

# Renewable resources and inequality aversion: what consequences for the future?\*

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## Abstract

This paper addresses intragenerational and intergenerational issues about a renewable natural resource exploitation. In particular, we analyze how different equity views, represented through a change in the intragenerational inequality aversion, influence the possible development paths for future generations. We suppose an agent has access to a renewable resource and works to exploit it, while another agent does not have access to it. A social planner implements a transfer mechanism from the former to the latter. We show that if the worker is originally better-off than the receiver, inequality aversion has a negative effect on the resource stock with a lump-sum transfer, but potentially a positive effect with a proportional tax. Reciprocally, the higher the stock the higher the possibilities for future consumptions. These links strongly suggest to deal jointly with the two equity dimensions in order to design consistent environmental policies.

**Keywords:** renewable resource, intragenerational equity, intergenerational equity.

**JEL Classification:** D63, Q20, Q56

## 1 Introduction

Emphasis has been put on futurity in the sustainability debate. Economists translated this question on how weighting the future compared to the present in the evaluation of different development paths. Between them, some can be considered as fair if generations that arise

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at different dates count equally. This view is often summarized under the vocable *intergenerational equity* (IE). But if society cares about the “difference of well-being” between two generations, it cares also about that of two individuals from the same generation. When they count equally, we talk about *intragenerational equity* (AE). The economics literature has generally separated these two equity dimensions, but there are several arguments for dealing with them together. First, it can be argued that an unjust society is likely to be unsustainable, either on the political side (revolution) or on the environmental side (degradation) (Haughton, 1999). Second, it seems curious to attach more importance to future generations, thus unborn, than to the current one (Solow, 1991; Anand and Sen, 2000). Finally, from a policy view, one may wonder if intragenerational and intergenerational concerns can be designed independently. On the contrary, one should be interested in how they interact to formulate consistent policies.

The interactions between these two dimensions are actually ever-present in economics. Think, for example, of a fiscal policy that aims to reduce the public debt. But alongside this debt there is the environmental debt, which can reduce the possibility of development for future generations. In this respect, three major dimensions can be taken into account: the climate change, with the question of burden-sharing between generations, but also into each one of them between expected losers and winners; the exhaustion of non-renewable resources, but this requires an understanding of the industrial processes. Especially, to what extent these resources can be substitutes for manufactured capital; and, finally, the management of renewable resource stocks. We are interested in renewable resources in particular.

The linkages between the two equity dimensions seem not to have been extensively studied in the economics literature, and in the environmental and resources economics literature in particular. Nonetheless, some authors have highlighted this question in the climate change debate for some time. For example, Schelling (1992) argues that the best way for developing countries to fight against the negative effects of climate change is to continue to develop. Heal (2009), on the opposite, explains that a preference for equality between generations and the preference for equality within each one of them may be opposed. If one expects consumption will grow, one can further discount the future, but this does not incite us to take preventive measures against the negative effects of climate change. Conversely, as developing countries are more vulnerable, more concerns about them would incite us to take actions more quickly. Kverndokk et al. (2014) proposed a model to analyze the burden-sharing between North and South in the reduction of greenhouse gas emissions through clean and dirty investments. These two dimensions are also present in the explanation of climate negotiations (e.g. Lecocq and Hourcade, 2012). Baumgärtner et al. (2012) proposed a framework to summarize the possible links between the two equity dimensions; independence, facilitation and/or rivalry. These features are detailed in the context of ecosystem services by Glotzbach and Baumgärtner (2012).

As renewable resources have not a market price, the choice of the welfare criterion is of

all importance. More precisely this may allow us to express the implicit values of stocks of natural resources. The *shadow prices* are essential to compute *genuine savings* (Hamilton, 1994; Pearce et al., 1996; Asheim, 2007). The expenditures that enhance the environment are seen as savings and depletion of natural resources and environmental degradations as dissavings. It generalizes the traditional concept of savings, and its positivity indicates that welfare, however defined, is currently non-declining. Renewable resources have to be managed on the long run, but compared to non-renewable ones; they are generally directly consumed, they may have amenities and one can have “win-win” solutions. Further, some can be “essential” for life. For all of these reasons, the State intervention can be justified.

We are not aware of any work that builds analytically the welfare possibility between individuals that depends on a renewable resource and has consequences on future generations. The purpose of this study is to analyze the intragenerational and the intergenerational equity trade-off. The question we are asking is what are the consequences of intragenerational concern on current and future welfares, when only some individuals have access to a renewable resource and a social planner orders a redistribution, and does the type of redistribution matters in such a context. The IE may be represented by the maximin criterion since it allows each generation to benefit from the same highest possible welfare (Solow, 1974). The AE is more contentious. The main ones are; the utilitarianism<sup>1</sup>, the liberal egalitarianism, the libertarianism and the marxism (Arnsperger and Van Parijs, 2003). We assume here a utilitarianism view (in a strict sense). For that, society is assumed to have an inequality aversion; it restrains the substitution of the “well-being” of one agent for the well-being of another. This approach is not as restrictive as it could seem since it allows to deal with different theories of justice as special cases. Indeed, the utilitarianism (broad sense) assumes a nil inequality aversion; only the total of utility matters (Vickrey, 1945; Harsanyi, 1955, 1977). The liberal egalitarianism can take a “*maximin*” form, inspired by Rawls (1971), if utilities are considered as “primary goods”. Here, inequality aversion is infinite; no substitution is possible between the individual utilities. “Right-libertarians” would promote no transfer and “left-libertarians” would promote a transfer such as to perfectly equalize utilities. The marxist approach would determine thresholds of utility representing “needs”. According to the intergenerational view, the intragenerational fulfillment may be constrained. Studying both together allows us to determine all possible choices of policy and to estimate opportunity costs. The renewable resource is a parable to links the two dimensions.

We use a social objective that allows us to deal with the two main doctrines (utilitarianism and maximin) as well as all intermediary cases. Besides, we build a set of possibilities for the utilities. This tells us to what extend one can take from one agent to give to another agent. Then, efficient allocations will be represented by Pareto frontiers. From an allocation situated

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<sup>1</sup>The term utilitarianism may also simply refer to the utilization of the utility concept

on that frontier, one cannot increase any more the utility of one agent without decreasing that of another. Afterward, we will be able to choose between these optimal allocations using the social criterion. To respond to our issue, we need, at least, two different agents. One agent has access to a resource and works to extract a part of it. The other agent, who does not have access to it, entirely devotes his/her time to leisure. We introduce a redistribution mechanism that takes a part of the harvest from the worker to give it to the receiver. The utility of the worker depends on his/her leisure time and on his/her available consumption. The utility of the receiver depends only on the amount s/he receives. We analyze the utility distribution possibilities offered by two redistribution mechanisms. The first one is a lump-sum transfer; whatever the effort of the worker, the receiver will get the same amount of the resource caught. The second one is a proportional tax. Whatever the effort of the first agent, the other will get the same proportion of the resource caught. We introduce a renewable resource which varies according to the catch. As the resource may affect the potential catch, it may impact the utility possibility sets. First, we will see how the optimal transfer (lump-sum and tax) evolve according to a change in the inequality aversion. We will also be interested in its impact on the stock. Second, we will analyze the consequences of the evolution of the resource on the agents.

Our contribution is having stated clearly the conditions underlying the construction of well-known utility possibility frontiers in the context of two heterogeneous agents. When the transfer is absolute (through a lump-sum), this does not pose any particular difficulties as long as leisure and consumption are normal goods. When the transfer is relative (proportional tax), it depends on the labor supply of the worker in reaction to the tax, which is given by his/her disposition to substitute leisure for consumption. In particular, if s/he works less when taxed, the utility possibility set is bounded by a "Laffer-like curve" (from Laffer, 2004); the amount received from a proportional tax is increasing, then decreasing, with respect to its rate. Knowing that, we were able to analyze the impact of a change in the inequality aversion on the transfers. If the worker is better-off than the receiver, one can expect that the higher the inequality aversion, the higher the transfer (absolute or relative). Regarding the redistribution of a renewable resource over time, the increasing of intragenerational inequality aversion when lump-sums are used favor the future if the stock is low or very large, since the harvest rise (compensation effect). And it favor the future when taxes are used, under the restriction of an increasing labor supply, when the stock is intermediary and decreasing, since the global consumption decreases (discouragement effect). It worsen the future in all other cases. More simply, a higher inequality aversion that implies to take more from the worker to give to the receiver affects negatively the resource stock in the case of lump-sum transfers, but *potentially* positively with proportional taxes.

The intragenerational dimension is built upon a framework proposed by Mas-Collel et al. (1995), borrowed itself from Atkinson (1973). We extend their example in two dimensions. First, we state clearly the conditions under their results, especially the construction of utility

possibility frontiers. And second, we introduce a resource to take into account the intergenerational concern in a Gordon-Schaefer model (Clark, 1990). The welfare analysis is based upon social welfare functions (Bergson, 1938; Samuelson, 1966).

The next Section presents the model. We solve it and we present some welfare analyses. Section 3 exhibits the links between the two equity dimensions. Section 4 concludes.

## 2 The Model

### 2.1 The Framework

We consider an economy with two (group of) heterogeneous agents. Virtually, each representative agent lives one instant. We assume one type of agent has access to a renewable natural resource and works to extract a part, while the other one does not have access to it. A social planner implements a mechanism of transfer at each period between the two agents. We consider two options: the first one is a lump-sum transfer and the second one a tax. We note  $i$  the worker and  $j$  the receiver. We normalize their available time to unity. At a given date, the resource stock  $X$  is given. Its law of evolution is given by the gap between renewal and harvesting;  $\dot{X} \equiv \frac{dX}{dt} = \phi(X) - H(l_i, X)$ . The function  $\phi$  describes a “bell curve” ( $\phi(0) = 0 = \phi(X_{sup})$ ) and reaches a maximum for  $\bar{X}$  (the Golden Rule stock). Total consumption equals the sum of individuals ones;  $c \equiv c_i + c_j$ . The utility of the worker depends on his/her leisure time and his/her effective consumption;  $u_i(l_i, c_i)$ ; while the utility of the receiver depends only on his/her consumption;  $u_j(1, c_j)$ . The utility functions are assumed to be interpersonally comparable, strictly concave and homothetic (see the Appendix A for details). Besides, each good is assumed to be essential;  $\lim_{c_k \rightarrow 0} \frac{\partial u_k(\cdot)}{\partial c_k} = \lim_{c_k \rightarrow 0} \frac{\partial u_k(\cdot)}{\partial l_k} = \infty$ ,  $k = i, j$ .

The supply of this economy is given by the production (catch-effort) function of the worker;  $H(l_i, X)$ . We assume it depends linearly on labor (or leisure), and it is convex with respect to the resource;  $H(l_i, X) = (1 - l_i)h(X)$ , where  $h(X)$  represents the catchability. The production is bounded between zero (no work) and  $h(X)$  (no leisure).<sup>2</sup> We assume no loss, so that the whole production is consumed;  $c = H(l_i, X)$ . By definition, the transfer amounts to  $c_j \equiv c - c_i \geq 0$ .

To represent inequality aversion in a simple way, we use an ordinal social welfare function with a constant elasticity of substitution (CES) between individuals;  $W(u_i, u_j) = \left(\frac{1}{2} \cdot u_i^\eta + \frac{1}{2} \cdot u_j^\eta\right)^{\frac{1}{\eta}}$ . From the parameter  $\eta$  (inferior to unity but non-zero) we obtain the elasticity of substitution  $\theta \equiv \frac{1}{1-\eta}$ . A decrease of the elasticity of substitution represents an increase of inequality aver-

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<sup>2</sup>The agent  $i$  earns a shadow wage  $w$  by hour worked;  $w(1 - l_i)$ ; and can consume the value of his/her net production (the harvest minus the amount transferred);  $pc_i$  (we normalize the price of the good to unity). As his/her utility is increasing with respect to consumption and leisure and that there is no savings, s/he will spend all his/her income;  $c_i = w(1 - l_i)$ .

sion.<sup>3</sup> Further, it is assumed to be nonpaternalist (only utility matters), paretian, symmetric and concave.<sup>4</sup>

The intergenerational dimension is dealt with through a value function of any maximal intertemporal welfare;  $V = V(X, t)$ . We do not solve such a problem, we simply assume its value function is non-declining with the stock.

## 2.2 Utility Possibility Frontier with Lump-Sum Transfer

The social planner can take an amount from what the worker harvests, to give it to the receiver. For each amount of transfer, we can determine the optimal utility of each agent. And making the amount transfered vary, we can construct the “first-best Pareto frontier”, which gives the maximum of utility of  $i$ , given that of  $j$  (or vice versa). This frontier bounds the utility possibility set. As leisure time of the worker is the only decision variable, we will be able to express the problem and solve it in only that variable. In the end, the frontier will be parametrized by the amount transfered.

The transfer being absolute, the worker can consume what s/he harvests minus a constant transfer;  $c_i = c - \bar{c}_j$ . The budget constraint can thus be rewritten as the maximal possible consumption minus the transfer:  $c_i = (1 - l_i)h(X) - \bar{c}_j$ .

### 2.2.1 Construction of the First-Best Frontier

For a given utility of the agent  $j$  (i.e. a given transfer), the agent  $i$  makes his/her trade-off labor-leisure so as to maximize his/her utility. We substitute the budget constraint with the available consumption in the utility, so as to maximize it in  $l_i$ .<sup>5</sup>

$$\max_{l_i} u_i(l_i, (1 - l_i)h(X) - \bar{c}_j). \quad (1)$$

A necessary condition to maximize the utility is that the marginal productivity of labor is equal to the marginal rate of substitution (MRS) of leisure for consumption;  $h(X) = \frac{\partial u_i(\cdot)/\partial l_i}{\partial u_i(\cdot)/\partial c_i}$ .

As we assumed the utility function of the agent  $i$  is homothetic, for a given consumption per leisure time, the MRS is constant. Therefore, we can express the optimal MRS as a function of this ratio. We note it  $\Omega$ , which is an increasing function. In this way, we can explicit the optimal consumption of the worker as a function of his/her leisure time.

$$\frac{c_i}{l_i} = \Omega(h(X)) \Leftrightarrow c_i = l_i \cdot \Omega(h(X)) \quad (2)$$

<sup>3</sup>The cases of (symmetric) minimum and pure utilitarian (mean) are obtained in the limit when, respectively,  $\theta$  tends to zero or to infinity (see the Appendix B).

<sup>4</sup>For more details, see for example Mas-Collel et al. (1995, pp. 825-826).

<sup>5</sup>Figures of the construction of the frontiers are plotted in the Appendix C.

With some substitutions (see the Appendix C), we get:

$$u_i^* = u_i \left( \frac{h(X) - \bar{c}_j}{h(X) + \Omega(h(X))}, \frac{\Omega(h(X))(h(X) - \bar{c}_j)}{h(X) + \Omega(h(X))} \right) \quad \text{and} \quad u_j^* = u_j(1, \bar{c}_j). \quad (3)$$

We obtain two functions parametrized by the amount transferred. Making it vary, we can draw the first-best frontier of the utility possibility set in the  $(u_i, u_j)$  space.

### 2.2.2 Shape of the First-Best Frontier

Once we were able to construct the first-best frontier, a natural question that arises is about its shape. For that, we have to study the evolution of the optimal utility of each agent according to a virtual variation of the amount transferred.

Here, it makes no difficulty to determine the signs of variation. It can indeed be shown (see the Appendix C) that the optimal utility of the agent  $i$  decreases with the transfer, while the one of the agent  $j$  increases. Hence, the first-best possibility frontier is downward-sloping in  $(u_i, u_j)$ . The utility possibility set is convex.

## 2.3 Utility Possibility Frontier with Tax

We study now the case where a social planner cannot transfer an absolute amount from the harvest of the agent  $i$  to the agent  $j$ . s/he decentralizes the transfer through a proportional tax, at the rate  $\tau$ . This problem can be related to a second-best approach. This case is comparable to the previous one, but relatively different in its implications. Here we make the tax rate vary to construct the “second-best Pareto frontier”. This frontier bounds the utility possibility set as before. We will explicit every variable as a function of the tax to solve the problem in that variable. The frontier will now be parametrized by the tax rate.

The transfer being proportional, the worker can consume what s/he harvests minus the taxed part;  $c_i = (1 - \tau)c$  (the receiver obtains  $c_j = \tau c$ ). The budget constraint can thus be rewritten as the maximal possible consumption net of the tax;  $c_i = (1 - \tau)(1 - l_i)h(X)$ .

### 2.3.1 Construction of the Second-Best Frontier

Let us consider a given utility of the agent  $j$ , i.e. a given tax rate. The agent  $i$  makes his/her trade-off labor-leisure so as to maximize his/her utility. We substitute, here also, the budget constraint with the consumption in the utility, so as to maximize it with respect to leisure time.

$$\max_{l_i} u_i(l_i, (1 - \tau)(1 - l_i)h(X)). \quad (4)$$

A necessary condition to maximize the utility is that the *net* marginal productivity of labor is equal to the MRS of leisure for consumption;  $(1 - \tau)h(X) = \frac{\partial u_i / \partial l_i}{\partial u_i / \partial c_i}$ .

With the assumption of homothecy of the utility function, we can express the optimal consumption as a function of leisure and of the tax rate.

$$\frac{c_i}{l_i} = \Omega((1 - \tau)h(X)) \Leftrightarrow c_i = l_i \cdot \Omega((1 - \tau)h(X)). \quad (5)$$

Here again, with some substitutions (see the Appendix C), we get:

$$u_i^{**} = u_i \left( \frac{(1 - \tau)h(X)}{(1 - \tau)h(X) + \Omega((1 - \tau)h(X))}, \frac{\Omega((1 - \tau)h(X))(1 - \tau)h(X)}{(1 - \tau)h(X) + \Omega((1 - \tau)h(X))} \right) \quad (6)$$

$$\text{and } u_j^{**} = u_j \left( 1, \frac{\Omega((1 - \tau)h(X))\tau h(X)}{(1 - \tau)h(X) + \Omega((1 - \tau)h(X))} \right). \quad (7)$$

We obtain two functions parametrized by the tax rate. Making it vary, we can draw the second-best frontier of the utility possibility set in the  $(u_i, u_j)$  space.

### 2.3.2 Shape of the Second-Best Frontier

Here, the evolution of the tax rate allows us to determine the shape of the frontier. It is shown in the Appendix C that the tax is always negative for the worker. The tax reducing his/her budget constraint, it reduces the utility reached. But, for the receiver, it depends. If the worker works more when s/he is taxed more, the catch received by the second agent increases; one taxes more a higher basis. If s/he works less (what should happen more generally), the effect is *a priori* ambiguous; one taxes more a lower basis. And if the worker is indifferent, the receiver is also better-off; one taxes more a constant basis.

Not surprisingly, the reaction of the worker to the tax depends on his/her elasticity of substitution of leisure for consumption (see the Appendix C). In particular, if the worker considers his/her leisure and his/her consumption as quite complementary, the more s/he is taxed, the more s/he works, and then the agent  $j$  is getting better and better as the tax rate grows. If they are quite substitutable, the amount received by the agent  $j$  increases if and only if the tax rate is not too high (see the Appendix C). The limit being given by the inverse of the difference between the elasticity of substitution;  $\sigma_{l_i, c_i}$ ; and the cross-price elasticity of leisure (with respect to the shadow net price of consumption);  $\varepsilon_{l_i, \frac{1}{1-\tau}}$ ; we note it  $\tilde{\tau}$ .

$$\frac{du_j^{**}}{d\tau} > 0 \Leftrightarrow \tau < \tilde{\tau} \equiv \frac{1}{\sigma_{l_i, c_i} - \varepsilon_{l_i, \frac{1}{1-\tau}}}. \quad (8)$$

Note that this is close to the concept of the Laffer curve (see Laffer, 2004), the receiver playing the role of the State. If the threshold is strictly inferior to unity, the receiver takes advantage of a



higher tax until a certain point ( $\bar{\tau}$ ), from which on, a higher rate reduces the perceived amount.

As the cross-elasticity depends on the substitution, we may expect the two parts of the denominator of the threshold be linked. Indeed, if, and only if, the elasticity of substitution is superior to unity, the threshold is lower than one (Appendix C). As this seems to be the more plausible case, we assume thereafter the worker has a high elasticity of substitution. This implies that the worker works less when s/he is taxed more.

To sum up, under the assumption of an increasing labor supply, the utility of the worker is decreasing with respect to the tax rate while the utility of the receiver increases until a threshold and decreases afterward. Thus, we have a “bell curve” utility frontier. But only the decreasing part matters, since the increasing one represents states where both agents can be better-off (Pareto-dominated).

### 2.4 Comparison of the Frontiers

As the two instruments have different implications, it is natural to wish to compare them. It is shown in the Appendix C that the second-best utility possibility set is included in the first-best one (see Figure 1). That is to say, for a given utility of the worker, the tax is always less favorable for the receiver than the lump-sum transfer. Indeed, the tax on the production discourage the worker and can even lead to worsen their both situations. From a social planner point of view, the lump-sum is always better than the tax since it permits more choice.

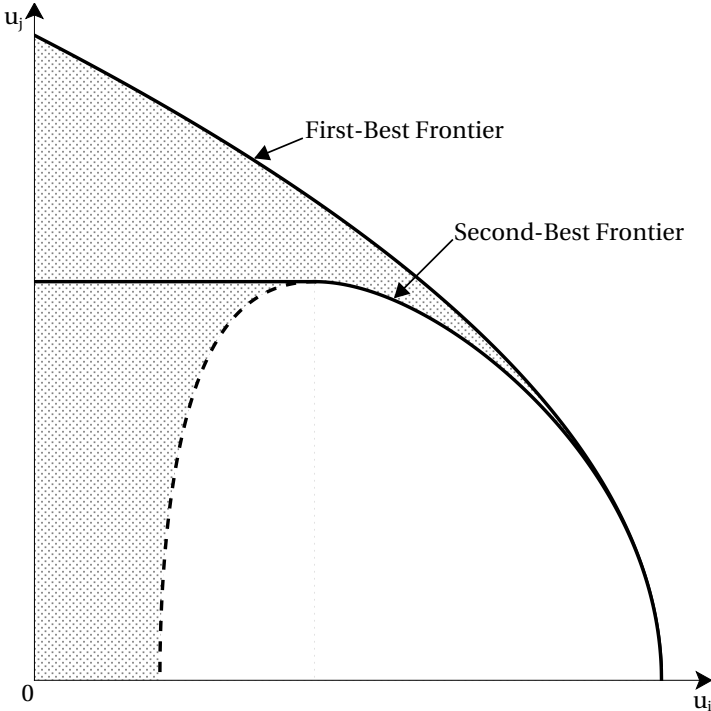


Figure 1: First and Second-Best Frontiers

Since the impact of the tax depends on the reaction of the worker, we may expect the difference between the two frontiers, i.e. the inefficiency of the tax (represented by the gray area in Figure 1), being dependent on the worker preferences. Indeed, if they are characterized by a constant elasticity of substitution, it is shown in the Appendix C that the lower it is, the more likely the gap between the two instruments is low<sup>6</sup>. At the limit, if leisure and consumption are perfectly complementary, the instrument used does no longer matter since the two frontiers merge together.<sup>7</sup> This latter result is not surprising since in this case the (implicit) relative prices do not matter for the worker choice.

## 2.5 Welfare Analysis

We built our utility possibility sets (bounded by the frontiers). We are now interested in finding the optimal allocations of utility, which indicate the optimal transfer (absolute or proportional). We will also see the consequence of a change in the inequality aversion on such an allocation. Let us recall that a higher inequality aversion corresponds to a lower elasticity of substitution. The welfare analysis is analytically the same with both mechanisms. We present it only with the lump-sum.

In general terms, we have to seek the maximal welfare subject to the fact that utilities are elements of the utility possibility set. Here, as we solved already a maximization problem, we can maximize directly the welfare through the amount transferred. We will always be situated on the frontier.

$$\max_{\bar{c}_j} W^*(\bar{c}_j) = \left( \frac{1}{2} \cdot u_i^*(\bar{c}_j)^\eta + \frac{1}{2} \cdot u_j^*(\bar{c}_j)^\eta \right)^{\frac{1}{\eta}}. \quad (9)$$

Let us define the marginal social rate of substitution (MSRS) as the willingness of society to increase marginally the utility of the agent  $j$  taking from the utility of the agent  $i$ , keeping its global satisfaction equal. A necessary condition for the welfare to be maximal implies that the utility ratio (the one of  $j$  over the one of  $i$ ) is a function of the MSRS and of inequality aversion (Appendix C).

At the optimum, the higher the slope of the frontier in absolute value, the higher the utility ratio. The easier is to get a high increasing of the utility of the agent  $j$  taking from the utility of the agent  $i$ , the more we will move toward a situation where the agent  $j$  has a high utility compared to that of the agent  $i$ .

The impact of inequality aversion depends on the initial situation (Appendix C). Two types of frontier illustrate it in Figure 2. If the agent  $j$  is relatively better-off in the original situation, the higher the inequality aversion (the elasticity of substitution pass from  $\theta_1$  to  $\theta_2$ ) the lower

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<sup>6</sup>Rigorously speaking, the more the worker is efficient (high  $h(X)$ ) and weights consumption compared to leisure, the more we are likely to be in a situation where a lower elasticity of substitution implies *always* a lower inefficiency of the tax. Otherwise, this is true only if the elasticity of substitution is under a threshold.

<sup>7</sup>We exclude the perfect substitutes case to avoid unrealistic corner solutions.

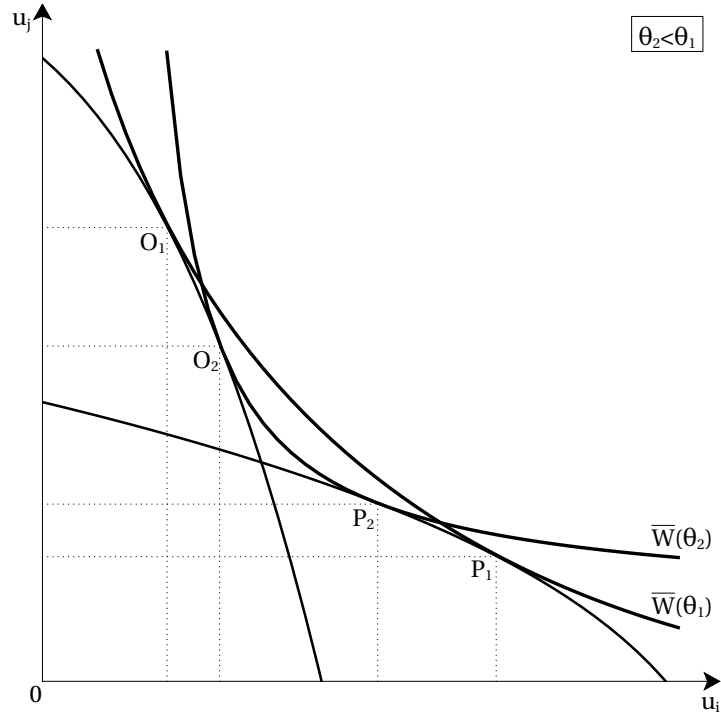


Figure 2: Variation of inequality aversion with two types of frontier

the utility ratio (from  $O_1$  to  $O_2$ ). This implies to reduce the optimal transfer from the worker to the receiver. On the contrary, if the agent  $i$  is relatively better-off at the beginning, the higher the inequality aversion the higher the utility ratio (from  $P_1$  to  $P_2$ ). This implies, thus, to rise the optimal transfer. And finally, if we have a perfect equality between individuals (ratio equals to one), inequality aversion has no influence. Anyways, a higher inequality aversion implies always a more egalitarian situation, but according to the shape of the frontier, a perfect equality may never be obtained.

### 3 Intragenerational and Intergenerational Considerations

From a social planner point of view, we can express the intergenerational consideration through a value function ( $V$ ), non-declining with respect to the resource stock. This minimalist view allows us to simply answer the question: does the *possibilities* of development have increased after the intragenerational redistribution?

Formally, the evolution of the value function over time is (for an autonomous problem);

$$\dot{V}(X) = \underbrace{\frac{\partial V(X,t)}{\partial X}}_v \dot{X}. \quad (10)$$

The value function is increasing over time if the genuine savings is positive (shadow price  $v$

times the evolution of the stock).

We will see first how inequality aversion impacts both the current stock (through the current consumption) and its evolution. Second, we will see how the resource transforms the welfare possibilities.

### 3.1 Inequality Aversion, the Current Consumption and the Evolution of the Stock

#### 3.1.1 Inequality Aversion and the Current Consumption

To understand the consequences of a variation of inequality aversion, say an increase, on the current global consumption, we need to analyze several stages. Let us see it for our two redistribution mechanisms.

**Lump-sum Transfer** Inequality aversion has ambiguous effects on the repartition. From the previous section, we know that we will converge toward a more egalitarian situation. That is, the evolution of the utility ratio depends on the initial situation. If the utility of the agent  $i$  is high compared to the utility of the agent  $j$  in the initial situation, the utility ratio grows with a higher inequality aversion. In this case, the agent  $i$  works more and gets a lower utility, while the agent  $j$  gets a higher utility.

As the global catch increases with work, global consumption rises.

**Tax** This case is much more harder since the reaction to the tax may be ambiguous (contrary to the transfer). Be that as it may, we still restrict ourselves to the case of an increasing labor supply (decreasing with respect to the tax).

Inequality aversion has still ambiguous effect here. According to the initial situation, a higher inequality aversion may advantage one or the other agent. Let us analyze, as before, the case that benefits to the receiver. In this case, the tax rate rises. With our restriction, the agent  $i$  works less and gets a lower utility, while the agent  $j$  gets a higher utility.

As the global catch decreases with leisure, global consumption reduces here.

Whatever the mechanism, if the utilities are equal, as inequality aversion has no influence on the redistribution, it has no influence on the global consumption too. The table 1 summarizes the different situations.<sup>8</sup>

We know that a higher catch will affect negatively the stock. It will also converges to a lower steady state.

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<sup>8</sup>— $d\theta > 0$  describes a higher inequality aversion.

$\text{sign} \left( \frac{dc_{\text{opt}}}{-d\theta} \right)$	$\frac{u_j}{u_i} < 1$	$\frac{u_j}{u_i} = 1$	$\frac{u_j}{u_i} > 1$
Lump-sum	+	0	-
Tax	-	0	+

Table 1: Evolution of the global consumption when inequality aversion rises

### 3.1.2 Inequality Aversion and the Evolution of the Resource Stock

We analyze now the impact of a variation of inequality aversion on the growth rate of the resource. Let us recall that this one is given by the gap between its natural growth and the optimal consumption;  $\dot{X} = \phi(X) - c_{\text{opt}}$ , and let us differentiate it with respect to  $-\theta$ ;

$$-\frac{d\dot{X}}{d\theta} = - \left( \phi'(X) \frac{dX}{d\theta} - \frac{dc_{\text{opt}}}{d\theta} \right) \Leftrightarrow -\frac{d\dot{X}}{d\theta} = -\frac{dc_{\text{opt}}}{d\theta} \left( \frac{\phi'(X)}{\frac{dc_{\text{opt}}}{dX}} - 1 \right). \quad (11)$$

As long as the marginal renewal of the resource is higher than the marginal contribution of the stock to the optimal consumption (which might be expected if the stock is low), the term inside the parentheses is positive. Thus, the evolution of the growth rate has the same sign as the evolution of consumption due to a change of the inequality aversion.

## 3.2 Evolution of the Resource and the Possibilities for Futures Welfares

The current consumption has consequences on the resource stock. And reciprocally, the resource stock has consequences on the possibilities of consumption.

If the resource stock varies, we have to determine beforehand to whom will go that supplement. This should be done according to the social welfare criterion chosen. But, to begin, we are interested in the “deformation” of the frontier due to the variation of the resource stock. Let us see it for our two mechanisms.

### 3.2.1 Lump-sum Transfer

To study the evolution of the frontier due to the evolution of the resource stock, we consider that an agent keeps constant his/her utility, while the other one maximizes his/hers. For example, we may consider the receiver is indifferent whatever the stock. In the case of an increasing stock, the worker has access to more possibility of couple (leisure, consumption). And not surprisingly, the higher the stock the higher the possible utility of the worker (Appendix C). That is to say, the agent *i can* take advantage of a higher resource for every level of the utility of the agent *j* (and vice versa). However, we cannot conclude that everybody will be better-off with

more resource, but that they have the possibility to, since the first-best frontier is expanding.<sup>9</sup>

### 3.2.2 Tax

Due to the (virtual) backward-bending relation between both utilities with the tax, it is easier here to fix the well-being of the worker. To make the worker indifferent when the stock evolves, we have to modify the tax rate. It is not surprising to rise it if the resource grows. Let us do it so as the budget constraint remains the same. Consequently, the receiver gets more. The frontier is therefore always expanding in the second-best approach too.

### 3.2.3 Welfare

As the frontiers are shifting outwards with an increase of the resource stock, the welfare is unambiguously increasing too.<sup>10</sup>

Hence, a variation of the inequality aversion has a direct effect on the current generation through the transfer and an indirect one on future generations through the evolution of the stock.

## 3.3 Can we reconcile intragenerational and intergenerational equity?

Let us recall that an increasing inequality aversion implies a rising of the global consumption in two situations; using a lump-sum when the worker is better-off and using a tax when the receiver is better-off (and vice versa). Is this good for the future? Two cases are to be distinguished here according to the initial stock level compared to the Golden Rule one.

- In an abundant resources world a higher consumption is desirable since every future generations will be able to benefit from it. This is true as long as the Golden Rule level is not overtook.
- If the initial stock is relatively low, the answer is more difficult. Let us note  $X^*$  the steady state stock. If the initial stock is higher than  $X^*$ , a higher consumption is negative for the future since it accelerate the decreasing of the stock, and hence its exhaustion. On the contrary, if the stock is lower than the steady state one, it can be desirable (if the consumption does not overtake the renewal). This can be compared to a catch up of the current generation. This would be dictate by the maximin criterion for example (see Cairns and Martinet, 2014).

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<sup>9</sup>Strictly speaking, as individuals are supposed to live “one instant”, more resources allows more possibilities for their descendants.

<sup>10</sup>At this stage, we were not able to prove that the optimal utilities are effectively increasing. We suppose they are.

## 4 Conclusion

If one type of agent has access to a resource and another does not have access to it, society may want to implement a transfer of a part of the harvest from the first one to the second one. We studied the effects of an absolute transfer and of a relative one based on the resource caught. We constructed utility possibility frontiers in each case. They represent the necessary trade-off society has to make when it wants to make one agent better off at the expense of another. Not surprisingly, the utility possibility set with a tax is lower than the one with a lump-sum transfer. The inefficiency of the tax depends on the preferences of the worker. Besides, the social criterion allowed us to find the social optimum, and we were particularly interested in its variation due to a change in the inequality aversion. This depends on the initial situation of the economy and on the mechanism of transfers used. In particular, we found that society may afford a higher inequality aversion without worsening the future in two situations. Let us say the worker is better-off. First, using a lump-sum that makes the global consumption rises if either the stock is low (catch up) or if it is very large (higher productivity). Second, using a proportional tax that makes the global consumption decreases if the stock is higher than its steady state but lower than the Golden Rule one (a lower decreasing).

Our heterogeneity was intentionally radical; one agent has access to a resource and another does not. Nonetheless it could be interesting to analyze the effects of a transfer between two agents having a different access. The redistribution could have influences on both of them.

## A Assumptions on utilities

- The assumption of concavity needs no justification since its extensive use in the economic literature (note that it implies cardinality<sup>11</sup>).
- The homothecy (only necessary for the agent  $i$ ) is purely for mathematical convenience. Nonetheless many common utility functions have this feature.<sup>12</sup>
- The assumption of interpersonal comparison needs more comments. To explain it, we think that the utility function has to be differently understood. Here, it is not only an individualistic measure of well-being, but an objective evaluation of a ‘legitimate request’. It can be obtained by revealed preferences for example. Further, we think that society is able to say whom is worse-off between to types of individuals. Here, it can be justifiable since natural resources may be inclusive of “primary goods”.

Besides, it should be recalled that avoiding any utility comparison lead to dictatorship according to the Arrow’s classical theorem. And the assumption of comparison permits to compare utilitarianism and leximin (lexicographic maximin) (d’Aspremont and Gevers, 1977).

## B Proof of special cases of CES

It could seem to be a pointless exercise to demonstrate a very well-known result. But, to our knowledge, a rigorous demonstration of the minimum is not present in the literature.

*Proof.* Let us consider a continuously differentiable function with a constant elasticity of substitution ( $\theta$ );  $f(x_1, \dots, x_n) = (\alpha_1 x_1^\rho + \dots + \alpha_n x_n^\rho)^{\frac{1}{\rho}}$ . With  $\sum_{i=1}^n \alpha_i = 1$ ,  $\alpha_i > 0 \forall i$  and  $\rho < 1, \rho \neq 0$ . Recall  $\theta \equiv \frac{1}{1-\rho}$ .

- Case 1: the elasticity tends to positive infinity. Trivially, if  $\theta \rightarrow \infty$  ( $\rho \rightarrow 1$ ), the CES function tends to the perfect substitutes function.

$$\lim_{\rho \rightarrow 1} f(x_1, \dots, x_n) = \alpha_1 x_1 + \dots + \alpha_n x_n. \quad (12)$$

- Case 2: the elasticity tends to zero.<sup>13</sup> Let us rewrite the CES function as:

$$f(x_1, \dots, x_n) = x_k \left( \alpha_1 \left( \frac{x_1}{x_k} \right)^\rho + \dots + \alpha_k + \dots + \alpha_n \left( \frac{x_n}{x_k} \right)^\rho \right)^{\frac{1}{\rho}}. \quad (13)$$

<sup>11</sup>I thank Ludovic Julien for having pointed it to me.

<sup>12</sup>Formally,  $-\frac{dc_i}{dl_i} \Big|_{U(c_i, l_i)} = -\frac{dc_i}{dl_i} \Big|_{U(\beta c_i, \beta l_i)}$ , ( $\beta > 0$ ).

<sup>13</sup>I am grateful to Jean-Baptiste Michau for having provided to me this proof. I also thank Théo Benonnier for having informed me of its existence.



Let  $\min\{x_1, \dots, x_n\} = x_k$ . Then

$$\lim_{\rho \rightarrow -\infty} \left( \frac{x_i}{x_k} \right)^\rho = 0, \forall i. \quad (14)$$

Thus

$$\lim_{\rho \rightarrow -\infty} x_k \left( \alpha_1 \left( \frac{x_1}{x_k} \right)^\rho + \dots + \alpha_k + \dots + \alpha_n \left( \frac{x_n}{x_k} \right)^\rho \right)^{\frac{1}{\rho}} = x_k. \quad (15)$$

As  $x_k$  can be any good;

$$\lim_{\rho \rightarrow -\infty} f(x_1, \dots, x_n) = \min\{x_1, \dots, x_n\}. \quad (16)$$

□

## C Details on the calculations

### C.1 First-Best Frontier

#### C.1.1 Construction of the First-Best Frontier

Figure:

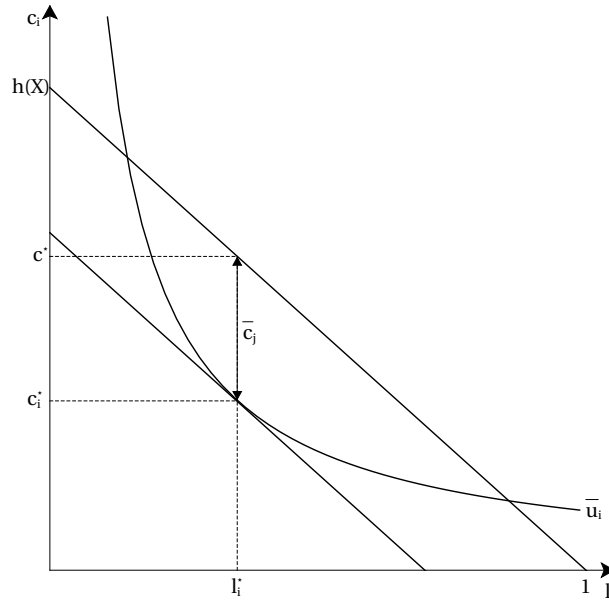


Figure 3: Construction of the utility possibility frontier with lump-sum transfers

Solution:

Substituting the expansion path (eq. (2)) into the budget constraint to get  $l_i^*$  and substituting

$l_i^*$  into the expansion path to get  $c_i^*$ , we have:

$$l_i^* = \frac{h(X) - \bar{c}_j}{h(X) + \Omega(\cdot)} \quad \text{and} \quad c_i^* = \frac{\Omega(\cdot) (h(X) - \bar{c}_j)}{h(X) + \Omega(\cdot)}. \quad (17)$$

Trivially;  $c_j^* = \bar{c}_j$ .

### C.1.2 Shape of the First Best-Frontier

$$\frac{du_i^*}{d\bar{c}_j} = \frac{\partial u_i^*}{\partial l_i} \frac{dl_i^*}{d\bar{c}_j} + \frac{\partial u_i^*}{\partial c_i} \frac{dc_i^*}{d\bar{c}_j} = \underbrace{\frac{\partial u_i^*}{\partial l_i}}_{>0} \underbrace{\left( -\frac{1}{h(X) + \Omega(\cdot)} \right)}_{<0} + \underbrace{\frac{\partial u_i^*}{\partial c_i}}_{>0} \underbrace{\left( -\frac{\Omega(\cdot)}{h(X) + \Omega(\cdot)} \right)}_{<0} < 0. \quad (18)$$

$$\frac{du_j^*}{d\bar{c}_j} = \frac{\partial u_j^*}{\partial c_j} \frac{dc_j^*}{d\bar{c}_j} = \frac{\partial u_j^*}{\partial c_j} > 0. \quad (19)$$

## C.2 Second-Best Frontier

### C.2.1 Construction of the Second-Best Frontier

Figure:

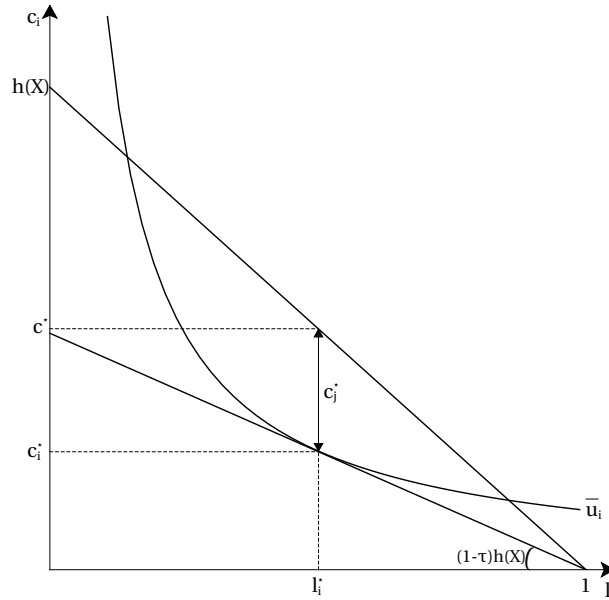


Figure 4: Construction of the utility possibility frontier with a proportional tax

Solution:

Using the same methodology than with the absolute transfer, we find:

$$l_i^{**} = \frac{(1-\tau)h(X)}{(1-\tau)h(X) + \Omega(\cdot)} \quad \text{and} \quad c_i^{**} = \frac{\Omega(\cdot)(1-\tau)h(X)}{(1-\tau)h(X) + \Omega(\cdot)}. \quad (20)$$

Recall that  $c_j = \tau c$  and  $c_i = (1-\tau)c$ , hence  $c_j = \frac{\tau}{1-\tau}c_i$ . We have:

$$c_j^{**} = \frac{\Omega(\cdot)\tau h(X)}{(1-\tau)h(X) + \Omega(\cdot)}. \quad (21)$$

## C.2.2 Shape of the Second-Best Frontier<sup>14</sup>

**Evolution of the Utilities According to the Tax Rate** For the worker:

$$\frac{du_i^{**}}{d\tau} = \underbrace{\frac{\partial u_i^{**}}{\partial l_i}}_{>0} \underbrace{\frac{dl_i^{**}}{d\tau}}_{?} + \underbrace{\frac{\partial u_i^{**}}{\partial c_i}}_{>0} \underbrace{\frac{dc_i^{**}}{d\tau}}_{?}. \quad (22)$$

Actually, since the constraint set is diminishing with the tax, the utility of the worker will reduce.

$$-\left. \frac{dc_i}{dl_i} \right|_{\tau_1} > -\left. \frac{dc_i}{dl_i} \right|_{\tau_2 > \tau_1} \Rightarrow -\frac{dc_i^{**}}{dl_i} > \frac{\frac{\partial u_i^{**}}{\partial l_i}}{\frac{\partial u_i^{**}}{\partial c_i}} \Leftrightarrow \frac{du_i^{**}}{d\tau} < 0. \quad (23)$$

For the receiver:

$$\frac{du_j^{**}}{d\tau} = \underbrace{\frac{\partial u_j^{**}}{\partial c_j}}_{>0} \underbrace{\frac{dc_j^{**}}{d\tau}}_{?}. \quad (24)$$

Recall that  $c_j = \tau c$ . We have:

$$\frac{du_j}{d\tau} = \frac{\partial u_j}{\partial c_j} \frac{dc_j}{d\tau} = \underbrace{\frac{\partial u_j}{\partial c_j}}_{>0} \left( \underbrace{(1-l_i)h(X)}_{>0} - \underbrace{\tau h(X)}_{>0} \underbrace{\frac{dl_i}{d\tau}}_{?} \right). \quad (25)$$

It is straightforward to see that if  $\frac{dl_i}{d\tau} \leq 0$  then  $\frac{du_j^{**}}{d\tau} > 0$ . But we cannot directly conclude.

**Evolution of the Utility of the Receiver When the Tax Rate Varies** The analyze of the sign of  $\frac{dc_j^{**}}{d\tau}$  is meaningless.<sup>15</sup> To obtain an explicable condition, we use the consumption of the

<sup>14</sup>Proofs do not appear in the same order than the results presented in the text, it is therefore advised to read the whole section.

<sup>15</sup>We have  $\frac{dc_j^{**}}{d\tau} > 0 \Leftrightarrow \tau < \frac{h(X) + \Omega((1-\tau)h(X))}{\sigma_{l_i, c_i} h(X)}$ . The sinificance of  $\sigma_{l_i, c_i}$  is given thereafter.

worker from the expansion path (eq. (5)). We have  $c_j(l_i) = \frac{\tau}{1-\tau}c_i(l_i)$ . Then;

$$\begin{aligned}
\frac{dc_j}{d\tau} > 0 &\Leftrightarrow \frac{c_i}{(1-\tau)^2} + \frac{\tau}{1-\tau} \frac{dc_i}{d\tau} > 0 \Leftrightarrow -\tau \frac{dc_i}{d\tau} < \frac{c_i}{1-\tau} \Leftrightarrow -\frac{dc_i}{d\tau} \frac{1-\tau}{c_i} < \frac{1}{\tau} \\
&\Leftrightarrow -\left(\frac{dl_i}{d\tau} \Omega(\cdot) - l_i \cdot h(X) \Omega'(\cdot)\right) \times \frac{1-\tau}{l_i \cdot \Omega(\cdot)} < \frac{1}{\tau} \\
&\Leftrightarrow \frac{1-\tau}{l_i} \frac{dl_i}{d(1-\tau)} + \Omega'(\cdot) \frac{(1-\tau)h(X)}{\Omega(\cdot)} < \frac{1}{\tau}.
\end{aligned} \tag{26}$$

The first term of the left-hand side is the elasticity of leisure with respect to  $(1-\tau)$ . We note it  $\varepsilon_{l_i, (1-\tau)}$ . It can be easily shown<sup>16</sup> that  $\varepsilon_{l_i, (1-\tau)} = -\varepsilon_{l_i, \frac{1}{1-\tau}}$ . So, the first term is the opposite of the cross-price elasticity of leisure.<sup>17</sup> The second term of the left-hand side is equal to the elasticity of substitution of leisure for consumption (evaluated at the optimum), we note it  $\sigma_{l_i, c_i} > 0$ .<sup>18</sup> Thus, we have (the positivity of the denominator is shown later):

$$\frac{du_j^{**}}{d\tau} > 0 \Leftrightarrow \tau < \tilde{\tau} \equiv \frac{1}{\sigma_{l_i, c_i} - \varepsilon_{l_i, \frac{1}{1-\tau}}}. \tag{27}$$

**Links of the Two Parts of the Threshold** As the cross-price elasticity depends on the reaction of the worker to the tax, we can directly link the threshold with the elasticity of substitution. For that, let us compute the cross-price elasticity at the optimum:

$$\begin{aligned}
\varepsilon_{l_i, \frac{1}{1-\tau}} \Big|_{l_i=l_i^{**}} &= -\varepsilon_{l_i, (1-\tau)} \Big|_{l_i=l_i^{**}} \\
&= -\frac{1-\tau}{l_i^{**}} \frac{dl_i^{**}}{d(1-\tau)} \\
&= -\frac{1-\tau}{\frac{(1-\tau)h(X)}{(1-\tau)h(X)+\Omega(\cdot)}} \times \\
&\quad \frac{h(X)((1-\tau)h(X)+\Omega(\cdot)) - (1-\tau)h(X)(h(X)+h(X)\Omega'(\cdot))}{((1-\tau)h(X)+\Omega(\cdot))^2} \\
&= -((1-\tau)h(X)+\Omega(\cdot)) \frac{(1-\tau)h(X)+\Omega(\cdot) - (1-\tau)h(X) - (1-\tau)h(X)\Omega'(\cdot)}{((1-\tau)h(X)+\Omega(\cdot))^2} \\
&= \frac{(1-\tau)h(X)\Omega'(\cdot) - \Omega(\cdot)}{(1-\tau)h(X)+\Omega(\cdot)} \\
&= \left(\frac{(1-\tau)h(X)\Omega'(\cdot)}{\Omega(\cdot)} - 1\right) \frac{\Omega(\cdot)}{(1-\tau)h(X)+\Omega(\cdot)} \\
&= (\sigma_{l_i, c_i} - 1)(1-l_i^{**}).
\end{aligned} \tag{28}$$

<sup>16</sup>  $\frac{d(1-\tau)}{1-\tau} = \frac{1}{1-\tau} \frac{d\left(\frac{1}{1-\tau}\right)}{\frac{1}{d(1-\tau)}} = -\frac{d\left(\frac{1}{1-\tau}\right)}{\frac{1}{1-\tau}}$ .

<sup>17</sup> Note that  $c_i = (1-\tau)(1-l_i)h(X) \Leftrightarrow \frac{1}{1-\tau}c_i = (1-l_i)h(X)$ , so  $\frac{1}{1-\tau}$  is the shadow net price of consumption.

<sup>18</sup> At the optimum:  $\sigma_{l_i, c_i} = \frac{d\Omega(\cdot)}{d\text{MRS}} \frac{\text{MRS}}{\Omega(\cdot)}$ .

What implies (for a non-zero work time):

$$\sigma_{l_i, c_i} \leq 1 \quad \Leftrightarrow \quad \varepsilon_{l_i, \frac{1}{1-\tau}} \Big|_{l_i=l_i^{**}} \leq 0. \quad (29)$$

Let us recall that  $\frac{dl_i}{d\tau} \leq 0 \Rightarrow \frac{du_j^{**}}{d\tau} > 0$ , and note that  $\varepsilon_{l_i, \tau}$  has the same sign as  $\varepsilon_{l_i, \frac{1}{1-\tau}}$ .<sup>19</sup> We have then

$$\sigma_{l_i, c_i} \leq 1 \quad \Rightarrow \quad \frac{du_j^{**}}{d\tau} > 0. \quad (30)$$

Besides, using the equation (28), the threshold of the inequality (27) becomes;

$$\tilde{\tau} \equiv \frac{1}{1 + l_i^{**}(\sigma_{l_i, c_i} - 1)}. \quad (31)$$

It is always positive. We can directly see that it is lower than one if and only if the elasticity of substitution is higher than one.

The table 2 summarizes the results.

$\text{sign} \left( \frac{du_j^{**}}{d\tau} \right)$	$\tilde{\tau} \geq 1$	$\tilde{\tau} < 1$
$\sigma_{l_i, c_i} \leq 1$	+	Impossible
$\sigma_{l_i, c_i} > 1$	Impossible	+/-

Table 2: Evolution of the utility of the receiver when the tax rate rises

### C.3 Comparison of Utility Possibility Sets

**General** We want to compare the two utility possibility sets. Let us do it through their frontier.

For a given utility of the agent  $i$ , let us seek the maximum of amounts one can transfer to the agent  $j$ , whatever the mechanism used. If we note  $\tilde{c}^i(l_i)$  the image of an indifference curve of the agent  $i$ , we have to maximize the gap between the production and that curve.

$$\max_{l_i} c_j = (1 - l_i)h(X) - \tilde{c}^i(l_i). \quad (32)$$

It is not hard to show that a necessary condition of this problem is the equalization of the MRS with the marginal productivity;  $h(X) = \text{MRS}$ . As this is done with the lump-sum transfer, this mechanism is then the most favorable for the agent  $j$ . For a given utility of the agent  $i$ , the first-best frontier corresponds to the highest utility for the agent  $j$ . Thus, the utility possibility

<sup>19</sup> $\text{sign}(\varepsilon_{l_i, \tau}) = -\text{sign}(\varepsilon_{l_i, (1-\tau)}) = -\text{sign}(-\varepsilon_{l_i, \frac{1}{1-\tau}}) = \text{sign}(\varepsilon_{l_i, \frac{1}{1-\tau}})$ .

set with the lump-sum transfer contains the set with the tax. The two frontiers coincide well when no transfer occurs, since net and gross productivity are equivalent.

**Constant Elasticity of Substitution** For a strictly convex indifference curve, the difference between the budget constraint (a straight line) and the indifference curve is strictly concave in  $(l_i, c_i)$ . We know that the optimum is reached at the first-best optimal consumption-leisure ratio;  $\Omega(h(X))$ . So, for a given utility of the worker, the higher is the difference between the first-best and the second-best optimal ratios, the higher is the distance between the two frontiers.

Let us assume here the worker has a constant elasticity of substitution of leisure for consumption<sup>20</sup>;  $u_i(l_i, c_i) = (\gamma \cdot l_i^\rho + (1 - \gamma) \cdot c_i^\rho)^{\frac{1}{\rho}}$ . The function that links the MRS and the optimal consumption-leisure ratio is then of the form;  $\Omega(z) = \left(\frac{1-\gamma}{\gamma}z\right)^\sigma$ .

When the elasticity of substitution evolves, we want to determine the evolution of the difference between the two optimal ratios;  $D(\sigma) \equiv \Omega(h(X)) - \Omega((1 - \tau)h(X))$ . Let us note  $r_1 \equiv \frac{1-\gamma}{\gamma}h(X)$  and  $r_2 \equiv \frac{1-\gamma}{\gamma}(1 - \tau)h(X)$ . We have:  $D(\sigma) \equiv r_1^\sigma - r_2^\sigma$ . Then,  $\frac{dD(\sigma)}{d\sigma} \geq 0 \Leftrightarrow \ln(r_1)r_1^\sigma - \ln(r_2)r_2^\sigma \geq 0$ . There are three cases (with  $\tau > 0$ ):

- $r_1, r_2 \geq 1$ .  $\frac{dD}{d\sigma} \geq 0 \Leftrightarrow \frac{r_1^\sigma}{r_2^\sigma} \geq \frac{\ln(r_2)}{\ln(r_1)}$ . The inequality always holds.
- $r_1 \geq 1, r_2 < 1$ . The inequality always holds.
- $r_1, r_2 < 1$ .  $\frac{dD}{d\sigma} \geq 0 \Leftrightarrow \frac{r_1^\sigma}{r_2^\sigma} \leq \frac{\ln(r_2)}{\ln(r_1)} \Rightarrow \sigma \leq \bar{\sigma} \equiv \frac{\ln\left(\frac{\ln(r_2)}{\ln(r_1)}\right)}{\ln\left(\frac{r_1}{r_2}\right)}$ . The inequality holds as long as the elasticity of substitution is not too high.

Besides,  $\lim_{\sigma \rightarrow 0} D(\sigma) = 0$ .

## C.4 Welfare Analysis

We do it with the lump-sum case, but it would be equivalent to do it with the tax case, substituting  $\bar{c}_j$  for  $\tau$ .

### Welfare Optimization

$$\max_{\bar{c}_j} W^*(\bar{c}_j) = \left( \frac{1}{2} \cdot u_i^*(\bar{c}_j)^\eta + \frac{1}{2} \cdot u_j^*(\bar{c}_j)^\eta \right)^{\frac{1}{\eta}}. \quad (33)$$

---

<sup>20</sup>Given by  $\sigma \equiv \frac{1}{1-\rho}$ .

First-order condition:

$$\begin{aligned}
& \frac{\partial W^*(\bar{c}_j)}{\partial \bar{c}_j} = 0; \\
\Leftrightarrow & \frac{1}{\eta} \left( \frac{1}{2} \cdot u_i^{*\eta} + \frac{1}{2} \cdot u_j^{*\eta} \right)^{\frac{1-\eta}{\eta}} \left( \frac{\eta}{2} u_i^{*\eta-1} u_i^{*'} + \frac{\eta}{2} u_j^{*\eta-1} u_j^{*'} \right) = 0; \\
\Leftrightarrow & u_i^{*\eta-1} u_i^{*'} + u_j^{*\eta-1} u_j^{*'} = 0; \\
\Leftrightarrow & \left( \frac{u_j^*}{u_i^*} \right)^{1-\eta} = -\frac{u_j^{*'}}{u_i^{*'}}; \\
\Leftrightarrow & \frac{u_j^*}{u_i^*} = \left( -\frac{u_j^{*'}}{u_i^{*'}} \right)^\theta. \quad (34)
\end{aligned}$$

The MSRS is by definition the marginal increase of the utility of the agent  $j$  when one decreases marginally the utility of the agent  $i$ , along a social indifference curve;  $\text{MSRS} \equiv \left| -\frac{du_j}{du_i} \right|$ . It makes no difficulties to show that it equals the ratio of social marginal welfare with respect to  $i$  above the one with respect to  $j$ ;  $\text{MSRS} = \frac{\frac{\partial W}{\partial u_i}}{\frac{\partial W}{\partial u_j}}$ .

Besides, let us differentiate the optimal welfare;

$$dW^*(\bar{c}_j) = \frac{\partial W(\cdot)}{\partial u_i} \frac{\partial u_i}{\partial \bar{c}_j} d\bar{c}_j + \frac{\partial W(\cdot)}{\partial u_j} \frac{\partial u_j}{\partial \bar{c}_j} d\bar{c}_j. \quad (35)$$

Along a social indifference curve, the welfare is constant, then:

$$\begin{aligned}
& \frac{\partial W(\cdot)}{\partial u_i} \frac{\partial u_i}{\partial \bar{c}_j} d\bar{c}_j + \frac{\partial W(\cdot)}{\partial u_j} \frac{\partial u_j}{\partial \bar{c}_j} d\bar{c}_j = 0; \\
\Leftrightarrow & \frac{\frac{\partial W}{\partial u_i}}{\frac{\partial W}{\partial u_j}} = -\frac{u_j^{*'}}{u_i^{*'}}. \quad (36)
\end{aligned}$$

Let  $\lambda$  be the utility ratio;  $\lambda \equiv \frac{u_j}{u_i}$ .  $\lambda$  is therefore a function of the optimal MSRS;  $\lambda = \text{MSRS}^{*\theta}$ .

**Variation of inequality aversion** Let us analyze the sign of the differential of the optimal utility ratio with respect to the elasticity of substitution;

$$\frac{d\lambda^*}{d\theta} > 0 \Leftrightarrow \text{MSRS}^{*\theta} \ln(\text{MSRS}^*) > 0 \Leftrightarrow \text{MSRS}^* > 1. \quad (37)$$

Note that a MSRS higher than one corresponds to a  $\lambda$  higher than one too. Then, the optimal utility ratio depends positively on the elasticity of substitution (negatively on inequality aversion) if it is originally higher than one, and vice versa. If the optimal MSRS is equal to one,

the utility ratio is equal to one too. The optimal ratio is then indifferent to inequality aversion in such a case.

## C.5 Evolution of the Resource

### C.5.1 Lump-Sum Transfer

We are considering that the receiver keeps a fixed utility, and we study how evolves the optimal utility of the worker.<sup>21</sup> Here also, we use the consumption of the worker from the expansion path (eq. (5)).

**The receiver keeps a constant utility**

$$\frac{du_j^*}{dX} = 0 \Leftrightarrow \frac{\partial u_j^*}{\partial c_j} \frac{dc_j}{dX} = 0 \Leftrightarrow \frac{\partial u_j^*}{\partial c_j} \left( \underbrace{\frac{dc}{dX}}_{>0} - \frac{dc_i}{dX} \right) = 0 \Leftrightarrow \frac{dc}{dX} = \frac{dc_i}{dX}. \quad (38)$$

Recall that  $c = (1 - l_i)h(X)$  and  $c_i = l_i\Omega(h(X))$ . The previous equation becomes

$$\begin{aligned} -\frac{dl_i}{dX}h(X) + (1 - l_i)h'(X) &= \frac{dl_i}{dX}\Omega(\cdot) + l_i \cdot \Omega'(\cdot)h'(X); \\ \Leftrightarrow \frac{dl_i}{dX} &= \frac{h'(X)(1 - l_i - l_i \cdot \Omega'(\cdot))}{h(X) + \Omega(\cdot)}. \end{aligned} \quad (39)$$

**Evolution of the utility of the worker**

$$\begin{aligned} \frac{du_i^*}{dX} &> 0; \\ \Leftrightarrow \frac{\partial u_i^*}{\partial l_i} \frac{dl_i}{dX} + \frac{\partial u_i^*}{\partial c_i} \frac{dc_i}{dX} &> 0; \\ \Leftrightarrow \frac{\partial u_i^*}{\partial l_i} \frac{dl_i}{dX} + \frac{\partial u_i^*}{\partial c_i} \left( \Omega(\cdot) \frac{dl_i}{dX} + l_i \cdot \Omega'(\cdot)h'(X) \right) &> 0; \\ \Leftrightarrow \frac{dl_i}{dX} &> -\frac{\frac{\partial u_i^*}{\partial c_i} \cdot l_i \cdot \Omega'(\cdot)h'(X)}{\frac{\partial u_i^*}{\partial l_i} + \frac{\partial u_i^*}{\partial c_i}\Omega(\cdot)}. \end{aligned} \quad (40)$$

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<sup>21</sup>Note that doing the converse would be equivalent.



Substituting the eq. (39) into the eq. (40)

$$\begin{aligned}
&\Rightarrow \frac{h'(X)(1 - l_i - l_i \cdot \Omega'(\cdot))}{h(X) + \Omega(\cdot)} > -\frac{\frac{\partial u_i^*}{\partial c_i} \cdot l_i \cdot \Omega'(\cdot) h'(X)}{\frac{\partial u_i^*}{\partial l_i} + \frac{\partial u_i^*}{\partial c_i} \Omega(\cdot)}; \\
&\Leftrightarrow \frac{1 - l_i}{l_i} > \Omega'(\cdot) \left( 1 - \frac{\frac{\partial u_i^*}{\partial c_i} (h(X) + \Omega(\cdot))}{\frac{\partial u_i^*}{\partial l_i} + \frac{\partial u_i^*}{\partial c_i} \Omega(\cdot)} \right). \tag{41}
\end{aligned}$$

Note that at the optimum;  $\frac{\frac{\partial u_i}{\partial l_i}}{\frac{\partial u_i}{\partial c_i}} = h(X)$ . Thus,

$$\Leftrightarrow \frac{1 - l_i}{l_i} > \Omega'(\cdot) \left( 1 - \frac{h(X) + \Omega(\cdot)}{h(X) + \Omega(\cdot)} \right). \tag{42}$$

In the end,

$$\left. \frac{du_i^*}{dX} \right|_{u_j^* = \bar{u}_j} > 0 \quad \Leftrightarrow \quad 0 < l_i < 1. \tag{43}$$

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