

The Impact of Subsidy Bidding Wars on the Optimal Investment Decisions of Multi-Establishment Firms

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Abstract: This paper studies the competition between regions to attract a firm's investment. An important takeaway from earlier papers is that such bidding wars can improve welfare by allocating new plants to the regions that value them the most. By explicitly modeling endogenous investment choice, we show that the firm strategically chooses an investment allocation different from the profit-maximising allocation. Specifically, the firm invests more and differentiates the plants, in turn increasing subsidies. Despite these distortions, such a bidding war retains the welfare-maximising properties of simpler models. In addition, it implements the optimal mechanism from the viewpoint of the firm.

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1 Introduction

In 2014, automotive company Tesla selected Nevada as the location for a “gigafactory” to build batteries for its electric cars, after that state offered a record-breaking \$1.5 billion in subsidies and incentives. The selection process involved a bidding war between counties across several states in the US, during which CEO Elon Musk stated that the company was considering selecting multiple production sites, even breaking ground in more than one location during the selection process. Tax incentives such as this one represent an appreciable amount of government spending each year. In the United States alone, state and local governments award approximately \$80 billion in tax incentives each year to companies.¹

Furthermore, these subsidies are often the result of bidding wars between many local or regional governments. Some see these bidding wars as wasteful, but they can also play an important role in eliciting private information and improving allocation efficiency (Menezes, 2003). In fact, despite paying subsidies to the firm, the winning region may actually benefit from the presence of the new plant. Greenstone and Moretti (2003), for example, compare the outcomes for winning and losing counties in contests for “million dollar plants”, and find that winning counties experience greater increases in land value as well as in the total wage bill of other firms in the industry of the new plant.

While the relatively overt stance of Tesla on the rules of the bidding war is rather unusual, especially on the possibility of multiple establishments,² this example highlights some important characteristics of firms vying for subsidies to build new plants. First, the firms running these bidding wars are frequently multi-establishment companies. For example, between 2007 and 2012, Boeing received at least \$327 million in incentives from 11 US states for multiple plants. In the same period, Procter & Gamble received at least \$128 million from 10 states.³ Put differently, firms make multiple investments in short periods of time, such that the bidding wars for these plants might interact with each other. Second, the firm controls much of the process in these bidding wars, setting the rules and retaining much of the bargaining power. Regional governments, on the other hand, can only decide whether to take part in these bidding wars or not, in which case their offers could be accepted, or not.

¹The New York Times, “As Companies Seek Tax Deals, Governments Pay High Price,” available at the following address: <http://www.nytimes.com/2012/12/02/us/how-local-taxpayers-bankroll-corporations.html>.

²However, at the time of the writing of this paper, Tesla ended up opening only one such “gigafactory” in the United States.

³Other examples are available from the New York Times, at the following URL: <http://www.nytimes.com/interactive/2012/12/01/us/government-incentives.html>

Owing to their prominence, economists have previously studied the behaviour of firms and governments participating in these location contests. However, they have generally considered a single firm opening a single establishment of an exogenously given size. Is the assumption of an exogenous amount of capital restrictive? If the firm can choose a level of capital to invest, she may be able to do so strategically, thereby increasing her total profits. In this paper, our objective is to investigate this strategic behaviour, and its effect on the amount of capital invested and on the welfare properties of bidding wars. The focus is thus mostly on the firm's strategic behaviour, instead of the governments'.

Generally, one way to study these bidding wars is to use models from auction theory (e.g., King et al., 1993). As a simple example, take a firm that wants to build a new plant by investing an exogenous amount of capital K in one of n regions. Regions receive a payoff from hosting the firm that depends on some privately-known value placed on the firm by that region. The firm chooses where to invest by conducting a simple open ascending auction. Regional governments make subsidy offers to the firm, and the winning region is the one offering the highest subsidy package. Such a bidding war has interesting properties in terms of social welfare. First, the firm is able to allocate its plant to the region that values it the most (as argued in Menezes, 2003). In contrast, without a bidding war, the firm is unable to gather information on the regions, and is thus likely to locate the plant in a region with a lower valuation. Second, even though these subsidies can be costly, through the social cost of public funds, if this cost is not too high, it is compensated by the increase in welfare coming from the location of the plant in the "correct" region.

In our model, we relax the assumption of fixed capital in two steps. First, we allow the firm to choose an endogenous level of capital to invest. Our first main finding is that the bidding war incites the firm to invest a larger amount of capital than she would without a bidding war. In other words, the amount of capital invested is larger than the level dictated by simple profit maximisation. These lower operating profits are, however, compensated by increased subsidies. In terms of welfare, the conclusion is straightforward: the plant is allocated to the region that values it the most, just as in the fixed capital case. While subsidies are larger due to the higher level of capital invested, the region also enjoys greater benefits from hosting a larger plant.

Second, we allow the firm to invest in more than one location, essentially making multiple plants available in the bidding war. Formally, we model this competition as a multi-unit open ascending auction. With this model, our second main finding is that the firm can allocate

investment across its production sites strategically, in order to increase the subsidies she receives from regional governments. The firm invests more than it would without a bidding war (as in the single-plant model), but also differentiates the plants, investing more in one than the other. That differentiation allows the firm to attract larger total subsidies. By taking into account the linkages between the bidding wars, our model captures the infra-marginal competition between the last two remaining bidding regions for the largest plant in the auction.⁴

Our paper also considers the implications of the firm's strategic choice of capital on social welfare. To do so, we first sophisticate our model to find the firm's optimal mechanism to "sell" her plants. We find that this optimal mechanism would allocate the plants to the same winners, for the same subsidies, as the open ascending auction, under some restrictions on the reserve subsidies. Then, we solve a similar model, but from the point of view of a social planner. We find that a social planner would choose an optimal mechanism with the same allocation and payment rules as the firm. However, the social planner would allocate the plants more often; the conditions for reserve subsidies to be non-binding in the social planner's problem are less restrictive. In conclusion, even if the multi-plant bidding war does distort the investment behaviour of the firm, our paper shows that the desirable welfare properties of such bidding wars persist despite the distortions, at least if the value put on the firm's plants by regional governments corresponds to the social value of hosting the firm.

This paper contributes to the literature on fiscal competition, and, more particularly, to the subset of papers that consider competition for a single large firm. Keen and Konrad (2014) offer a short overview of this literature, which includes early contributions by, e.g., Black and Hoyt (1989), Doyle and van Wijnbergen (1994), and King *et al.* (1993). This is in contrast to the larger stream of that literature that considers the competition between regions or countries to attract units of homogeneous and perfectly divisible capital. Wilson (1999) and Keen and Konrad (2014) offer extensive surveys of these models. Moreover, in contrast to many of these papers, we are primarily interested in how bidding wars affect the strategy and the behaviour of the firm, instead of governments.

Many authors investigating bidding wars for a single large firm use models similar to auctions. Indeed, auctions are a useful tool for sellers who do not know the value potential buyers

⁴Cowie *et al.* (2007) previously considered infra-marginal competition in the context of an auction. They analyse how a seller can divide the units for sale in multiple lots in order to receive higher offers from the bidders. They find that differentiating the lots can lead to higher bids due to the infra-marginal competition for the largest lot. We have a similar reasoning in the auction stage of our model.

place on the product sold. Moreover, as suggested by Klemperer (2004, Chapter 2), auction theory can also provide a rich set of tools to study a number of problems in economics and social sciences. Location contests are a good example of a context in which auctions are a useful theoretical tool; many bidders (governments) place some private value on a good (investment), and a seller (the firm) does not know how to price it, thus choosing to accept bids (subsidies).

Our model is particularly related to the analysis of Haaparanta (1996), who uses a menu auction model. This author considers two regions competing for investment from a firm, under the assumption that this investment is divisible. However, while Haaparanta (1996) considers a model under perfect information, we instead assume that the regions' private benefits from hosting the firm are private knowledge. In fact, such information asymmetry is a justification to use a mechanism similar to an auction in the first place. As the model will show formally, analysing the question under information asymmetry will reveal new insights about the bidding war and the allocation of investment. First, when establishments are asymmetric, infra-marginal competition takes place between the last two remaining bidders, increasing the subsidy on the large plant, and allowing the firm to benefit from higher total subsidies. Second, in contrast to Haaparanta (1996), who finds that the firm captures the whole rent from the regions, in our model the information asymmetry curbs the firm's ability to extract rents from the regions.

Other closely related papers include Black and Hoyt (1989) who were, to our knowledge, the first to explicitly model the firm's location choice as an auction. Before that, Doyle and van Wijnbergen (1994), in a paper first published in 1984, considered a bargaining game between one firm and a government over taxation. Doyle and van Wijnbergen (1994) assumed that firms negotiate with a single government at a time. However, firms have no reason not to negotiate simultaneously with multiple governments. Recognising this fact, Bond and Samuelson (1986) investigate a situation in which a firm has to decide between two locations. An important feature of their model is information asymmetry. Similarly to Bond and Samuelson (1986), information asymmetry is an important of our model, although our results are derived without productivity differences between the regions.

King and Welling (1992) explore the consequences of allowing the firm to relocate in later periods. They consider a two-period model, in which the firm conducts an auction to decide on its location in each period.⁵ King, McAfee and Welling (1993) generalise the model of King and

⁵In this paper, we do not consider a two-period model. The firm installs new production facilities in one period, but we do not model the interactions in the following periods. We do so deliberately, to focus instead on how the firm decides to allocate across regions in multiple establishments in a single period. If we did consider many

Welling (1992), but with a continuum of local productivities. They also consider an extension in which regions can invest in infrastructure in a previous stage, thus increasing their productivity potential. King et al. (1993) assume some information asymmetry, but it is the firm who does not know its productivity in each region. In this paper, we instead assume that the regions hold some private information, while productivity is the same everywhere. This modelling choice reflect the fact that not all regions value the firm's presence identically.

Another closely related paper is that of Martin (1999). This author studies two firms in the same industry who use bidding wars sequentially to decide where to locate. Martin (1999) shows that agglomeration effects incite regions to overbid in the first auction, expecting it will increase their probability of winning in the second period. Indeed, winning the investment in the first period from the first firm increases the attractiveness of the region to other firms in the same industry. In this paper, we also find that regions offer greater subsidies for one plant. However, we consider how a single firm can entice greater subsidies by modifying her allocation of production between two plants. In addition, we do so without considering agglomeration economies. Finally, other more recent papers include Martin (2000); Scoones (2001); Furusawa, Hori and Wooton (2010); and Ferrett and Wooton (2010).

The next section presents the framework of the model, including the timing of the game. Section 3 presents the results of a benchmark model restricted to one plant, while Section 4 solves the three stages of the full model and discusses its implications on the firm's investment choices and the bidding behaviour of the regional governments. Section 5 derives the optimal mechanism from the viewpoint of the firm, with different assumptions of the firm's commitment. Section 6 discusses the welfare conclusions from our model. The last section concludes.

2 The Model

Consider a firm that plans to build new production facilities in some of n regions, indexed by $i \in 1, \dots, n$. To decide the location of these plants, the firm puts the n regional governments in competition against each other. The governments submit offers of subsidies to attract the firm to their territory. In contrast to most of the previous literature, however, the firm can divide

periods, our results could be related to those of Janeba (2000), for example, who considers a firm that installs excess production capacity in multiple regions in order to avoid the problem of hold-up by the regions. Indeed, in subsequent periods, regions could increase taxes or renege on their commitment to tax breaks (i.e., subsidies). By having excess capacity, the firm could credibly threaten to decrease production, and thus employment in the region that increased taxes, to increase it in the other.

her production in multiple locations, either in symmetric or asymmetric establishments. For simplicity and tractability, we limit the model to the case of two establishments, indexed by $j \in 1, 2$.⁶ Without loss of generality, we label the largest plant by $j = 1$, so that $K_1 \geq K_2$.

2.1 The Firm

We consider a multinational firm that already produces elsewhere, and wants to increase production by installing new establishments among the n regions. Once she decided where to install the new plants, the firm produces, in each establishment, according to the production function $f(K_j, L_j)$, with K_j the capital invested in location j , and L_j the labour employed in that establishment. We make the usual assumptions that the production function exhibits decreasing returns to scale in both inputs ($\frac{\partial f(K_j, L_j)}{\partial K_i} > 0$, $\frac{\partial f(K_j, L_j)}{\partial L_i} > 0$ and $\frac{\partial^2 f(K_j, L_j)}{\partial K_i^2} < 0$, $\frac{\partial^2 f(K_j, L_j)}{\partial L_i^2} < 0$).⁷ The firm sells the product on a global market for a price p , acting as a price-taker. We deliberately do not model the goods market explicitly, to instead focus on the firm's location decision and the bidding war between regions. The production costs are identical in every region (w, r). Therefore, the firm's operating profits in each establishment $j = 1, 2$ are equal to

$$\pi_j = pf(K_j, L_j) - wL_j - rK_j \quad (2.1)$$

In addition to the profits from production, the firm also receives subsidies from the regions (resulting from the bidding war), so that her total *ex post* profits are equal to:

$$\Pi = s_1^* + s_2^* + \pi_1 + \pi_2 \quad (2.2)$$

where s_j^* is the equilibrium subsidy for establishment j .

⁶As will be evident following the exposition of the model, without restrictions on the number of plants, the firm would prefer operating an infinity of equally-sized establishments. Such organisation of production is obviously impractical, thus we limit the model to a finite number of establishments. We choose to focus on 2 plants to keep the model mathematically tractable, but the main mechanisms we wish to highlight in this paper would also be present with larger numbers of establishments.

⁷This assumption implies, in the model, that the firm has incentives to produce in more than one establishment.

2.2 The Regions

These subsidies depend on the regions' valuation of the firm's investments. In particular, if regional government i wins establishment j , it receives a payoff equal to

$$V_{ij} = L_j \cdot b_i - s_{ij} \quad (2.3)$$

where L_j is the number of persons employed by the firm in establishment j , b_i is the level of private benefits from hosting the firm for region i 's government, and s_{ij} is the subsidy (bid) offered to the firm by region i when winning establishment j . The subsidy can be interpreted as a total "fiscal package" offered to the firm.⁸

A region's private benefits b_i are private knowledge, and they capture, for example,⁹ an increase in labour taxation from workers who will be employed by the firm, as well as spillovers to domestic firms, but also the affinity between the firm and the region. Indeed, if the industry of the firm has a bad reputation in one region, the regional government would put only a small value on the firm's investment (due to, for example, re-election concerns).¹⁰ The private benefits are identically and independently distributed according to a distribution $g(\cdot)$ on some interval $[\underline{b}, \bar{b}]$ (with $\underline{b} \geq 0$).

2.3 The Auction Process

The equilibrium subsidies are then determined by an auction in which the firm takes the role of the auctioneer, and the regional governments submit their bids to host the firm's plants. Since there are two establishments available, the firm conducts a multi-unit auction, with both establishments available simultaneously.

The formal mechanism is an open ascending auction. More specifically, the firm runs an ascending clock, representing the current price for the lowest-value establishment still available (the one with the lowest investment). Regional governments still in the running are ready to offer a bid equal to the current price. The winning bid is determined from the price on the clock when the previous bidder withdrew from the auction. In particular, if the two establishments

⁸In effect, our model assumes that all regional governments have the same basic tax rate, but differentiate themselves with targeted tax holidays that may differ. This assumption may not be unreasonable in the case of sub-national jurisdictions. Even when considering countries, we are mostly interested in the competition taking place in subsidies, and abstracting from tax competition allows us to focus on our variables of interest.

⁹Ferrett and Wooton (2013) use a similar justification for private benefits, while Martin (2000:6) provides a more thorough list of potential explanation for these benefits.

¹⁰More generally, Buts, Jegers, and Jottier (2012) find that subsidies to firms increase support for incumbent politicians.

are still available, then when there are only two regions left bidding, the price for the lowest-valued establishment will be determined from the clock price at which the third-to-last region withdrew from the auction.¹¹ These two remaining regions will then continue bidding until one of them exits. The clock price at which the second-to-last region withdrew will be the price for the highest-valued establishment. Formally, this mechanism is a type of second-price auction.

2.4 Timing

We can summarize the timing of the whole game as follows.

Stage 0: Nature picks the set of $\{b_i\}_{i=1,\dots,n}$. Regional governments learn their b_i .

Stage 1: The firm chooses and commits to an allocation of capital (K_1, K_2) , anticipating the subsidies offered by governments resulting from the auction in Stage 2, and the firm's own profit maximization in the last stage.

Stage 2: The multi-unit auction takes place. Winning regions offer s_1^* and s_2^* , based on their expectation of the labour that will be employed by the firm (from profit maximization in the last stage).

Stage 3: The firm invests capital K_1 and K_2 , as determined in Stage 1, in the winning regions. She then maximizes her profits, taking capital fixed, choosing L_1 and L_2 .

In the first stage, the firm commits to a certain allocation of capital. One could reasonably argue that the firm has incentives to deviate from that allocation once she receives the subsidies from the region. However, in that case, regions would anticipate these deviations and bid accordingly. To facilitate the analysis, we make the assumption that the firm can credibly commit to her allocation.

3 A Simple Benchmark: Endogenous Capital in a Single-Plant Bidding War

The model in this paper makes two additions to the usual discussion on bidding wars. First, the firm can choose the amount of capital to invest endogenously. Second, we consider the possibility for the firm to make multiple new investments. In this section, we provide a benchmark by

¹¹Note that regions who withdraw from the auction without winning one establishment do not pay anything.

investigating the endogenous investment decision of the firm when the decision is restricted to one plant. To that end, we restrict the model described above to only one new establishment. The set-up of the model is identical, except that the firm only chooses $K_1 = K_s$.

In this restricted model, the solution in Stage 3 is simple. The firm maximizes profits in her plant by choosing L , with a fixed K since it is chosen in Stage 1. Her maximization problem in the plant is as follows.

$$\max_L \quad pf(K^*, L) - wL - rK^* \quad (3.1)$$

The first-order condition is

$$pf'(K^*, L) - w = 0$$

This condition is standard, and defines the optimal choice of L given the amount of capital invested in the earlier stages. We define the function $L(K)$, determining the amount of labour employed for each possible equilibrium level of capital invested in the first stage.

At the auction stage, the equilibrium winning bid will be

$$s^*(K) = L(K_s) \cdot b_{(2)} \quad (3.2)$$

where $b_{(2)}$ is the second-highest private benefits among the n competing regions (i.e., the second-order statistic).

In the first stage, then, the firm's optimisation problem is the following:

$$\max_K \Pi = E(\pi(K) + s^*) = E \left[pf(K, L(K)) - wL(K) - rK + L(K) \cdot b_{(2)} \right] \quad (3.3)$$

where π and s^* are, respectively, the operating profits of the firm's plant and the equilibrium subsidy. The result of that problem leads to the following lemma.

Lemma 1. *A single bidding war for a new plant increases the firm's investment compared to a situation without a bidding war.*

Proof. The first-order condition is:

$$\begin{aligned} \frac{\partial E(\Pi)}{\partial K} &= L'(K)E(b_{(2)}) + p \left(\frac{\partial f(K, L(K))}{\partial K} + \frac{\partial f(K, L(K))}{\partial L(K)} \cdot L'(K) \right) - wL'(K) - r = 0 \\ \frac{\partial E(\Pi)}{\partial K} &= L'(K)E(b_{(2)}) + p \frac{\partial f(K, L(K))}{\partial K} - wL'(K) - r = 0 \end{aligned}$$

It simplifies to

$$p \frac{\partial f(K, L(K))}{\partial K} = r + L'(K)(w - E(b_{(2)})) \quad (3.4)$$

This first-order condition implies that when using a bidding war, the firm chooses to invest an amount of capital K_s greater than would be invested without a bidding war. Indeed, the subsidies received effectively reduce the cost for the firm's labour ($w - E(b_{(2)}) < w$). \square

Lemma 1 shows that a single bidding war with endogenous capital already distorts the firm's investment decision, by increasing capital invested compared to a situation without a bidding war. In fact, the firm sacrifices some operating profits, but gains a larger amount in term of subsidies.

4 The Full Model: Equilibrium Subsidies and Firm Location Choice

The second question in this paper is whether the assumption that the firm conducts two bidding wars further modifies the allocation of investment across establishments. A crude way to model such a multi-plant contest would be to simply repeat the single-plant bidding war. The firm essentially conducts one auction, then a second one, with regions acting as though the contests are independent of each other. With this simple modelling choice, the firm would simply invest a larger amount in both plants (i.e., the firm makes a similar decision for both plants).

However, these assumptions are unrealistic. When participating in this repeated bidding war, regional governments expect that the firm will have multiple plants available. This section investigates the complete model as described in Section 2, which takes into account the linkages between the bidding wars. We solve it by backwards induction.

4.1 Stage 3: Production

We first solve the last stage of the game, to find the firm's optimal labour input demand in each firm for each level of capital invested. At this stage of the game, the firm already knows the identity of the winning regions, and invests the capital in these two regions as determined in the first stage. She also knows how the amount of the subsidies conditional on the amount of labour she will employ.

The firm thus maximizes her profits in each plant, choosing L . At this last stage of the

game, the firm already decided on (K_1, K_2) , so it is fixed. Her maximization problem in each plant is thus as follows.

$$\max_{L_j} \quad pf(K_j^*, L_j) - wL_j - rK_j^* \quad (4.1)$$

The first-order condition is

$$pf'(K_j^*, L_j) - w = 0$$

implying that the firm chooses L_j to equalize the marginal product that input, $f'(K_j^*, L_j)$, with the ratio of w and p . Therefore, the optimal L_j will depend on the amount of capital invested, K_j^* . We define the function $L(K_j)$, determining the amount of labour employed for each possible equilibrium level of capital invested in the first stage.

Since the regions' valuation depends on the amount of labour employed, we want to know how L varies with K . By totally differentiating the first-order condition, we can obtain the sign of $\frac{dL}{dK}$:

$$\frac{dL}{dK} = -\frac{\frac{\partial^2 f(K^*, L^*)}{\partial K^2}}{\frac{\partial^2 f(K^*, L^*)}{\partial K \partial L}} > 0$$

This derivative is greater than zero as long as the cross partial derivatives in K and L are positive (e.g., increasing capital increases the marginal product of labour). Therefore, a greater investment by the firm in an establishment translates into a greater valuation of that establishment by the regions.

As an example, take a simple Cobb-Douglas production function $f(K, L) = K^\alpha L^\beta$ with $\alpha + \beta < 1$. At that stage, K is fixed in each establishment and the firm already received the subsidies. Therefore, the firm chooses L in each plant to maximise her operating profits. In that case, for each level of K , she chooses an optimal amount of labour L equal to

$$L(K) = \left(\frac{p\beta}{w}\right)^{\frac{1}{1-\beta}} K^{\alpha/(1-\beta)} \quad (4.2)$$

In this example, larger investments by the firm translate in more labour employed ($L'(K) > 0$), but at a decreasing rate ($L''(K) < 0$).

4.2 Stage 2: Auction and Equilibrium Subsidies

In the auction stage, the firm puts up two plants for sale of sizes K_1 and K_2 . The regional governments expect the firm to employ $L(K_1)$ and $L(K_2)$, respectively, and bid according to their valuation functions V_{ij} . More specifically, regions decide when to withdraw from the

auction, in which the current price is indicated on an ascending clock. The following lemma describes the equilibrium subsidies resulting from the auction.

Lemma 2. *The equilibrium bids for the two establishments will be equal to*

$$s_2^*(K_1, K_2) = L^*(K_2) \cdot b_{(3)} \quad (4.3)$$

$$s_1^*(K_1, K_2) = (L^*(K_1) - L^*(K_2))b_{(2)} + L^*(K_2)b_{(3)} \quad (4.4)$$

where $b_{(z)}$ is the z^{th} -highest signal among the n regions.

Proof. See Appendix. □

First note that, as expected, if $K_1 = K_2$, then $s_1^*(K_1, K_2) = s_2^*(K_1, K_2)$. In the more interesting case of asymmetric establishments, the value of $s_1^*(K_1, K_2)$ is determined through infra-marginal competition between the last two remaining bidders. At that stage of the auction, both regional governments still standing are guaranteed to win one of the establishments. Therefore, the decision is based on a comparison of the payoffs from winning each plant, and paying the corresponding subsidy (i.e., $L(K_1)b_{(2)} - s_1^*(K_1, K_2) = L(K_2)b_{(2)} - s_2^*(K_1, K_2)$). Notably, increasing K_2 has an ambiguous effect on the total subsidies received by the firm. On the one hand, it increases the value of the smaller plant, so the corresponding subsidy increases. On the other hand, it increases the opportunity cost to the winner of the large plant, thus reducing the subsidy for the largest plant.

4.3 Stage 1: The Firm's Optimal Location Choice

In the first stage, the firm's optimisation problem is the following:

$$\max_{K_1, K_2} E(s_1^* + s_2^* + \pi_1 + \pi_2) \quad (4.5)$$

where $\pi_j = pf(K_j, L_j) - wL_j - rK_j$ and s_j^* are, respectively, the operating profits in each establishment and the equilibrium subsidies as determined in Lemma 2.1. The firm thus chooses K_1 and K_2 to maximise her total expected profits, anticipating the bids of the regions, as well as her profit maximisation in the last stage. The solution to this optimisation problem leads to the following proposition.

Proposition 1. *When the firm allocates her production units through a multi-unit auction, she always chooses to differentiate the two establishments ($K_1 \neq K_2$).*

Proof. See Appendix. □

We can rearrange the first-order conditions for profit maximisation as such:

$$p \frac{\partial f(K_1, L(K_1))}{\partial K_1} = L'(K_1)(w - E(b_{(2)})) + r \quad (4.6)$$

$$p \frac{\partial f(K_2, L(K_2))}{\partial K_2} = L'(K_2)(w + E(b_{(2)}) - 2E(b_{(3)})) + r \quad (4.7)$$

This formulation is informative of the trade-offs at play. In each establishment, the firm's choice of K_j reflects the usual trade-off of marginal revenues and marginal costs. However, the marginal cost of labour is not simply equal to the wages paid. In fact, the firm receives subsidies that depend on the level of employment, effectively lowering the firm's marginal labour costs. Therefore, when the firm increases K_j , her labour costs increase not simply by $L'(K_j) \cdot w$, but by an amount with wages "adjusted" by the marginal subsidies. Denoting total equilibrium subsidies by s_t^* , we find that $\frac{\partial s_t^*}{\partial K_1} = E(b_{(2)})$ and $\frac{\partial s_t^*}{\partial K_2} = -E(b_{(2)}) + 2E(b_{(3)})$.

An increase in K_1 has an unambiguous effect: it increases subsidies for the largest plant, and thus reduces the "adjusted" marginal cost for that plant. An increase in K_2 , however, could increase or decrease the "adjusted" marginal cost for the smaller plant, for the reasons outlined in the discussion on subsidies. Indeed, while increasing K_2 increases the value of the small plant, and thus the corresponding subsidy, it also increases the foregone value for the winner of the largest plant.

Having solved all the stages of the game, we can describe the sub-game perfect Nash equilibrium. In it, the firm commits in Stage 1 to (K_1^*, K_2^*) , defined by the first-order conditions (4.6) and (4.7). In Stage 2, the regions bid until the price on the clock passes their valuation. The region with the highest private benefits wins the largest establishment and offers subsidies of $s_1^*(K_1, K_2) = (L^*(K_1) - L^*(K_2))b_{(2)} + L^*(K_2)b_{(3)}$. The region with the second-highest private benefits wins the smaller establishment, paying subsidies equal to $s_2^*(K_1, K_2) = L^*(K_2) \cdot b_{(3)}$. In Stage 3, the firm invests the amounts (K_1^*, K_2^*) , employs labour $L(K_j)$ in each establishment j , and produces according to $f(\cdot)$.

4.4 Equilibrium Amount of Investment: Bidding War vs. No Bidding War

For comparison purposes, without a bidding war, the firm chooses to invest an equal amount of capital in two random regions. Indeed, the firm's revenues are then simply equal to $\pi_1(K_1) +$

$\pi_2(K_2)$. The first-order conditions are

$$p \cdot \frac{\partial f(K_1, L(K_1))}{\partial K_1} = wL'(K_1) + r$$

$$p \cdot \frac{\partial f(K_2, L(K_2))}{\partial K_2} = wL'(K_2) + r$$

Put differently, the firm's optimal allocation in this case simply results from equating marginal revenues and marginal costs in each establishment. The assumptions on the production function imply that the firm chooses an identical investment in both plants: K_{nbw} .

Since the firm has no information about the private benefits of the regions, and since regions are identical in terms of productive capacity, the firm chooses to invest an equal amount K_{nbw} in two regions. She can just choose two regions at random, since her production costs and profits will be identical with any set of two regions.

This comparison begs the question whether the firm invests more in total when allocating through a bidding war than when she randomly chooses two regions to invest in. Intuitively, one might suspect that the firm always chooses a larger K_1 when using a bidding war. We prove this intermediary result in the following lemma.

Lemma 3. *The capital investment in the first establishment (K_1) is always greater under a bidding war than without a bidding war.*

Proof. As long as $E(b_{(2)}) > 0$, adjusted wages (the firm pays wages w , but the subsidy effectively lowers them) are lower than w . Indeed, $w > w - E(b_{(2)})$. Therefore, $K_1^* > K_{nbw}$. \square

The intuition is less clear in the case of the second establishment. Indeed, $E(b_{(2)}) - 2E(b_{(3)})$ could be greater or smaller than zero, depending on the distribution of the private benefits. In turn, investment could be lower or higher than without a bidding war. With a uniform distribution, it is easy to see that K_2^* will be greater than (with $n > 3$) or equal to (with $n = 3$) K_{nbw} . In the following lemma, we prove that the opposite is possible.

Lemma 4. *There exists some distribution of private benefits for which the firm invests less in the second establishment under a bidding war than without a bidding war.*

Proof. We prove this lemma by constructing an example. Take the following cumulative distribution function: $G(b) = b^{1/3}$ on the interval $[0, 1]$. With such a distribution, $E(b_{(2)}) = \frac{n(n-1)}{(n+2)(n+3)}$

and $E(b_{(3)}) = \frac{n(n-1)(n-2)n!}{(n+3)!}$. Consequently, $E(b_{(2)}) - 2E(b_{(3)}) > 0$ if and only if:

$$\frac{n+1}{n-2} > 2$$

$$n < 5$$

For this distribution function, if $n < 5$, we have $w + E(b_{(2)}) - 2E(b_{(3)}) > w$, and the firm has larger effective marginal labour costs in the second establishment than she would under a situation with no bidding war. Consequently, she chooses a level of K_2^* lower than the no-bidding-war amount ($K_2^* < K_{nbw}$). \square

This distribution function is strongly skewed to the right, giving more weight to values closer to zero. Therefore, for low values of n , $b_{(3)}$ is sufficiently close to the lower bound, and thus smaller than $b_{(2)}$, for the wage adjustment to be positive. In economic terms, such a distribution would translate in a situation where one or few regions put a great value on the firm's presence, while the great majority of regions put little to no value. In such a case, the firm might be able to extract a large subsidy from one government, but the differentiation comes at the cost of lower production in, and lower subsidies for, the second plant.

We are ultimately interested in the comparison of $K_1^* + K_2^*$ and $2 \cdot K_{nbw}$. From Lemma 3, we know that $K_1^* > K_{nbw}$, so for total investment to be lower under a bidding war, it is necessary that $K_2^* < K_{nbw}$ by an amount large enough to counter-balance the increase in the first establishment.

Proposition 2. *Under conditions on the relative concavity of the production ($f(\cdot)$) and labour demand ($L(\cdot)$) functions, the total amount invested by the firm under a bidding war is always larger than the amount she would invest without a bidding war.*

Proof. See Appendix. \square

In particular, one condition for this proposition to be true imply that the production function must “bend more” than the labour demand function. Notably, this is also a condition to obtain an interior solution in the first place. However, for Proposition 2 to be true, we do need additional conditions on the third derivatives of the production and labour demand functions. However, note that the proof makes no assumption on the distribution of the regions' private benefits, other than they are always non-negative.

To illustrate the proposition, we follow Haaparanta (1996) and use a Cobb-Douglas production function. We show that in this case, the decrease in K_2 is never large enough to counter-balance the increase in K_1 . In other words, total investment is always larger when using a bidding war than without a bidding war.

Corollary 1. *Assuming a Cobbs-Douglas production function with decreasing returns to scale ($\alpha + \beta < 1$), the total amount invested by the firm under a bidding war is always larger than the amount she would invest without a bidding war.*

Proof. See Appendix. □

4.5 A Numerical Example

To illustrate some results of the model, let's continue with the simple Cobbs-Douglas production function introduced previously: $f(K, L) = K^\alpha L^\beta$, with $\alpha + \beta < 1$. With specific functional forms, we can find the optimal investment allocation given a set of parameters $\{\alpha, \beta, p, w, r, n\}$. The analytical solutions are omitted here, as they are not informative. Instead, we describe graphically how the firm behaves facing different conditions.

One interesting question is whether the number of regions in the bidding war affects the firm's investment choices. In the more general model, note that when $E(b_{(2)})$ and $E(b_{(3)})$ are closer together, the differentiation between K_1 and K_2 diminishes. In the extreme case of $E(b_{(2)}) = E(b_{(3)})$, we have that $K_1^* = K_2^*$. In turn, the number of regions n participating in the bidding war affects the difference between $E(b_{(2)})$ and $E(b_{(3)})$. For example, if the distribution of private benefits is uniform on $[0, 1]$, then a low number of regions (e.g., 3 regions) will translate in a large difference between the expected private benefits of the regions, while a larger number of regions will translate in lower differences. For that reason, we should see decreasing differentiation with an increasing number of competitors. Figure 1 illustrates this relationship for specific values of the parameters and a uniform distribution. It also shows how both K_1^* and K_2^* are larger than K_{nbw}^* .

Lemma 4 showed that for some distributions of the private benefits, the value of K_2 may behave differently. Figure 2 shows how K_1 and K_2 vary with n for the distribution $b^{\frac{1}{3}}$ on the interval $[0, 1]$, along with the value of K_{nbw} as reference. It shows how investment in the second establishment may actually be lower than without a bidding war, but that even in this example, *total* investment is higher.

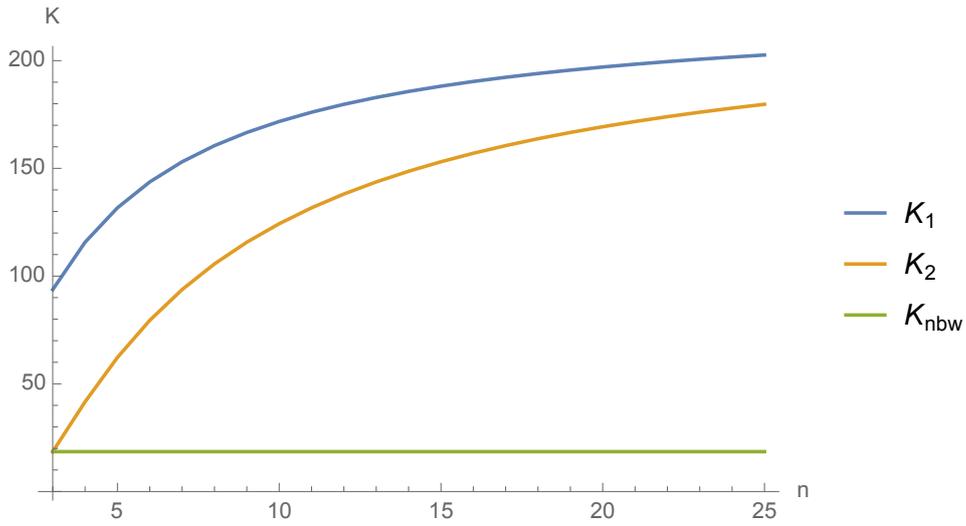


Figure 1: K_1 vs K_2 , with a uniform distribution

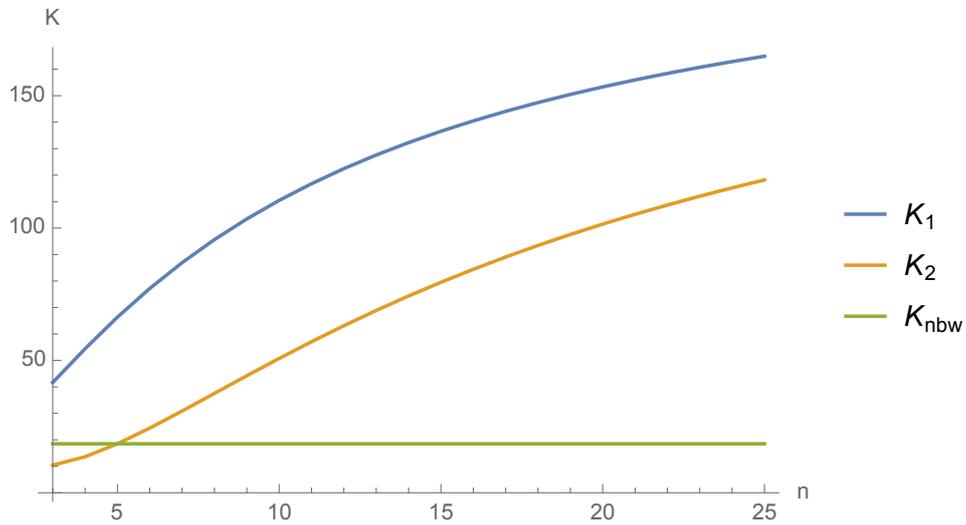


Figure 2: K_1 vs K_2 , with distribution of private benefits respecting Lemma 4

5 Optimal Mechanisms

So far, this paper considered that the firm allocated the plants using an open ascending auction. However, the firm might prefer another type of selling mechanism. This section determines the optimal mechanism under two alternative assumptions on the commitment by the firm, then compares it to the open ascending auction.

5.1 Optimal Mechanism with Pre-Commitment in Investment Quantities

First, we find the optimal mechanism under the assumption that the firm first chooses and commits to values of K_1 and K_2 , as in the model of previous sections, and then chooses the selling

mechanism. We have n regional governments, each willing to buy up to 1 unit of production from a firm. Each regional government i has a private valuation per labour unit employed in the firm's production of b_i . The b_i are identically and independently distributed according to $g(\cdot)$ on the interval $[\underline{b}, \bar{b}]$. Define $b = (b_1, \dots, b_n)$, $B_i = [\underline{b}, \bar{b}]$, and $B = \Pi_i B_i = [\underline{b}, \bar{b}]^n$. The firm has two units of production available, $j = 1, 2$. We define $x_i(b) = (x_{i,1}(b), x_{i,2}(b))$ as the allocation function vector, with $x_{i,j}(b) \in [0, 1]$, and $s_i(b_i, b_{-i})$ as the payment from the region to the firm. Then, the expected payoff to a regional government is equal to $V_i = x_{i,1}(b)L(K_1)b_i + x_{i,2}(b)L(K_2)b_i$, and the expected utility is:

$$EU_i(x_i, b, s_i) = \int_{B_{-i}} (x_{i,1}(b_i, b_{-i})L(K_1)b_i + x_{i,2}(b_i, b_{-i})L(K_2)b_i - s_i(b_i, b_{-i}))g_{-i}(b_{-i})db_{-i} \quad (5.1)$$

The firm wants to maximise

$$E(\Pi) = \pi_1(K_1) + \pi_2(K_2) + \sum_{i=1}^n \int_B s_i(b)g(b)db \quad (5.2)$$

The firm plays a two-stage game. She chooses (K_1, K_2) in a first stage, and then chooses and implements a mechanism to allocate these two plants while receiving subsidies from regional governments.

In the second stage, the firm's objective is to choose a mechanism to maximise her profits. By the revelation principle, we can restrict our attention to direct mechanism characterised by a set of functions $\{x_i(b), s_i(b)\}_{i=1, \dots, n}$ where the x_i 's reflect the allocation rule and the s_i 's reflect the payment rule, when b is the vector of types reported by the regions. Formally, the firm then solves the following problem:

$$\begin{aligned} \max_{x(b), s(b)} \quad & \int_B \left(\sum_{i=1}^n s_i(b) + \sum_{j=1}^2 \left(\pi(K_j) \sum_{i=1}^n x_{ij} \right) \right) g(b)db \\ \text{s.t.} \quad & EU_i(x_i, b_i, s_i) \geq EU_i(x_i, \tilde{b}_i, b_{-i}, s_i) \quad \forall i && ICC \\ & EU_i(x_i, b_i, s_i) \geq 0 \quad \forall i \forall j && IRC \\ & \sum_{i=1}^n x_{ij} \leq 1 \quad \forall j = 1, 2 && FC1 \\ & x_{ij}(b) \geq 0 && FC2 \\ & x_{i1}(b) + x_{i2}(b) \leq 1 && FC3 \end{aligned}$$

The Incentive Compatibility Constraint (ICC) states that it must be optimal for each region to report its true private benefits (b_i). The Individual Rationality Constraint (IRC) states that it must be optimal for each region to participate in the mechanism. The other three constraints are feasibility constraints. FC1 states that for each plant, the allocation probabilities for all regions must sum to one or less. FC2 states that these probabilities must be non-negative. FC3 ensures that regions can, at the equilibrium, receive only one plant. Defining $\beta_i(b_i) = b_i - \frac{1-G(b_i)}{g(b_i)}$ as the virtual benefits of region i , we can express the solution to this optimisation problem as such:

Lemma 5. *Let $x^*(b)$ be the solution to the following simplified optimisation problem:*

$$\begin{aligned} \max_{x(b)} \quad & \sum_i \int_B \left[\beta_i(b_i) x_i(b) L + \sum_{j=1}^2 \pi(K_j) x_{ij}(b) \right] g(b) db \\ \text{s.t.} \quad & EU_i(x_i, \underline{b}, s_i) = 0 \quad \forall i \\ & (\tilde{b}_i - b_i) [p_i(x_i, b_i) \cdot L] \geq (\tilde{b}_i - b_i) [p_i(x_i, \tilde{b}_i) \cdot L] \quad \forall b_i < \tilde{b}_i \\ & \sum_{i=1}^n x_{ij} \leq 1 \quad \forall j = 1, 2 \\ & x_{ij}(b) \geq 0 \quad \forall i, j \\ & x_{i1}(b) + x_{i2}(b) \leq 1 \quad \forall i \end{aligned}$$

Let $s_i^*(b)$ be given by:

$$s_i^*(b) = b_i(x_i^*(b))L - \int_{\underline{b}}^{b_i} x_i^*(t, b_{-i}) dt$$

Then, (x^*, t^*) is the optimal mechanism, with reserve subsidies corresponding to region types b_{r1} and b_{r2} .

Proof. See Appendix. □

How does this optimal mechanism compare to the open ascending auction used in our previous model? Abstracting from the reserve subsidies for now, thus implicitly assuming that they are non-binding, we find that the optimal mechanism allocates to two plants to the same two regions as the open ascending auction, with the same equilibrium subsidies.

Proposition 3. *Assume the firm first commits to the sale of (K_1, K_2) , and subsequently chooses a mechanism to allocate these two plants. When reserve subsidies are non-binding, the optimal mechanism found in Lemma 5 results in the same allocation $(x^*(b))$ and subsidies $(s^*(b))$ as the multi-unit open ascending auction.*

Proof. Similarly to standard problems in mechanism design, the optimal allocation function is deterministic: $x^*(b)$ takes value of 0 or 1. In particular, the firm will allocate the first production unit ($L(K_1)$) to the region with the highest virtual valuation (equivalently, to the one with highest private benefits), and the second one ($L(K_2)$) to the region with second-highest virtual valuation. Defining $b_{(k)}$ as the k -th highest private benefits, we thus have

$$x^*(b) = (x_1^*(b), x_2^*(b)) = \begin{cases} (1, 0) & \text{if } b = b_{(1)} \\ (0, 1) & \text{if } b = b_{(2)} \\ (0, 0) & \text{otherwise} \end{cases} \quad (5.3)$$

The optimal payment rule depends on this allocation function. First note that the first term in $s_i^*(b)$ is simply the value to the region of hosting the firm. For the region hosting K_1 , it is equal to $b_{(1)}L(K_1)$, while for the region hosting K_2 it is equal to $b_{(2)}L(K_2)$. The second term can be interpreted as the informational rent going to the regions. Define $z_{ij}(b_{-i})$ as the lowest value of private benefits that a region i can announce and still win establishment j . The integral in the second term then takes the following values:

$$\int_{\underline{b}}^{b_i} x_i^*(t, b_{-i}) L dt = \begin{cases} L(K_1)b_i - L(K_1)b_{(2)} + L(K_2)b_{(2)} - L(K_2)b_{(3)} & \text{if } b_i > z_{i1}(b_{-i}) \\ L(K_2)b_i - L(K_2)b_{(3)} & \text{if } z_{i1}(b_{-i}) > b_i > z_{i2}(b_{-i}) \\ 0 & \text{otherwise} \end{cases}$$

The first case warrants some discussion. If a region's private benefits are greater than $z_{i1}(b_{-i})$, such that they win the first establishment, we also need to take into account the fact that by winning the first establishment, that region also renounces to the smaller establishment. We can see this more clearly when developing the expression to integrate: $\int_{\underline{b}}^{b_i} x_i^*(t, b_{-i}) L dt = \int_{\underline{b}}^{b_i} (x_{i1}^*(t, b_{-i})L(K_1) + x_{i2}^*(t, b_{-i})L(K_2)) dt$. When calculating the integral over the interval $[\underline{b}, b_i]$, we have to take into account that $x(b)$ takes non-null values not only over the interval in which the region wins the first establishment, but also over the interval over which the regions wins

the second establishment. These results lead to the following payments

$$s_i^*(b) = \begin{cases} (L(K_1) - L(K_2))b_{(2)} + L(K_2)b_{(3)} & \text{if } b_i > z_{i1}(b_{-i}) \\ L(K_2)b_{(3)} & \text{if } z_{i1}(b_{-i}) > b_i > z_{i2}(b_{-i}) \\ 0 & \text{otherwise} \end{cases} \quad (5.4)$$

These are exactly the same payments found in the auction in the previous sections. Since that auction led to the same allocation and the same payments, we can conclude that the auction implemented the optimal mechanism (albeit without reserve prices), from the point of view of the firm. Moreover, we can see that the *ex ante* choice of K_1 and K_2 will be identical. \square

This proposition indicates that the open ascending auction chosen in the first part of the paper is actually optimal from the firm's point of view. In other words, she can do no better, when committing to K_1 and K_2 beforehand, than the open ascending auction.

In the solution to the problem, we saw that the firm allocated the plants to the regions with the highest *virtual valuations*. In our model, the regions have information on their own benefits while the firm does not. In turn, they receive some informational rents (as seen by the payments). In setting the optimal mechanism, the firm tries to extract some of that rent. In fact, the firm could decide not to allocate the plant at all even if it is efficient to do so. Indeed, at some positive level of b_i , $\beta_i(b_i)$ can be negative. If all signals are such that $\beta_i(b_i)$ is negative, the firm maximises her objective function by not allocating the plants.

In fact, the optimal mechanism should also define reserve prices: threshold values of the regions' private benefits under which the firm would not allocate her plants. In a simpler model, the reserve prices would simply be defined by the level of private benefits under which the virtual valuation is negative, $b_r = \beta^{-1}(0)$. Indeed, if the revealed b_i 's are all lower, then the objective function would also be negative, thus choosing not to allocate the units at all.

In our model, however, the reserve prices must take the technological profits into account. Indeed, by not allocating the plants, the firm actually reduces her own profits. The intuition is similar. The firm selects the regions to allocate the plant by choosing $x(b)$, and her payoff must be positive: $\pi(K_1) + \beta(b_{r1})L(K_1) > 0$ and $\pi(K_2) + \beta(b_{r2})L(K_2) > 0$.

With reserve prices, the optimal mechanism would differ from the open ascending auction of the previous sections. Therefore, we find the conditions under which the reserve prices are non-binding.

Lemma 6. For $K_j \in [0, \bar{K}] \forall j$ and $p > \underline{p}$, the reserve prices in the optimal mechanism are non-binding.

Proof. If the private benefits revealed through the mechanism are equal to \underline{b} , the lowest possible level, the payoff to the firm must respect the following condition:

$$\pi(K_j) + \beta(\underline{b})L(K_j) = pf(K_j, L(K_j)) - L(K_j)(w - \beta(\underline{b})) - rK_j \geq 0$$

Notably, $K_j = 0$ respects this condition. Moreover, given the assumptions on the production function, we know that the slopes of $pf(K_j, L(K_j))$ and $L(K_j)(w - \beta(\underline{b})) + rK_j$ are positive. Therefore, two cases are possible at $K_j = \epsilon$ (i.e., an arbitrary small level of investment):

- The firm makes positive profits: $pf(K_j, L(K_j)) - L(K_j)(w - \beta(\underline{b})) - rK_j > 0$
- The firm does not make profit: $pf(K_j, L(K_j)) - L(K_j)(w - \beta(\underline{b})) - rK_j < 0$

In the second case, we can conclude that for any $K_j > \epsilon$, she never makes profits if the winning region has private benefits \underline{b} . In the first case, we can conclude that the firm will make positive profits up to a certain point \bar{K} (where $pf(\bar{K}, L(\bar{K})) = L(\bar{K})(w - \beta(\underline{b})) + r\bar{K}$). Assuming that p is large enough, we are always in the first situation.

Under these assumptions, the firm always makes profits at \underline{b} . If the private benefits revealed in the mechanism are higher, she also necessarily makes profits. Indeed, a larger b implies a lower $w - \beta(\underline{b})$, and thus higher profits at the same level of capital invested and prices.

Therefore, even if the firm sets reserve prices, they would never affect the decision if the pre-determined K_j are always in the interval $[0, \bar{K}]$. □

5.2 Optimal Mechanism with Investment Choices Endogenous to the Mechanism

In the auction model and in the derivation of the optimal mechanism thus far, we have assumed that the firm commits to levels of capital investment (K_1, K_2) . After the reveal of the private benefits, however, it is possible that the firm would like to modify her allocation of capital.

In this section, we investigate how the allocation and payments would differ under an optimal mechanism without such commitment, where the firm chooses the amounts to invest simultaneously with the allocation and payments. Also, we show that at least in expected values,

the optimal mechanism with commitment leads to the same allocations as mechanism without commitment.

To do so, we will modify the problem above slightly. Instead of only choosing a vector of probabilities $x_i(b)$, the firm chooses, in addition, a vector of investments $k_i(b) = (k_{i1}(b), k_{i2}(b))$.

The firm's problem becomes:

$$\begin{aligned}
\max_{x(b), k(b), s(b)} \quad & E(\Pi) = \sum_{i=1}^n \int_B [x_{i1}(b)\pi(k_{i1}(b)) + x_{i2}(b)\pi(k_{i2}(b)) + s_i(b)] g(b) db \\
s.t. \quad & EU_i(k_i, b_i, s_i) \geq EU_i(k_i, \tilde{b}_i, b_{-i}, s_i) \quad \forall i && ICC \\
& EU_i(k_i, b_i, s_i) \geq 0 \quad \forall i \forall j && IRC \\
& \sum_{i=1}^n x_{ij} \leq 1 \quad \forall j = 1, 2 && FC1 \\
& x_{ij}(b) \geq 0 && FC2 \\
& x_{i1}(b) + x_{i2}(b) \leq 1 && FC3 \\
& k_{ij}(b) \geq 0 && FC4
\end{aligned}$$

By using similar manipulations on the constraints as in the constrained mechanism problem, we can transform the firm's objective function as such

$$E(\Pi) = \sum_{i=1}^n \int_B [x_{i1}(b)\pi(k_{i1}(b)) + x_{i2}(b)\pi(k_{i2}(b)) + \beta_i(b_i) (x_{i1}(b)L(k_{i1}(b)) + x_{i2}(b)L(k_{i2}(b)))] g(b) db \tag{5.5}$$

As in the previous sections, the solution for $x(b)$ is deterministic:

$$x^*(b) = (x_{i1}^*(b), x_{i2}^*(b)) = \begin{cases} (1, 0) & \text{if } b = b_{(1)} \\ (0, 1) & \text{if } b = b_{(2)} \\ (0, 0) & \text{otherwise} \end{cases} \tag{5.6}$$

What are the values of $k_1^*(b)$ and $k_2^*(b)$? The firm will choose these investment amounts after observing the signals. She only commits to a function $k(b)$. From the objective function, we can find the first-order condition, assuming the b_i 's are observed, and the plants are assigned to the respective winners.

$$p \frac{\partial f(k_1^*, L(k_1^*))}{\partial k_1^*} = L'(k_1^*)(w - \beta(b_{(1)})) + r$$

$$p \frac{\partial f(k_2^*, L(k_2^*))}{\partial k_2^*} = L'(k_2^*)(w - \beta(b_{(2)})) + r$$

These conditions define functions $k^*(b)$.¹² The actual values of capital investment are not decided until the end of the mechanism. However, as long as $b_{(1)} \neq b_{(2)}$, $\beta(b_{(1)}) \neq \beta(b_{(2)})$. In turn, we can conclude that $k_1^* \neq k_2^*$, just as in the mechanism with commitment and in the auction model of the previous sections.

Obviously, since the firm does not commit to ex ante optimal values of investment, but chooses the amount only when observing the private benefits of the regions, the firm does better *ex post* in the situation without commitment. However, in expected value, the optimal mechanism with commitment leads, *ex ante* to the same solution.

Proposition 4. *If the regions' private benefits are uniformly distributed, then the optimal mechanism with pre-commitment leads, in expected value, to the same amounts of investment from the firm as in the optimal mechanism without commitment.*

Proof. Assume that the b_i 's are distributed uniformly on the interval $[0, \bar{b}]$. Then,

$$E(\beta(b_{(1)})) = E[b_{(1)} - (\bar{b} - (b_{(1)}))] = E[2b_{(1)} - \bar{b}] = \frac{n-1}{n+1}\bar{b} = E(b_{(2)})$$

Similarly,

$$E(\beta(b_{(2)})) = E[b_{(2)} - (\bar{b} - (b_{(2)}))] = E[2b_{(2)} - \bar{b}] = \frac{n-3}{n+1}\bar{b}$$

We also know that $E(b_{(3)}) = \frac{n-2}{n+1}\bar{b}$. Therefore,

$$-E(b_{(2)}) + 2E(b_{(3)}) = \frac{n-3}{n+1}\bar{b}$$

Consequently,

$$E(\beta(b_{(2)})) = -E(b_{(2)}) + 2E(b_{(3)})$$

¹²For this optimal mechanism, a discussion on reserve subsidies is unnecessary. Indeed, reserve subsidies will be endogenously determined in the $k^*(b)$ functions. The firm can simply set $k^*(b) = 0$ for some values of b , which is equivalent to a reserve subsidy.

Recall the first-order conditions from the auction model:

$$p \frac{\partial f(K_1, L(K_1))}{\partial K_1} = L'(K_1)(w - E(b_{(2)})) + r$$

$$p \frac{\partial f(K_2, L(K_2))}{\partial K_2} = L'(K_2)(w + E(b_{(2)}) - 2E(b_{(3)})) + r$$

In expected value, we thus have the exact same first-order conditions, leading to the same investment decisions from the firm. \square

The previous result holds for other distributions as well. This finding is not surprising. Indeed, virtual valuations can be interpreted as the marginal revenues on the sale for the seller. In the auction model, the “adjustments” on the marginal cost of labour, $E(b_{(2)})$ and $-E(b_{(2)}) + 2E(b_{(3)})$, are simply the marginal revenues from the subsidies. Similarly, $\beta(b_{(1)})$ and $\beta(b_{(2)})$ are the marginal revenues from the sale of the 2 investments to the seller in the unconstrained optimal mechanism.

6 Welfare

The previous discussion shows that the optimal mechanism (at least under some conditions on the price p and with the pre-determined $K_j \in [0, \bar{K}] \forall j$) is equivalent to the open ascending auction of the previous sections. An interesting question, then, is whether this mechanism is also optimal in terms of social welfare.

6.1 The Social Planner Problem

To investigate the social welfare question, we can replace the firm as the decision-maker by a social planner trying to find a mechanism to allocate K_1 and K_2 (decided ex ante) to maximise a social welfare function. Would the allocations and payments be the same as in the firm’s optimal mechanism? The set-up is similar to the one of Lemma 5. The difference is that the objective function includes not only the firm’s welfare, but also that of the regions. It also considers the marginal cost of public funds (λ). We also assume that the social planner is

uninformed about the regions' signals.¹³

$$\begin{aligned}
\max_{x(b), s(b)} \quad & E(W) = \int_B \left[\alpha E(\Pi) + \gamma \sum_{i=1}^n EU_i - \lambda \sum_{i=1}^n s_i(b) \right] g(b) db \\
\text{s.t.} \quad & EU_i(x_i, b_i, s_i) \geq EU_i(x_i, \tilde{b}_i, b_{-i}, s_i) \quad \forall i && ICC \\
& EU_i(x_i, b_i, s_i) \geq 0 \quad \forall i \forall j && IRC \\
& \sum_{i=1}^n x_{ij} \leq 1 \quad \forall j = 1, 2 && FC1 \\
& x_{ij}(b) \geq 0 && FC2 \\
& x_{i1}(b) + x_{i2}(b) \leq 1 && FC3
\end{aligned}$$

The values for α and γ are the social weights placed on the welfare of the firm and the regions, respectively, and they sum to one ($\alpha + \gamma = 1$). In this section, we assume that the firm chooses the amounts of capital to invest in a previous step, and that the mechanism is used to allocate these amounts. Therefore, $E(\Pi) = \pi(K_1) \sum_{i=1}^n x_{i1}(b) + \pi(K_2) \sum_{i=1}^n x_{i2}(b) + \sum_{i=1}^n s_i(b)$. Since the firm has no information to reveal, she does not appear in the incentive compatibility constraints.

Lemma 7. *For a given K_1 and K_2 , an uninformed social planner would locate the plants in the same regions as the firm.*

Proof. We can simplify the objective function as such:

$$E(W) = \int_B \left[\gamma \sum_{i=1}^n (x_i(b) b_i L) + (\alpha - \gamma - \lambda) \sum_{i=1}^n s_i(b) + \alpha \sum_{j=1}^2 x_{ij}(b) \cdot \pi(K_j) \right] g(b) db \quad (6.1)$$

By using the same manipulations on the constraints as in the previous problem, we find that

$$\begin{aligned}
E(W) &= \sum_{i=1}^n \int_B \left[(\alpha - \lambda)(x_i(b) b_i L) - (\alpha - \gamma - \lambda)(x_i(b) L \frac{1 - G_i(b_i)}{g_i(b_i)}) + \alpha \sum_{j=1}^2 x_{ij}(b) \cdot \pi(K_j) \right] g(b) db \\
E(W) &= \sum_{i=1}^n \int_B \left[\left(b_i - \frac{\alpha - \gamma - \lambda}{\alpha - \lambda} \cdot \frac{1 - G_i(b_i)}{g_i(b_i)} \right) (\alpha - \lambda) x_i(b) L + \alpha \sum_{j=1}^2 x_{ij}(b) \cdot \pi(K_j) \right] g(b) db \\
E(W) &= \sum_{i=1}^n \int_B \left[\beta'_i(b_i) (\alpha - \lambda) x_i(b) L + \alpha \sum_{j=1}^2 x_{ij}(b) \cdot \pi(K_j) \right] g(b) db
\end{aligned}$$

The function $\beta'_i(b_i)$ differs from $\beta_i(b_i)$ by the multiplication of the inverse of the hazard function

¹³If the social planner has perfect information, the problem is trivial. It allocates the plants to the regions that value them the most, with no payments.

by a combination of the model parameters. We make the assumption that $\alpha - \gamma \geq \lambda$, implying also that $\alpha \geq \lambda$, so that welfare is non-negative. Intuitively, from Equation 6.1, this assumption implies that for the transfers from the regions to the firm, the additional social weight placed on the firm versus the regions (i.e., $\alpha - \gamma$) is large enough to cover the marginal cost of public funds (λ).¹⁴

Notably, with these assumptions, the social planner chooses the same deterministic $x^*(b)$:

$$x^*(b) = (x_1^*(b), x_2^*(b)) = \begin{cases} (1, 0) & \text{if } b = b_{(1)} \\ (0, 1) & \text{if } b = b_{(2)} \\ (0, 0) & \text{otherwise} \end{cases} \quad (6.2)$$

□

While the allocation function may be the same as the firm, the social planner's optimal mechanism is not entirely identical to the firm's. Since $\frac{\alpha - \gamma - \lambda}{\alpha - \lambda} \leq 1$, then $\beta'_i(b_i) \geq \beta_i(b_i) \forall i$. We can see that this has implications for the reserve prices. In particular, the social planner allocates the plants more often.

As in the discussion on the optimal mechanism from the firm's viewpoint, we do not discuss reserve prices but the conditions under which the reserve prices are not binding. However, to illustrate how the reserve prices would change, we first look at an example. Abstracting from the technological profits, we would find the following b_r :

$$\begin{aligned} 0 &= \beta'_i(b_r^*) & 0 &= \beta_i(b_r^*) \\ 0 &= \frac{\alpha - \gamma - \lambda}{\alpha - \lambda} \cdot \frac{1 - G_i(b_r^*)}{g_i(b_r^*)} & 0 &= \frac{1 - G_i(b_r^*)}{g_i(b_r^*)} \end{aligned}$$

With a uniform distribution on $[0, 1]$, for example,

$$\begin{aligned} b_r^* &= \frac{\alpha - \gamma - \lambda}{\alpha - \lambda} \cdot (1 - b_r^*) & b_r^* &= 1 - b_r^* \\ b_r^* &= \frac{\frac{\alpha - \gamma - \lambda}{\alpha - \lambda}}{1 + \frac{\alpha - \gamma - \lambda}{\alpha - \lambda}} < \frac{1}{2} & b_r^* &= \frac{1}{2} \end{aligned}$$

Therefore, although the allocations under a social planner and the firm are the same, the

¹⁴The assumption $\alpha - \gamma \geq \lambda$ has additional implications for the marginal cost of public funds. Indeed, if the weight placed on the firm and regions is the same ($\alpha = \gamma$), then if $\lambda > 0$, social welfare is negative. Conversely, if we allow $\alpha - \gamma < \lambda$, then the shape of $\beta'_i(b_i)$ is uncertain. For a uniform distribution, it is merely necessary that $\left| \frac{\alpha - \gamma - \lambda}{\alpha - \lambda} \right| \leq 1$, but this is not true in the general case.

social planner would allocate more often. We show now that this result translates in a looser condition on the possible investment interval ($[0, \bar{K}']$).

Proposition 5. *For any distribution such that $\beta(\underline{b}) < 0$, $\bar{K}' > \bar{K}$. In other words, the social planner allocates the plants more often.*

Proof. Similar to the firm's problem, we have that

$$\alpha\pi(K_j) + \beta'(\underline{b})(\alpha - \lambda)L(K_j) = \alpha pf(K_j, L(K_j)) - L(K_j)(\alpha w - (\alpha - \lambda)\beta'(\underline{b})) - \alpha r K_j \geq 0$$

Like in the firm's problem, if $K = 0$, then $\alpha\pi(K_j) + \beta'(\underline{b})(\alpha - \lambda)L(K_j) = 0$. Assuming that p is high enough so that with an arbitrary small amount of capital ϵ , $\alpha\pi(K_j) + \beta'(\underline{b})(\alpha - \lambda)L(K_j) > 0$, we want to find the maximum amount of investment such that profits are positive at the lowest level of private benefits \underline{b} .

$$\begin{aligned} \alpha pf(\bar{K}', L(\bar{K}')) - L(\bar{K}')(\alpha w - (\alpha - \lambda)\beta'(\underline{b})) - \alpha r \bar{K}' &= 0 \\ \alpha \left[pf(\bar{K}', L(\bar{K}')) - L(\bar{K}') \left(w - \frac{\alpha - \lambda}{\alpha} \beta'(\underline{b}) \right) - r \bar{K}' \right] &= 0 \\ \alpha \left[pf(\bar{K}', L(\bar{K}')) - L(\bar{K}') \left(w - \left(1 - \frac{\gamma + \lambda}{\alpha} \right) \beta(\underline{b}) \right) - r \bar{K}' \right] &= 0 \end{aligned}$$

In the third line, we used the definitions of $\beta'(\cdot)$ and $\beta(\cdot)$. This equation defines a level \bar{K}' . How does that amount differ from \bar{K} ? The difference between the two conditions is in the multiplier in front of $L'(\cdot)$ (essentially the effective marginal cost of labour). We therefore compare these costs. $\bar{K}' > \bar{K}$ if and only if:

$$\begin{aligned} w - \beta(\underline{b}) &> w - \left(1 - \frac{\gamma + \lambda}{\alpha} \right) \beta(\underline{b}) \\ \beta(\underline{b}) &< 1 - \frac{\gamma + \lambda}{\alpha} \beta(\underline{b}) \\ \underline{b} \left(\frac{\lambda}{\alpha} \right) - \frac{1 - G(\underline{b})}{g(\underline{b})} \left(\frac{\gamma + \lambda}{\alpha} \right) &< 0 \\ \underline{b} - \frac{\gamma + \lambda}{\lambda} \frac{1 - G(\underline{b})}{g(\underline{b})} &< 0 \end{aligned}$$

This last expression will be true for low levels of \underline{b} . Notably, it is always true for $\underline{b} = 0$. However, more generally, we can prove that for any distribution $f(\cdot)$,

$$\underline{b} - \frac{1 - G(\underline{b})}{g(\underline{b})} < 0 \implies \underline{b} - \frac{\gamma + \lambda}{\lambda} \frac{1 - G(\underline{b})}{g(\underline{b})} < 0$$

Therefore, if the virtual valuation, from the firm's point of view, of a region with private benefits \underline{b} is negative, then the social planner's condition is looser: $\bar{K}' > \bar{K}$. \square

The condition ($\beta(\underline{b}) < 0$) for this result is not very restrictive. Indeed, if from the firm's point of view, virtual valuations are all positive, the firm allocates all the time, so a discussion on welfare is not as interesting. Since it is inefficient socially to not allocate the plants, the looser conditions set by the social planner actually increases welfare. The intuition for this result is that the social planner puts less importance on the firm capturing the regions' informational rent.

6.2 Social vs. Politician Welfare

In our model, b_i captures both the public value of the plant accruing to the region as a whole (e.g., to citizens) and the private value accruing to the regional politician who is making the subsidy decision. In the social planner problem discussed earlier, the social welfare function kept that assumption. However, a discussion on social welfare might need to focus on the public part of the benefits.

Let's instead assume that b_i has two components: a public portion accruing to the region (p_i) and a private portion accruing to the politician (P_i). So far, the analysis has focused on mechanisms that allocate the plants to the regions with the highest values of $b_i = p_i + P_i$. In terms of social welfare, however, one could be mostly interested in p_i .

If p_i and P_i are perfectly and positively correlated in rank, then the ordering of both are exactly the same: the region with the highest P_i also has the highest p_i . Consequently, the region with the highest combined level (b_i) also has the highest level of social benefits (p_i). In that case, the optimal mechanism described earlier allocates the plants to the regions with the highest p_i . However, the level of subsidies might be too high. The subsidy for the smaller plant, for example, is equal to $L(K_1) \cdot E(b_{(3)}) = L(K_1) \cdot E(p_{(3)} + P_{(3)})$. If, instead, the social planner could distinguish between the two sources of benefits to the regions, the subsidy for the small establishment would be equal to $L(K_1) \cdot E(p_{(3)})$.

If, instead, p_i and P_i are negatively correlated in rank, then the optimal mechanism might not allocate the plants to the regions that actually value it the most. Instead, the social planner might allocate the plant to a region where the politician places a large value on it (high P_i), but where the region as a whole sees only limited value from hosting the plant (low p_i).

The conclusion on social welfare thus depends on the assumptions we make on the correlation between the two components of benefits going to the region. If we believe that at least some of the politician's welfare results from being re-elected, a reasonable assumption would be that p_i and P_i are positively correlated in rank, but not necessarily perfectly. Indeed, a politician would not stand to gain much in terms of future re-election by subsidising a new plant for which voters see little or no value (i.e., a low p_i).

6.3 Are Regional Governments Better Off with a Bidding War?

Given the results above, one may wonder if it's in the regions' best interests that such a bidding war takes place. Without a bidding war, region i has the following expected utility:

$$E(W_{nbw,i}) = \frac{2}{n} L(K_{nbw}^*) \cdot b_i \quad (6.3)$$

where K_{nbw}^* is the investment from the firm in one establishment, without a bidding war.

With a bidding war, the same region has the following expected utility

$$\begin{aligned} E(W_{bw,i}) = & \int_{\underline{b}}^{b_i} \int_{\underline{b}}^{b_{(1),-i}} (L(K_1^*)b_i - s_1^*)h(b_{(1),-i}, b_{(2),-i}, n-1)db_{(2),-i}db_{(1)} \\ & + \int_{b_i}^{\bar{b}} \int_{\underline{b}}^{b_i} (L(K_2^*)b_i - s_2^*)h(b_{(1),-i}, b_{(2),-i}, n-1)db_{(2),-i}db_{(1),-i} \end{aligned} \quad (6.4)$$

where for region i , $db_{(k),-i}$ is the k -th highest benefit among the $n-1$ other regions. The expression $E(W_{nbw,i}) = E(W_{bw,i})$ defines a level of b_i over which a region prefers a bidding war. Conversely, it also defines a level of b_i under which regional governments are made worse off by a bidding war.

This preference results from 2 factors. First, with a bidding war, regions with large private benefits expect to win more often. Second, under a bidding war, the regions expect the firm to choose a higher level of capital K_1 , and, as seen in the discussion on Corollary 1, a higher level of capital K_2 as well, at least in some cases.

To illustrate, with a Cobb-Douglas production function of parameters $\alpha = \beta = 1/3$, a uniform distribution of benefits on $[0, 1]$, and $p = w = r = 1$, to illustrate, we find that a given region i prefers that the firm uses a bidding war as long as

$$b_i > 0.227$$

7 Conclusion

This paper investigates how a firm can allocate investment across multiple sites strategically to attract larger subsidies from regions who participate in a bidding war for these investments. It proposes a model in which a firm wishes to build new production facilities and puts regional governments in competition against each other to decide the location of those facilities. Regional governments submit bids, in the form of tax holidays or other financial packages, and the firm invests in the winning region(s). In contrast to previous models, the firm can split her production in two establishments. This split introduces new strategic choices for the firm, and modifies the bidding behaviour of the regional governments.

First, we find that equilibrium subsidies will depend on the firm's choice of capital amounts to invest. In particular, when she chooses asymmetric plants, total subsidies are larger. Second, we show that this bidding behaviour affects the optimal amounts of investment of the firm. More specifically, she always chooses to differentiate her establishments. Therefore, the firm departs from her profit-maximising production allocation, in order to attract larger subsidies. I then compare this result to a situation without a bidding war. We find that the effect of the bidding war is ambiguous in general, but when using a Cobb-Douglas form for the production function, we find that total investment increases. Notably, this result is true for any distribution function. Moreover, total subsidies also increase.

We also discuss the optimal mechanism to allocate the establishments under three different sets of assumptions: first from the point of view of the firm, second from the point of view of a social planner, and finally under more relaxed assumptions on the firm's commitment. We find that the open ascending auction used in the model implements the optimal auction, under certain conditions. Moreover, we find that a social planner would optimally choose the same allocation and payment rules as the firm. Finally, we describe the optimal mechanism under the more general assumption that the firm chooses amounts to invest endogenously through the mechanism. We show that in expected value, this optimal mechanism without prior commitment from the firm results in the same *ex ante* allocation and payments.

To summarise, this paper can be interpreted as two successive additions to the usual literature on bidding wars for firms. First, instead of considering a fixed investment amount, this paper allows the firm to choose the amount of capital to invest and make available in a bidding war. This addition changes the strategy of the firm, inciting her to over-invest in comparison

to a situation without a bidding war. Second, we add a multi-location component: the firm can allocate the total investment across two sites. This addition modifies the firm's behaviour further, by inciting her to differentiate the amounts of investment between the production sites. In doing so, she continues to over-invest in total.

In terms of social welfare, the paper shows that while the allocation of investment is distorted versus a situation without a bidding war, the positive effect on allocative efficiency resulting from bidding wars is preserved in a multi-plant bidding war. More specifically, the regions that value the investment the most win. However, while total investment is increased, which may have positive or negative implications, the increase is (under some conditions) for both hosts of the plants. Therefore, the increase in investment is not skewed only towards one of the winners at the expense of the other.

To conclude, the paper shows that the strategic behaviour of firms has important implications on the bidding wars for plants between regions. This distinction is important, since many bidding wars involve multi-national firms, and that such firms receive many subsidies in short periods by many local governments.

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A Proofs

Lemma 2

Proof. To see why these two bids are optimal, take a region i with private benefits b_i and assume that everyone else bids according to the following strategy: continue bidding until the clock reaches my private valuation. In that case, if the clock reaches $L_2 b_i$ and there are still 3 or more regions in the auction, then region i has no incentive to continue bidding. Indeed, if she does, whatever the stop price, she will need to pay more than her valuation if she wins. Therefore, at price $L_2 b_i$, she prefers to leave the auction. Now consider prices lower than $L_2 b_i$, for example $L_2 b_l$. At that clock price, region i has a positive valuation and would like to win. Therefore, she has no incentive to leave the auction. Therefore, the equilibrium bid for the small establishment will be equal to

$$s_2^*(K_1, K_2) = L(K_2) \cdot b_{(3)}$$

where $b_{(3)}$ is the third-highest signal among the n regions.

If the two plants are of symmetric sizes (i.e., $K_1 = K_2$), then the two remaining regions each pay $s_2^*(K_1, K_2)$ and each receive the same investment.

However, if the two plants are asymmetric (i.e., $K_1 \neq K_2$), we still have to determine which region receives the largest investment. Both regions know that their possibilities are now to pay $s_2^*(K_1, K_2)$ and receive the small establishment, or to pay more and receive the large establishment. The bid for the largest establishment will thus be determined by the infra-marginal competition between the two remaining bidders. Since at that point, the auction becomes a simple second-price auction between two bidders, it is optimal for both regions to simply withdraw once the clock price reaches their valuation of the large plant. If they continue past that price, they either win and pay a price higher than their valuation, or they lose and pay the price for the second establishment, which was already determined.

Take the decision problem of the region with the second-highest private benefits.¹⁵ It will be indifferent between the two establishments when

$$L(K_1)b_{(2)} - s_1^*(K_1, K_2) = L(K_2)b_{(2)} - s_2^*(K_1, K_2)$$

By rearranging this equation and substituting the value of $s_2^*(K_2)$ found earlier, we obtain the value of the highest bid

$$s_1^*(K_1, K_2) = (L(K_1) - L(K_2))b_{(2)} + L(K_2)b_{(3)}$$

□

Proposition 1

Proof. The firm does not know the private benefits of the regions in the competition, but knows that they are distributed according to $g(\cdot)$ on the interval $[b, \bar{b}]$. Her objective function can thus

¹⁵Given the monotonicity of the valuation function of the regions, for any level of private benefits, regions prefer the largest establishment to the small one.

be expressed as

$$E(\Pi) = \int_{\underline{b}}^{\bar{b}} \int_{\underline{b}}^{b_{(2)}} \left[(L(K_1) - L(K_2))b_{(2)} + 2L(K_2)b_{(3)} \right. \\ \left. + pf(K_1, L(K_1)) - wL(K_1) - rK_1 + pf(K_2, L(K_2)) - wL(K_2) - rK_2 \right] \\ h(b_{(2)}, b_{(3)}, n) db_{(3)} db_{(2)} \quad (\text{A.1})$$

where the last part $h(b_{(2)}, b_{(3)}, n) = n(n-1)(n-2) \cdot [1 - G(b_{(2)})] [G(b_{(3)})]^{n-3} g(b_{(2)})g(b_{(3)})$ is the joint distribution of $b_{(2)}$ and $b_{(3)}$, and $L(K_j)$ is the equilibrium amount of labour for a level of capital K_j . We obtain the following first-order conditions:

$$\frac{\partial E(\Pi)}{\partial K_1} = L'(K_1)E(b_{(2)}) \\ + p \left(\frac{\partial f(K_1, L(K_1))}{\partial K_1} + \frac{\partial f(K_1, L(K_1))}{\partial L(K_1)} \cdot L'(K_1) \right) - wL'(K_1) - r = 0$$

$$\frac{\partial E(\Pi)}{\partial K_2} = -L'(K_2)E(b_{(2)}) + 2L'(K_2)E(b_{(3)}) \\ + p \left(\frac{\partial f(K_2, L(K_2))}{\partial K_2} + \frac{\partial f(K_2, L(K_2))}{\partial L(K_2)} \cdot L'(K_2) \right) - wL'(K_2) - r = 0$$

Since $L(K)$ represents equilibrium values, the FOCs can be simplified using the Envelope Theorem. We then obtain:

$$\frac{\partial E(\Pi)}{\partial K_1} = L'(K_1)E(b_{(2)}) + p \frac{\partial f(K_1, L(K_1))}{\partial K_1} - wL'(K_1) - r = 0 \quad (\text{A.2})$$

$$\frac{\partial E(\Pi)}{\partial K_2} = -L'(K_2)E(b_{(2)}) + 2L'(K_2)E(b_{(3)}) + p \frac{\partial f(K_2, L(K_2))}{\partial K_2} - wL'(K_2) - r = 0 \quad (\text{A.3})$$

Combining the two FOCs, we see that

$$p \left(\frac{\partial f(K_2, L(K_2))}{\partial K_2} - \frac{\partial f(K_1, L(K_1))}{\partial K_1} \right) = L'(K_2) \left(w + E(b_{(2)}) - 2E(b_{(3)}) \right) - L'(K_1) \left(w + E(b_{(2)}) \right)$$

We want to show that $K_1 \neq K_2$. Let's first assume that $E(b_{(2)}) \neq E(b_{(3)})$ (i.e., we focus on the interesting cases where the firm expects regions to have different valuations). To prove that the firm has to optimally split in asymmetric establishments, we first assume that she does not, and show that it leads to an inconsistency. Indeed, if $K_1 = K_2 = K$, the previous equation reduces

to

$$0 = 2L'(K) \left(E(b_{(2)}) - E(b_{(3)}) \right)$$

Since the regions have different expected private benefits, this equation is true only if $L'(K) = 0$. However, that derivative is always positive. Therefore, we conclude that $K_1 \neq K_2$. □

Proposition 2

Proof. The first-order condition for profit maximisation in one arbitrary establishment is:

$$p \frac{\partial f(K, L(K))}{\partial K} = L'(K)(w - x) + r \quad (\text{A.4})$$

where x can be zero or the adjustment on marginal labour costs arising from subsidies. Rearranging this equation, we obtain:

$$\frac{\frac{\partial f(K, L(K))}{\partial K} - r}{L'(K)} = \Phi(K) = (w - x) \quad (\text{A.5})$$

We thus have a relationship between K and x , the adjustment on the marginal cost of labour. First, let's assume that $\frac{\partial \Phi(K)}{\partial K} < 0$ and $\frac{\partial^2 \Phi(K)}{\partial K^2} > 0$. Under these assumptions, we know that a decrease in $w - x$ of a given amount (e.g., e) increases K by more than an identical increase in $w - x$ would decrease K . Since we also know that $\left| E(b_{(2)}) - 2E(b_{(3)}) \right| < \left| -E(b_{(2)}) \right|$, the possible increase in K_2 is always lower than the decrease in K_1 .¹⁶

Therefore, under the assumptions that $\frac{\partial \Phi(K)}{\partial K} < 0$ and $\frac{\partial^2 \Phi(K)}{\partial K^2} > 0$, we can conclude that the increase in K_1 due to a bidding war is always larger than the decrease in K_2 . Consequently, the total amount invested is always larger in a bidding war.

When are these assumptions true? Since r does not change the sign of the derivatives, we can rewrite $\Phi(K)$ as

$$\Phi(K) = \frac{\frac{\partial f(K, L(K))}{\partial K}}{L'(K)} \quad (\text{A.6})$$

For $\frac{\partial \Phi(K)}{\partial K} < 0$ to be true, $f(\cdot)$ needs to be more concave (“bend more”) than $L(\cdot)$ (Cargo, 1965). Assuming as usual that $f(0) = 0$ and $L(0) = 0$, this condition is always respected if we have an interior solution. For $\frac{\partial^2 \Phi(K)}{\partial K^2} > 0$ to be true, we need positive third derivatives for $f(\cdot)$ and

¹⁶Or they are both decreases, in which case total investment is certainly increased

$L(\cdot)$, a positive second derivative for $L(\cdot)$, and that

$$\frac{\partial^3 f(K, L(K))/\partial K^3}{\partial f(K, L(K))/\partial K} > \frac{\partial^3 L(K)/\partial K^3}{\partial L(K)/\partial K} \quad (\text{A.7})$$

We therefore need to put conditions on the third derivatives of the production and labour demand functions. □

Corollary 1

Proof. If $f(K, L) = K^\alpha L^\beta$, and using the function $L(K)$ as in equation (4.2), we find that:

$$\frac{p\left(\frac{p\beta}{w}\right)^{\frac{\beta}{1-\beta}}\left(\frac{\alpha}{1-\beta}\right) \cdot K^{\frac{\alpha}{1-\beta}-1} - r}{\left(\frac{p\beta}{w}\right)^{\frac{1}{1-\beta}}\left(\frac{\alpha}{1-\beta}\right)K^{\frac{\alpha}{1-\beta}-1}} = w - x$$

This equation can be expressed as (with $A > 0$ and $B > 0$):

$$A - B \cdot K^{\frac{1-\alpha-\beta}{1-\beta}} = w - x \quad (\text{A.8})$$

Since $0 < \frac{1-\alpha-\beta}{1-\beta} < 1$, Figure 3 illustrates a stylised version of the left-hand side of Equation (A.8).

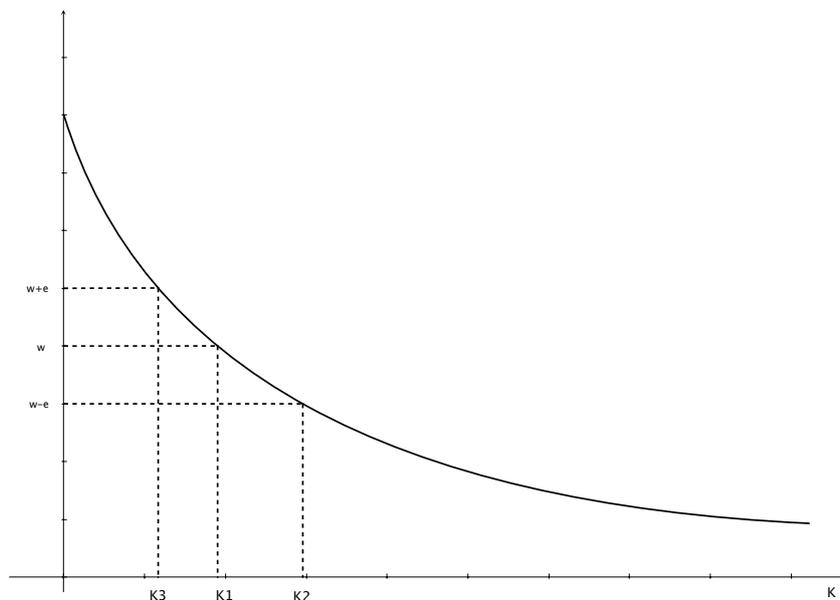


Figure 3: A stylised illustration comparing an upwards adjustment of wages to a downwards adjustment and their effects on the amount of capital invested.

In particular, since the second derivative is positive, a decrease in the right-hand side of a given amount (e) increases K by more than an identical increase in the right-hand side would decrease K . Since we know that $|E(b_{(2)}) - 2E(b_{(3)})| < |-E(b_{(2)})|$, the possible increase in the left-hand side (in the case of K_2) is always lower than the decrease in the left-hand side (in the case of K_1),¹⁷ Therefore, we can conclude that the increase in K_1 due to a bidding war is always larger than the decrease in K_2 . Consequently, the total amount invested is always larger in a bidding war, under a Cobb-Douglas production function. \square

Lemma 5

Proof. The solution to this problem in general is due to Myerson (1981). The solution here will follow Morand (2000), constrained to unit demand.

The incentive compatibility constraint (ICC) states that regional governments must have incentives to state their true private benefits. It has to be satisfied locally. Using the envelope theorem, it must be that

$$\begin{aligned} \frac{dEU_i(x_i, b_i, s_i)}{db_i} &= \frac{\partial EU_i(x_i, \tilde{b}_i, s_i, b_i)}{\partial b_i} \Big|_{\tilde{b}_i=b_i} \\ &= \int_{B_{-i}} (x_{i1}(b)L(K_1) + x_{i2}(b)L(K_2))g_{-i}(b_{-i})db_{-i} \end{aligned} \quad (\text{A.9})$$

Define the marginal probabilities as:

$$\begin{aligned} p_{i1}(x_i, b_i) &= \int_{B_{-i}} x_{i1}(b)g_{-i}(b_{-i})db_{-i} \\ p_{i2}(x_i, b_i) &= \int_{B_{-i}} x_{i2}(b)g_{-i}(b_{-i})db_{-i} \\ p_i &= (p_{i1}, p_{i2}) \end{aligned}$$

With these, we can rewrite Equation A.9 as

$$\frac{dEU_i(x_i, b_i, s_i)}{db_i} = p_{i1}(x_i, b_i)L(K_1)b_i + p_{i2}(x_i, b_i)L(K_2)b_i \quad \forall i \quad (\text{A.10})$$

From Equation A.10, we can find the expected utility of a regional government such that the

¹⁷Or they are both decreases, in which case total investment is certainly increased

incentive compatibility constraint is respected:

$$\begin{aligned}
\int_{\underline{b}}^{b_i} \frac{dEU_i(x_i, t, s_i)}{dt} dt &= \int_{\underline{b}}^{b_i} (p_{i1}(x_i, t)L(K_1)t + p_{i2}(x_i, t)L(K_2)t) dt \\
EU_i(x_i, b_i, s_i) - EU_i(x_i, \underline{b}, s_i) &= \int_{\underline{b}}^{b_i} (p_{i1}(x_i, t)L(K_1)t + p_{i2}(x_i, t)L(K_2)t) dt \\
EU_i(x_i, b_i, s_i) &= \int_{\underline{b}}^{b_i} (p_{i1}(x_i, t)L(K_1)t + p_{i2}(x_i, t)L(K_2)t) dt + EU_i(x_i, \underline{b}, s_i)
\end{aligned} \tag{A.11}$$

This expected utility is thus expressed in two terms. The first term depends on the marginal probabilities to win one of the production sites, while the second one is the expected utility of a regional government with the lowest private benefits (\underline{b}).

With the incentive compatibility constraint, we can also show that $p_{ij}(x_i, b_i)$ is non-decreasing $\forall i, j$. First, we can rewrite the expected utility of a region that announces private benefits \tilde{b}_i when he actually has private benefits b_i , and conversely, as

$$\begin{aligned}
EU_i(x_i, b_i, \tilde{b}_i, s_i) &= EU_i(x_i, \tilde{b}_i, s_i) - (b_i - \tilde{b}_i) [L(K_1)p_{i1}(x_i, \tilde{b}_i) + L(K_2)p_{i2}(x_i, \tilde{b}_i)] \\
EU_i(x_i, \tilde{b}_i, b_i, s_i) &= EU_i(x_i, b_i, s_i) - (\tilde{b}_i - b_i) [L(K_1)p_{i1}(x_i, b_i) + L(K_2)p_{i2}(x_i, b_i)]
\end{aligned}$$

From the incentive compatibility constraint, we thus have that

$$\begin{aligned}
EU_i(x_i, b_i, s_i) &\geq EU_i(x_i, \tilde{b}_i, s_i) - (b_i - \tilde{b}_i) [L(K_1)p_{i1}(x_i, \tilde{b}_i) + L(K_2)p_{i2}(x_i, \tilde{b}_i)] \\
EU_i(x_i, \tilde{b}_i, s_i) &\geq EU_i(x_i, b_i, s_i) - (\tilde{b}_i - b_i) [L(K_1)p_{i1}(x_i, b_i) + L(K_2)p_{i2}(x_i, b_i)]
\end{aligned}$$

A few manipulations show that

$$\begin{aligned}
EU_i(x_i, b_i, s_i) - EU_i(x_i, \tilde{b}_i, s_i) &\geq (\tilde{b}_i - b_i) [L(K_1)p_{i1}(x_i, \tilde{b}_i) + L(K_2)p_{i2}(x_i, \tilde{b}_i)] \\
EU_i(x_i, b_i, s_i) - EU_i(x_i, \tilde{b}_i, s_i) &\leq (\tilde{b}_i - b_i) [L(K_1)p_{i1}(x_i, b_i) + L(K_2)p_{i2}(x_i, b_i)] \\
(\tilde{b}_i - b_i) [L(K_1)p_{i1}(x_i, b_i) + L(K_2)p_{i2}(x_i, b_i)] &\geq (\tilde{b}_i - b_i) [L(K_1)p_{i1}(x_i, \tilde{b}_i) + L(K_2)p_{i2}(x_i, \tilde{b}_i)]
\end{aligned}$$

Therefore, if $\tilde{b}_i > b_i$, $L(K_1)p_{i1}(x_i, b_i) + L(K_2)p_{i2}(x_i, b_i)$ is non-decreasing in b_i . Defining $L = (L(K_1), L(K_2))$, we can express this equation as $L \cdot p_i(x_i, b_i)$. With this property, we can also

simplify the individual rationality constraint to a single one:

$$EU(x, \underline{b}, s) \geq 0 \quad (\text{A.12})$$

The problem of the firm can now be simplified. From Equations (5.1) and (A.11), we know that

$$\int_{\underline{b}}^{b_i} (p_i(x_i, t)L)dt + EU_i(x_i, \underline{b}, s_i) = \int_{B_{-i}} (x_i(b_i, b_{-i})Lb_i - s_i(b_i, b_{-i}))g_{-i}(b_{-i})db_{-i}$$

Therefore,

$$\begin{aligned} E\Pi &= \int_B \sum_{i=1}^n s_i(b)g(b)db \\ &= \sum_{i=1}^n \left[\int_B b_i x_i(b)Lg(b)db - \int_{B_i} \int_{\underline{b}}^{b_i} \int_{B_{-i}} (x_i(t, b_{-i})Lg_{-i}(b_{-i})db_{-i} \cdot dt \cdot g(b_i)db_i - EU_i(x_i, \underline{b}, s_i) \right] \\ &= \sum_{i=1}^n \left[\int_B b_i x_i(b)Lg(b)db - EU_i(x_i, \underline{b}, s_i) \right] - \sum_{i=1}^n \left[\int_{\underline{b}}^{\bar{b}} \int_{\underline{t}}^{\bar{t}} \int_{B_{-i}} (x_i(t, b_{-i})Lg_{-i}(b_{-i})db_{-i} \cdot g(b_i)db_i \cdot dt \right] \\ &= \sum_{i=1}^n \left[\int_B b_i x_i(b)Lg(b)db - EU_i(x_i, \underline{b}, s_i) \right] - \sum_{i=1}^n \left[\int_{\underline{b}}^{\bar{b}} \int_{B_{-i}} (x_i(t, b_{-i})Lg_{-i}(b_{-i})db_{-i} \cdot (1 - G(t)) \cdot dt \right] \\ &= \sum_{i=1}^n \left[\int_B b_i x_i(b)Lg(b)db - EU_i(x_i, \underline{b}, s_i) \right] - \sum_{i=1}^n \left[\int_{\underline{b}}^{\bar{b}} \int_{B_{-i}} (x_i(b_i, b_{-i})Lg_{-i}(b_{-i})db_{-i} \cdot (1 - G(b_i)) \cdot db_i \right] \\ &= \sum_{i=1}^n \left[\int_B b_i x_i(b)Lg(b)db - \int_B (x_i(b)Lg(b)db \frac{(1 - G(b_i))}{g_i(b_i)} \right] - \sum_{i=1}^n [EU_i(x_i, \underline{b}, s_i)] \\ &= \sum_{i=1}^n \left[\int_B (b_i - \frac{(1 - G(b_i))}{g_i(b_i)}) \cdot x_i(b)Lg(b)db - EU_i(x_i, \underline{b}, s_i) \right] \end{aligned}$$

Define the virtual benefits of region i as $\beta_i(b_i) = b_i - \frac{1-G(b_i)}{g(b_i)}$. We make the usual assumption that the distribution function is regular: $\beta_i(b_i)$ is increasing in b_i . We can write the firm's expected revenues as

$$\sum_i \int_B \beta_i(b_i)x_i(b)Lg(b)db \quad (\text{A.13})$$

In doing so, we assume that at the optimum, $EU_i(x_i, \underline{b}, s_i) = 0$. From this assumption, and

Equations (5.1) and (A.11), we then find:

$$\begin{aligned}
EU_i(x_i, b_i, s_i) &= \int_{\underline{b}}^{b_i} p_i(x_i, t) L dt \\
&= \int_{B_{-i}} \int_{\underline{b}}^{b_i} x_i(t, b_{-i}) dt g_{-i}(b_{-i}) db_{-i} \\
&= \int_{B_{-i}} [b_i(x_i(b_i, b_{-i}))L - s_i(b_i, b_{-i})] g_{-i}(b_{-i}) db_{-i}
\end{aligned}$$

With this equation, we can express the equilibrium payments $s_i^*(b)$:

$$\begin{aligned}
\int_B [b_i(x_i(b))L - s_i(b)] g(b) db &= \int_B \int_{\underline{b}}^{b_i} x_i(t, b_{-i}) dt g(b) db \\
\int_B s_i(b) g(b) db &= \int_B b_i(x_i(b))L g(b) db - \int_B \int_{\underline{b}}^{b_i} x_i(t, b_{-i}) dt g(b) db \\
s_i^*(b) &= b_i(x_i^*(b))L - \int_{\underline{b}}^{b_i} x_i^*(t, b_{-i}) dt
\end{aligned}$$

Assuming $x^*(b)$ is the allocation function that solves the firm's problem, we can then find the optimal payment function $s_i^*(b)$:

$$s_i^*(b) = b_i x_i^*(b) \cdot L - \int_{\underline{b}}^{b_i} x_i^*(t, b_{-i}) L dt \quad (\text{A.14})$$

The optimisation problem can therefore be expressed as follows. Let $x^*(b)$ be the solution to the following problem:

$$\begin{aligned}
\max_{x(b)} \quad & \sum_i \int_B \left[\beta_i(b_i) x_i(b) L + \sum_{j=1}^2 \pi(K_j) x_{ij}(b) \right] g(b) db \\
s.t. \quad & EU_i(x_i, \underline{b}, s_i) = 0 \quad \forall i \\
& (\tilde{b}_i - b_i) [p_i(x_i, b_i) \cdot L] \geq (\tilde{b}_i - b_i) [p_i(x_i, \tilde{b}_i) \cdot L] \quad \forall b_i < \tilde{b}_i \\
& \sum_{i=1}^n x_{ij} \leq 1 \quad \forall j = 1, 2 \\
& x_{ij}(b) \geq 0 \quad \forall i, j \\
& x_{i1}(b) + x_{i2}(b) \leq 1 \quad \forall i
\end{aligned}$$

Let $s_i^*(b)$ be given by:

$$s_i^*(b) = b_i(x_i^*(b))L - \int_{\underline{b}}^{b_i} x_i^*(t, b_{-i}) dt$$

Then, (x^*, t^*) is the optimal mechanism. □