the product  $\lambda\sigma$ . The risk to reach liquidation is higher. There is indeed a higher liquidation rate  $\nu$  in the economy (Table 2 Panel B). The industry mean level of cash, denoted by  $C$ , jumps, without affecting growth to a substantial extent yet. The slight effect on growth is mainly due to initial cash which is already at a level allowing for R&D close to the first best. These simulations are consistent with the vast literature relating cash holdings to cash flow risks at the firm level. In addition, this property is verified at the aggregate level too: industries with higher information asymmetry have substantially higher cash ratios and slightly slower growth. Growth differs by 0.05 point between the baseline economy and this scenario.

When liquidation losses are higher, there is a need to build more financial room to avoid this outcome. This is done when firms are created with higher initial cash levels (Table 2 Panel C compared to Panel A). Then, optimal initial cash dampens a large part of the effects arising from liquidation: simulations changing liquidation losses while initial cash is



Table 2 – Financial friction experiments

Panel A - Baseline											
$x_{fb}$	$q_{fb}$	$\tau$	$w_s$	$l$	$\bar{\omega}$	$\omega_0$	$z_i$	$z_e$	$\nu$	$C$	$g$
3.10	1.78	6.77	11.52	1.25	21.0	7.3	2.95	3.82	0.40	15.5	1.51
Panel B - $\lambda = 0.05$											
$x_{fb}$	$q_{fb}$	$\tau$	$w_s$	$l$	$\bar{\omega}$	$\omega_0$	$z_i$	$z_e$	$\nu$	$C$	$g$
3.03	1.74	6.99	11.51	1.28	5.8	2.5	2.80	4.19	0.13	2.8	1.56
Panel C - $\kappa = 0.03$											
$x_{fb}$	$q_{fb}$	$\tau$	$w_s$	$l$	$\bar{\omega}$	$\omega_0$	$z_i$	$z_e$	$\nu$	$C$	$g$
3.09	1.79	6.73	11.58	1.64	18.8	4.3	2.95	3.79	0.74	8.2	1.50
Panel D - $\zeta = 0.03$											
$x_{fb}$	$q_{fb}$	$\tau$	$w_s$	$l$	$\bar{\omega}$	$\omega_0$	$z_i$	$z_e$	$\nu$	$C$	$g$
3.06	1.76	6.90	11.51	1.22	21.0	9.7	2.91	3.99	0.26	14.2	1.54
Panel E - $\zeta = 1, \omega_0 = 4.3$											
$x_{fb}$	$q_{fb}$	$\tau$	$w_s$	$l$	$\bar{\omega}$	$\omega_0$	$z_i$	$z_e$	$\nu$	$C$	$g$
3.45	1.97	5.89	11.42	1.25	22.7	4.3	3.10	2.81	0.87	14.2	1.32

Note:  $x_{fb}$  and  $q_{fb}$  indicate optimal R&D effort and Tobin's  $q$  in the first best case without financial frictions.

endogenous do not display substantial changes in global outcomes.

When initial cash injection capacity is changed, i.e. just a part of optimal initial cash can be reached (from 75 percent to 97 percent), then optimal growth changes by 0.03 point (Table 2 Panel D compared to baseline). For these simulations, effects are relatively small as initial cash can however increase by around 25 percent.

Yet, liquidation losses and constraints on initial cash can interact. Compare two industries  $A$  and  $B$  with high and low liquidation losses. Industry  $B$  is less constrained and has an optimal initial cash value at a level  $\omega_{0B}^*$  which is lower than the one industry  $A$  would have naturally. Assume that industry  $A$  has the same initial cash conditions as industry  $B$ :  $\omega_{0A} = \omega_{0B}^*$ . So, the pure effect of lowering liquidation value is considered without any possibility to adjust initial cash. In this case, on the contrary, the effect on growth is massive (Table 2 Panel E). In this very stringent case, the growth rate could be diminished by 0.2

point.<sup>24</sup>

### 3.3 Experiments related to firm technology

The behavior of an industry depends on the structural properties of its innovation capacity. In this section, experiments consider changes with respect to R&D return and costs, for incumbents and entrants, and with cash flow volatility. In all these experiments, parameters are increased by 50 percent. These experiments can be thought as comparisons between industries, or between equilibria of a single industry with or without stylized targeted public intervention. These experiments are common for schumpeterian growth models but have additional implications from a financial point of view. Notably, specific factors of innovation do not have the same implications for firm financial management and the aggregate cash intensity of an industry. In particular, higher entry costs and higher cash flow volatility generates a stringent reliance on cash to alleviate increased liquidation risks. On the contrary, higher mean returns reduce the stock of cash required to insulate from liquidation risk. In the end, growth and aggregate cash behaviors crucially depend on the mix of many innovation features of a specific industry.

Considering higher costs of R&D would amount to a stylized way to address a lowering of R&D government subsidies directed to incumbent firms.<sup>25</sup> Compared to the baseline case, there are compensating effects (Table 3 Panel B). First, the incumbent innovation rate is reduced from 2.95 to 2.21 percent. This implies an overall fall in the rate of creative destruction from 6.77 to 6.41 percent, a lower overall demand for researchers and falling wages from 11.51 to 11.49. This favors more entry by new firms whose innovation increases from 3.82 to 4.20 percent. Note here that overall cash in the industry is slightly lower from 15.5 to 14.9. While optimal initial cash does not change substantially, the effect is purely due to the higher share of entrants in the distribution of firms. So, there is an adjustment between entrants and incumbents in line with Acemoglu et al. (2014) and generally with this kind of product line endogenous growth models.

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<sup>24</sup>In this scenario, initial cash values corresponds approximately to one quarter to one semester of pure cash accumulation, when compared to  $\mu$  at 0.20. Thus, if public funds were used to fill the gap, the amount per firm would correspond to a reasonable scale compared to realistic needs for liquidity management. If such a measure is adopted for all entrants, the total aid would amount to around 0.25 percent of GDP.

<sup>25</sup>If R&D costs were alleviated by government aids between Panel B and the Baseline, the financial effort would amount to around 0.5 percent of GDP. This amount can be derived from outcomes in Table 3 related to the labor markets equilibrium as the difference between the wage bill devoted to researchers by incumbents in the two scenarios.

Table 3 – Innovation capacity experiments

Panel A - Baseline											
$x_{fb}$	$q_{fb}$	$\tau$	$w_s$	$l$	$\bar{\omega}$	$\omega_0$	$z_i$	$z_e$	$\nu$	$C$	$g$
3.10	1.78	6.77	11.52	1.25	21.0	7.3	2.95	3.82	0.40	15.5	1.51
Panel B - $\theta = 0.30$											
$x_{fb}$	$q_{fb}$	$\tau$	$w_s$	$l$	$\bar{\omega}$	$\omega_0$	$z_i$	$z_e$	$\nu$	$C$	$g$
2.32	1.78	6.41	11.50	1.25	20.8	7.3	2.21	4.20	0.29	14.9	1.43
Panel C - $\psi = 0.20$											
$x_{fb}$	$q_{fb}$	$\tau$	$w_s$	$l$	$\bar{\omega}$	$\omega_0$	$z_i$	$z_e$	$\nu$	$C$	$g$
4.12	2.34	4.62	11.33	1.62	24.9	9.1	3.97	0.75	0.99	19.6	1.05
Panel D - $\mu = 0.266$											
$x_{fb}$	$q_{fb}$	$\tau$	$w_s$	$l$	$\bar{\omega}$	$\omega_0$	$z_i$	$z_e$	$\nu$	$C$	$g$
3.08	1.77	10.57	11.50	1.26	18.7	6.5	2.89	7.68	0.37	11.7	3.27
Panel E - $\sigma = 0.173$											
$x_{fb}$	$q_{fb}$	$\tau$	$w_s$	$l$	$\bar{\omega}$	$\omega_0$	$z_i$	$z_e$	$\nu$	$C$	$g$
3.13	1.80	6.67	11.52	1.24	27.6	9.1	2.95	3.73	0.51	19.5	1.49

Note:  $x_{fb}$  and  $q_{fb}$  indicate optimal R&D effort and Tobin's  $q$  in the first best case without financial frictions.

With higher entry costs, the picture is symmetric (Table 3 Panel C).<sup>26</sup> The innovation rate by entrants goes to 0.75 percent compared to 3.82 in the baseline case, which favors innovation by incumbents through reduced product line destruction and lower wages. Lower wages are a direct effect of higher entry costs through the free entry condition, explaining why the change in the wages is sharper here than in any other experiment. However, this direct impact is compensated by increased firm entry value, as first best  $q$  goes from 1.78 to 2.34. The impact on growth is lower than the fall of innovation from entrants because of fostered innovation by incumbents. Note that in line with a higher first best  $q$ , the proportional loss when a firm is liquidated is also higher, which implies a higher initial cash injection from 7.3

<sup>26</sup>In this case, if the difference in entry costs were paid by the government, the amount devoted to help entrants here would be around 5 percent of GDP. The scale of the scenario is the same as the most intensive ones in Acemoglu et al. (2014). While wages remain relatively unchanged, the innovation rate by entrants rises hugely.

to 9.1. This fact associated with lower entry implies an overall cash level rising by around 5 points from 15.5 to 19.6. Putting it differently, favoring entry as also an indirect positive effect in that it lowers the required initial equity injection, and alleviate financial constraints as less profitable projects imply less losses when liquidation occurs.

Changes with respect to mean returns, or equivalently to the innovation step size, lead to a sharp rise of innovation by entrants and to a corresponding rise in the destructive creation rate from 6.67 to 10.57 percent (Table 3 Panel D). So, the growth rate is more than doubled thanks to both the initial exogenous increase in the innovation step size, and the accompanying rise in product line destructive creation. If the creative destruction rate were fixed, a higher product line mean profitability would increase the inefficiency of liquidation and spur the cash threshold to insulate from cash flow risks. Yet, as the creative destruction rate increases a lot, product line profitability does not increase to such an extent, as reflected in the first best  $q$  which is almost unchanged. So, cash hoarding at the industry level in this case displays strong compensating effects between project profitability and creative destruction.

Finally, if the volatility of cash flows increases, there is a higher risk to reach liquidation rapidly and cash accumulation is spurred, with an increase of the initial cash level to 9.1 (Table 3 Panel E). As this initial target varies endogenously, the increased volatility does not affect much real variables. Only aggregate cash jumps by around 5 percent. So, in presence of relatively well functioning initial equity markets, the cash volatility risk can be largely alleviated. In addition, it replicates at the macroeconomic level the well established result that firms with higher cash flow volatility accumulate more liquidity, such as in Bates et al. (2009) notably.

Yet, any of the previous one-dimensional experiment should be associated to high-tech sectors as many of the structural parameters may differ simultaneously. Following the spirit of an exercise in Klette and Kortum (2004), an additional experiment is lead by proceeding to a proportional increase by 50 percent of all parameters affecting profits, for entrants and incumbents: mean profits, volatility, R&D cost, and entry cost (R&D costs for entrants). Here, higher innovation opportunities are associated with better returns but also higher cash uncertainty and costlier R&D.

In this scenario, initial cash rises by 25 percent (Table 4 Panel B). This is consistent with

Table 4 – High-technology experiments

Panel A - Baseline											
$x_{fb}$	$q_{fb}$	$\tau$	$w_s$	$l$	$\bar{\omega}$	$\omega_0$	$z_i$	$z_e$	$\nu$	$C$	$g$
3.10	1.78	6.77	11.52	1.25	21.0	7.3	2.95	3.82	0.40	15.5	1.51

Panel B - $\theta = 0.30, \psi = 0.20, \mu = 0.266, \sigma = 0.173$											
$x_{fb}$	$q_{fb}$	$\tau$	$w_s$	$l$	$\bar{\omega}$	$\omega_0$	$z_i$	$z_e$	$\nu$	$C$	$g$
3.09	2.29	7.14	11.12	1.59	27.7	9.7	2.94	4.21	0.43	19.6	2.21

Panel C - $\theta = 0.30, \psi = 0.20, \mu = 0.266, \sigma = 0.173,$ $\lambda = 0.05, \kappa = 0.03, \zeta = 0.03$											
$x_{fb}$	$q_{fb}$	$\tau$	$w_s$	$l$	$\bar{\omega}$	$\omega_0$	$z_i$	$z_e$	$\nu$	$C$	$g$
3.01	2.24	7.39	11.14	2.12	6.7	2.8	2.97	4.42	0.22	3.8	2.29

Note:  $x_{fb}$  and  $q_{fb}$  indicate optimal R&D effort and Tobin's  $q$  in the first best case without financial frictions.

Begenau and Palazzo (2016) who show that R&D intensive firms have been having higher and higher cash over asset ratios over the recent three decades. Aggregate cash also increases by around 25 percent. This general pattern is the sum of effects in Table 3. Increases in  $\psi$  and  $\sigma$  foster cash while the increase in  $\mu$  tends to dampen it, the increase in  $\theta$  being neutral. In this case, higher aggregate cash can be associated with higher growth. Yet, another experiment mix could provide differing results.

## Conclusion

This paper develops a micro-funded model of firm innovation, growth and liquidity management. It mixes a seminal framework of schumpeterian growth relying on product innovation, with standard information asymmetries about cash flows between managers and investors using continuous-time methods. The model displays tractable properties, notably for solving the firm problem at the product line scale, consistently with evidence on the relation between cash holdings and R&D at the firm level. It allows for experiments for a large set of parameters without restriction on their values. In particular, various factors increasing liquidation risks are disentangled, and initial equity funding happens to be crucial for dynamics. In addition, many dimensions related to innovation capacity affect cash holdings at the industry level differently. Some industries could have both higher growth and cash

holdings, yet identifying them as belonging to high-tech sectors is not straightforward.

This paper did not address several avenues of further research. First, the agency problem could rely on uncertainty affecting R&D success on top of uncertainty related to cash flows. This could provide a direct explanation for a lack of external funding due to insufficient visibility on projects. The framework could also be enriched with tangible capital providing safer returns. Second, a financial intermediation block could be added, as this paper abstracts from considerations in terms of inside and outside liquidity. Cash holdings accumulated by firms may affect the overall balance in the economy if safe assets are scarce.

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