Competency Trap, Optimal Contracts and Limited Liability*

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Abstract

We consider a principal-agent model in which the agent receives private information about an unknown state of the world and must then take an action accordingly. There are junior and senior agents, and seniors receive a bonus when taking an action that they already took in the past. We can think of this bonus as the result of learning-by-doing, as repeating an action is less costly than taking a new one. To avoid "competency trap", a situation in which the senior simply takes the same action than in the past, we characterize the optimal incentive schemes. We however find that under a wide range of parameters incentive contracts are too costly and the principal prefers to hire a junior, who is less efficient than a senior. This occurs for all ranges of parameters (whether learning-by-doing is low, intermediate or strong), depending on how precise the private information of the agent is.

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1 Introduction

Agents are generally tempted to repeat actions already taken in the past. An important reason for this is learning by doing, or experience: it is less costly to repeat a past process than to implement a new one, even if it seems better adapted to the present situation. This phenomenon is known as "competency trap," as suggested by Levitt & March (1988) and formalized analytically by Jovanovic & Nyarko (1996). An agent can be so skilled at some technology that he will never switch to a superior one. Competency trap is a well-identified perverse phenomenon, which may have dramatic consequences, especially for firms: they might overlook important opportunities, launch products that

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are born obsolete, and finally be pushed out of their core market by agile and innovative rivals. Famous examples include Kodak film being a late arrival to the digital industry, IBM allowing Microsoft to develop the PC operating platform (DOS and later Windows) or Chrysler focusing on minivans during the 1980's to the point of missing the rising popularity of SUVs. Avoiding competency trap is therefore one of the most important challenges that a firm faces, especially in evolving economic environments.

In this paper we introduce learning-by-doing in a Principal-Agent relationship. There is an unknown state of the world, and the agent must choose a technology (or, more generally an action) that matches the state of the world for the principal. The agent is better informed than the principal as to which technology should be chosen given the present environment.

There are two types of agents, juniors and seniors. Senior agents – contrary to junior ones, benefit from experience and, because of learning-by-doing, are more efficient in implementing an action already taken in the past. This takes the form of an added bonus to his utility, which one can think of as a reduction in the cost of effort in taking that action compared to another action. The problem faced by the principal is therefore to design an incentive contract such that the agent will take the "best" action rather than the "easiest" one - that is, the principal must use incentive contracts to avoid the competency trap.

We however find that in some circumstances it is too costly to avoid the competency trap by providing incentives to a senior agent. In that case, the optimal solution for the principal is to hire a junior agent, who is less efficient than a senior one as he does not benefit from any experience and learning-by-doing. This can occur when learning-by-doing is low and the expertise of the agent is also low, but can also happen when learning-by-doing is high and the expertise of the agent is high. When learning-by-doing is high, one of the main problems in hiring a senior will be that providing incentives will be too expensive as the principal reaches a corner solution because of limited liability.

We also consider the possibility of screening for senior agents who have performed in the past the action that is most likely going to be satisfactory for the principal (this can be done only when learning-by-doing is small), which allows to reduce the average wage that needs to be paid, but we still find that this may be too costly and so that a junior agent may be preferred.

The rest of the paper is organized as follows: in Section 2, we present the model; in Section 3 we show how agents revise their beliefs after receiving a signal; in Section 4 we consider flat wage schemes for junior and senior agents; in Section 5 we consider incentive contracts for seniors; in Section 6 we consider incentive contracts that screen agents who have performed in the past the task that is most likely suited currently for the principal; in Section 7 we compare the principal's profits from various contracts with the aim of understanding when a junior agent will be preferred; finally Section 8 summarizes the various cases in which incentive schemes to avoid the competency trap are too expensive and a junior will be hired.

2 Model

Players and actions. There is one period and there are two players, a principal P and an agent A. The agent must choose an action $a \in \{a_0, a_1\}$.

Information structure. The underlying state of the world is $\omega \in \Omega := \{\omega_0, \omega_1\}$ and the prior probability is given by $\mathbb{P}(\omega = \omega_0) = p$, where we can assume without loss of generality that $p \geq 1/2$. The agent receives a private signal $s \in \{s_0, s_1\}$, which has accuracy $\alpha > p$: f

$$\mathbb{P}(s = s_i \mid \omega = \omega_i) = \alpha. \quad i = 0, 1.$$

The action taken by A is not observed by P, and at the end of the game an observable and verifiable signal $y \in \{y_0, y_1\}$ is realized in the following way: $\mathbb{P}(y = y_1 \mid a_0, \omega_0) = \mathbb{P}(y = y_1 \mid a_1, \omega_1) = 1$ and $\mathbb{P}(y = y_1 \mid a_1, \omega_0) = \mathbb{P}(y = y_1 \mid a_0, \omega_1) = 0$. That is, the signal fully reflect whether the action chosen matches the state of the world.

Contracts. The principal offers a contract to the agent. A contract consists in a pair of wages (w_0, w_1), such that A will receive w_k if and only if the realized signal at the end of the game is y_k, $k = 0, 1$.

Juniors and seniors. There are junior and senior agents in the population. Seniors have already worked previously, having chosen the action a_0 with probability $\tilde{p}$ in the past.

Payoffs and learning-by-doing. The principal is risk neutral and his payoff is given by $\mathbb{E}(y - w)$. The agent is risk averse and has a utility $u(w)$ when given a wage w, where u is a strictly concave function.

Seniors receive a bonus b if they choose the same action than in the past, in which case their utility is $u(w) + b$. This models learning-by-doing; indeed taking the same action than in the past now requires less effort for a senior worker.[1]

The agent has an outside option of $\bar{u} > b$ (whether senior or junior).

3 Bayesian updating

In this section we compute the posterior belief of A over ω. By Bayes' rule, we have:

$$\begin{aligned}
\mathbb{P}(\omega = \omega_0 \mid s = s_0) &= \frac{\mathbb{P}(s_0 \mid \omega_0)\mathbb{P}(\omega_0)}{\mathbb{P}(s_0 \mid \omega_0)\mathbb{P}(\omega_0) + \mathbb{P}(s_0 \mid \omega_1)\mathbb{P}(\omega_1)} \\
&= \frac{\alpha p}{\alpha p + (1 - \alpha)(1 - p)},
\end{aligned}$$

[1]The benefit is outside of the utility function because we can think of it as a reduction in the cost of effort, and we can assume that the utility function of the agent takes the standard quasi-linear form $U(w, c) = u(w) - c$.

and

$$
\begin{aligned}
\mathbb{P}(\omega = \omega_1 \mid s = s_1) &= \frac{\mathbb{P}(s_1 \mid \omega_1)\mathbb{P}(\omega_1)}{\mathbb{P}(s_1 \mid \omega_1)\mathbb{P}(\omega_1) + \mathbb{P}(s_1 \mid \omega_0)\mathbb{P}(\omega_0)} \\
&= \frac{\alpha(1-p)}{\alpha(1-p) + (1-\alpha)p}.
\end{aligned}
$$

We denote by q the random variable which correspond to the signal received by A being correct. That is $q(s_0, \alpha) = \mathbb{P}(\omega = \omega_0 \mid s = s_0, \alpha)$ and $q(s_1, \alpha) = \mathbb{P}(\omega = \omega_1 \mid s = s_1, \alpha)$.

Note that when the signal s_1 (the less likely of the two signals) is received, the probability that the signal is accurate is greater than $1/2$ as long as $\alpha \geq p$. That is, the signal must be more precise that the prior if it is to reverse the prior.

Also note that Bayesian updating is easily expressed in terms of change in the log-likelihood. If we let $p' = \mathbb{P}(\omega^t = \omega_0 \mid s^t)$ be the posterior probability that the state is ω_0, then we have

$$
\ln \frac{p'}{1-p'} = \ln \frac{p}{1-p} \pm \ln \frac{\alpha}{1-\alpha},
$$

where the sign in front of $\ln(\alpha/(1-\alpha))$ will be positive if the signal is s_0 and negative if the signal is s_1.

4 Flat wage

Suppose that P offers a flat wage. Then the junior agent follows his signal (he is indifferent) while the senior agent takes the same action than in the past. Therefore the wage for a junior will be w_j such that $u(w_j) = \bar{u}$ while the wage for a senior will be w_s such that $u(w_s) = \bar{u} - b$.

The profit from hiring a junior is therefore given by

$$
\Pi_J = \alpha y_1 + (1-\alpha) y_0 - u^{-1}(\bar{u}). \tag{1}
$$

To find the profit resulting from hiring a senior with a flat wage, we must find the probability with which that senior will generate the successful outcome y_1. Recall that we assumed a senior had chosen the action a_0 with probability $\tilde{p}$ in the past. Such an agent will be successful if he chose a_0 in the past and the state today is w_0, which occurs with probability $\tilde{p}p$, or if he chose a_1 in the past and the state today is w_1, which occurs with probability $(1-\tilde{p})(1-p)$, so that the probability of success is given by $\tilde{p}p + (1-\tilde{p})(1-p)$. With the complementary probability the senior will not be successful. Hence the profit from hiring a senior with a flat wage is given by

$$
\Pi_{SN} = \Big[\tilde{p}p + (1-\tilde{p})(1-p)\Big] y_1 + \Big[1 - \tilde{p}p - (1-\tilde{p})(1-p)\Big] y_0 - u^{-1}(\bar{u} - b).
$$

In practice, we assume that the senior was a junior in the previous period and therefore followed his signal, so that he chose action a_0 when receiving the signal s_0, which occurred with probability $p\alpha+(1-p)(1-\alpha) = \tilde{p}$. Hence we have $\tilde{p}p+(1-\tilde{p})(1-p) = \alpha(2p-1)^2 + 2p(1-p)$ and Π_{SN} can be rewritten as

$$\Pi_{SN} = \Big[\alpha(2p-1)^2 + 2p(1-p)\Big]y_1 + \Big[1 - \alpha(2p-1)^2 - 2p(1-p)\Big]y_0 - u^{-1}\left(\bar{u} - b\right). \quad (2)$$

Note that because $\alpha > 1/2$, we have $\alpha(2p-1)^2 + 2p(1-p) < \alpha$, so that a senior with a flat wage is less likely to generate a success than a junior. However, the senior is cheaper to hire.

5 Incentive contract for seniors

5.1 Incentive compatibility constraints

There are four incentive compatibility constraints, depending on the action previously taken by the agent and the signal received.

First, assume that the signal s is the same than the action previously taken by the agent. In that case, if the agent follows his signal, he will get a bonus b. Therefore the agent follows his signal if

$$q(s,\alpha)u(w_1) + (1 - q(s,\alpha))u(w_0) + b \geq q(s,\alpha)u(w_0) + (1 - q(s,\alpha)u(w_1),$$

for $s \in \{s_0, s_1\}$.

If however A receives a signal s that differs from his past experience then A would not get a bonus when following his signal. A will therefore follow his signal if

$$q(s,\alpha)u(w_1) + (1 - q(s,\alpha))u(w_0) \geq q(s,\alpha)u(w_0) + (1 - q(s,\alpha))u(w_1) + b,$$

for $s \in \{s_0, s_1\}$.

First, note that for any s, if the constraint is satisfied when the signal is different than A's previous experience then it is trivially satisfied when s is similar to previous experience, since in the former case A receives the bonus when going against his signal, while in the latter case A receives the bonus when following his signal. Therefore we only need to consider the constraints

$$\begin{aligned} q(s_0,\alpha)u(w_1) + (1 - q(s_0,\alpha))u(w_0) &\geq q(s_0,\alpha)u(w_0) + (1 - q(s_0,\alpha))u(w_1) + b \\ q(s_1,\alpha)u(w_1) + (1 - q(s_1,\alpha))u(w_0) &\geq q(s_1,\alpha)u(w_0) + (1 - q(s_1,\alpha))u(w_1) + b \end{aligned}$$

Consider the second constraint. Because $q(s_1, \alpha) > 1/2$, for the lhs to be greater than the rhs, we must have $u(w_1) > u(w_0)$. Now note that because we must have $u(w_1) > u(w_0)$, and because $q(s_0, \alpha) > q(s_1, \alpha)$,[2] if the second constraint holds then so will the first.

Therefore the remaining incentive compatibility constraint to consider is:

$$q(s_1,\alpha)u(w_1) + (1 - q(s_1,\alpha))u(w_0) \geq q(s_1,\alpha)u(w_0) + (1 - q(s_1,\alpha))u(w_1) + b.$$

[2]Because state ω_0 is initially more likely.

That is, it is most difficult to give incentives to a worker who chose action a_0 in the past and now receives a signal that indicates he should choose a_1. This constraint rewritten as

$$u(w_1) - u(w_0) \geq \frac{b}{2q(s_1, \alpha) - 1} = \frac{p(1-\alpha) + (1-p)\alpha}{\alpha - p} b$$

5.2 Individual rationality constraint

There are two individual rationality constraints, depending on the previous action taken by the senior. We will only consider the constraint for seniors who took action a_1 in the past, as it is more stringent than for others.

The distribution over state of the worlds and outcomes is given by

$$\begin{aligned}
\mathbb{P}[(\omega, s) = (\omega_0, s_0)] &= p\alpha \\
\mathbb{P}[(\omega, s) = (\omega_0, s_1)] &= p(1-\alpha) \\
\mathbb{P}[(\omega, s) = (\omega_1, s_1)] &= (1-p)\alpha \\
\mathbb{P}[(\omega, s) = (\omega_1, s_0)] &= (1-p)(1-\alpha),
\end{aligned}$$

and since we assume that the past action of the agent was a_1, the individual rationality constraint is given by:

$$p\alpha u(w_1) + p(1-\alpha)[u(w_0) + b] + (1-p)\alpha[u(w_1) + b] + (1-p)(1-\alpha)u(w_0) \geq \bar{u},$$

which simplifies to[3]

$$\alpha u(w_1) + (1-\alpha)u(w_0) + b\Big[p(1-\alpha) + (1-p)\alpha\Big] \geq \bar{u}.$$

The wages will depend only on whether the signals are accurate or not, while the bonus will depend on both on the state of the world and the accuracy of the signal.

5.3 Profit maximization

The Principal's profit maximization problem, while giving incentives to the Agent to follow his signal, becomes:

$$\max_{w_0, w_1} \alpha(y_1 - w_1) + (1-\alpha)(y_0 - w_0)$$

subject to

$$u(w_1) - u(w_0) \geq \frac{p(1-\alpha) + (1-p)\alpha}{\alpha - p} b \qquad (IC)$$

$$\alpha u(w_1) + (1-\alpha)u(w_0) + b\Big[p(1-\alpha) + (1-p)\alpha\Big] \geq \bar{u} \qquad (IR)$$

[3] In the case of a senior having previously played action a_0, the probability of receiving a bonus would be given by $p\alpha + (1-p)(1-\alpha) \geq p(1-\alpha) + (1-p)\alpha$ as we have $p \geq 1/2$ and $\alpha \geq 1/2$.

Note that the individual rationality constraint must be binding, otherwise the principal could reduce w_0 without affecting IC.[4]

The Lagrangian is given by

$$\begin{aligned}\mathcal{L} = \alpha(y_1 - w_1) + (1-\alpha)(y_0 - w_0) \\ + \lambda\Big[u(w_1) - u(w_0) - \frac{p(1-\alpha)+(1-p)\alpha}{\alpha - p}b\Big] \\ + \mu\Big[\alpha u(w_1) + (1-\alpha)u(w_0) + b[p(1-\alpha)+(1-p)\alpha] - \bar{u}\Big],\end{aligned}$$

and first order conditions give

$$\begin{aligned}\frac{1}{u'(w_0)} &= \mu - \frac{1}{1-\alpha}\lambda \\ \frac{1}{u'(w_1)} &= \mu + \frac{1}{\alpha}\lambda\end{aligned}$$

along with the complementary slackness conditions.

Note that we must have $\lambda > 0$, otherwise we have $w_0 = w_1$ and IC cannot hold. Hence both constraints must be binding. This gives the following solutions:

$$\begin{aligned}u(w_0) &= \bar{u} - \alpha\frac{p(1-\alpha)+(1-p)\alpha}{\alpha - p}b - b[p(1-\alpha)+(1-p)\alpha] \\ u(w_1) &= \bar{u} + (1-\alpha)\frac{p(1-\alpha)+(1-p)\alpha}{\alpha - p}b - b[p(1-\alpha)+(1-p)\alpha]\end{aligned}$$

These solutions can be rewritten as:

$$\begin{aligned}u(w_0) &= \bar{u} - \Big[1 + \frac{\alpha}{\alpha - p}\Big]\Big[p(1-\alpha)+(1-p)\alpha\Big]b \\ u(w_1) &= \bar{u} + \Big[\frac{1-\alpha}{\alpha - p} - 1\Big]\Big[p(1-\alpha)+(1-p)\alpha\Big]b.\end{aligned}$$

Note that $u(w_1) < \bar{u}$ when $\alpha > (1+p)/2$. Assuming there is no problem of limited liability, the profit from hiring a senior while providing incentives is given by

$$\begin{aligned}\Pi_{SI} = \alpha y_1 + (1-\alpha)y_0 \\ - \alpha u^{-1}\Bigg[\bar{u} + \Big[\frac{1-\alpha}{\alpha - p} - 1\Big][p(1-\alpha)+(1-p)\alpha]b\Bigg] \\ - (1-\alpha)u^{-1}\Bigg[\bar{u} - \Big[1 + \frac{\alpha}{\alpha - p}\Big][p(1-\alpha)+(1-p)\alpha]b\Bigg]. \quad (3)\end{aligned}$$

[4]Provided it is possible to reduce w_0, so that we are not facing a binding limited liability constraint. We discuss the case with limited liability in the next subsection.

5.4 Limited liability

If $b > \frac{\bar{u}}{\left(1+\frac{\alpha}{\alpha-p}\right)(p(1-\alpha)+(1-p)\alpha)}$ then we would have a negative utility in the case of poor performance. Assuming this is not feasible because of limited liability, we would have

$$u(w_0) = 0,$$

and using the incentive compatibility constraint, we would have

$$u(w_1) = \frac{p(1-\alpha)+(1-p)\alpha}{\alpha - p} b.$$

The profit then becomes

$$\Pi_{SI} = \alpha y_1 + (1-\alpha)y_0 - \alpha u^{-1}\left[\frac{p(1-\alpha)+(1-p)\alpha}{\alpha-p} b\right] - (1-\alpha)u^{-1}(0). \quad (4)$$

6 Screening: hiring only seniors who performed a_0

When hiring a senior who performed a_0, the incentive constraint will be the same as above, that is $u(\tilde{w}_1) - u(\tilde{w}_0) \geq \frac{p(1-\alpha)+(1-p)\alpha}{\alpha-p} b$, but the individual rationality constraint now becomes $\alpha u(\tilde{w}_1) + (1-\alpha)u(\tilde{w}_0) + b[p\alpha + (1-p)(1-\alpha)] \geq \bar{u}$, which can be rewritten as $\alpha u(\tilde{w}_1) + (1-\alpha)u(\tilde{w}_0) + b[p(1-\alpha)+(1-p)\alpha] \geq \bar{u} - b(2\alpha-1)(2p-1)$. Hence the solution will be similar to the one in the previous section, if the agent were to have a lower $\bar{u}$ given by $\bar{u} - b(2\alpha-1)(2p-1)$. Thus we have

$$\begin{aligned} u(\tilde{w}_0) &= \bar{u} - b(2\alpha-1)(2p-1) - \left[1 + \frac{\alpha}{\alpha-p}\right]\left[p(1-\alpha)+(1-p)\alpha\right]b \\ u(\tilde{w}_1) &= \bar{u} - b(2\alpha-1)(2p-1) + \left[\frac{1-\alpha}{\alpha-p} - 1\right]\left[p(1-\alpha)+(1-p)\alpha\right]b, \end{aligned}$$

and the profit from hiring an a_0 senior while providing incentives is given by

$$\begin{aligned} \Pi_{SIa_0} &= \alpha y_1 + (1-\alpha)y_0 \\ &\quad - \alpha u^{-1}\left[\bar{u} - b(2\alpha-1)(2p-1) + \left[\frac{1-\alpha}{\alpha-p} - 1\right][p(1-\alpha)+(1-p)\alpha]b\right] \\ &\quad - (1-\alpha)u^{-1}\left[\bar{u} - b(2\alpha-1)(2p-1) - \left[1+\frac{\alpha}{\alpha-p}\right][p(1-\alpha)+(1-p)\alpha]b\right]. \quad (5) \end{aligned}$$

6.1 Limited liability

If $b > \frac{\bar{u}}{\left(1+\frac{\alpha}{\alpha-p}\right)(p(1-\alpha)+(1-p)\alpha)+(2\alpha-1)(2p-1)}$ then we would have a negative utility in the case of poor performance. Assuming this is not feasible because of limited liability, we would have to impose $u(w_0) = 0$ and use the incentive compatibility constraint to find $u(w_1)$. This is as in Section 5.4, and therefore if we run into a limited liability problem it is no longer possible to screen agents.

7 Hiring a junior: the competency trap

In what follows, α, p and $\bar{u}$ are fixed, and we study whether or not the principal would like to hire a junior worker, depending on the value of b. We need to consider three cases: (*i*) $b \in [\bar{b}, \bar{u}]$; (*ii*) $b \in [\underline{b}, \bar{b}]$; and (*iii*) $b < \underline{b}$, where the thresholds are given by

$$\underline{b} = \frac{\bar{u}}{\left(1 + \frac{\alpha}{\alpha - p}\right)(p(1-\alpha) + (1-p)\alpha) + (2\alpha - 1)(2p - 1)},$$
$$\bar{b} = \frac{\bar{u}}{\left(1 + \frac{\alpha}{\alpha - p}\right)(p(1-\alpha) + (1-p)\alpha)}.$$

We consider the following parametric specification: $u(x) = x^{1/2}$, which yields $u^{-1}(x) = x^2$, $y_1 = y > 0$ and $y_0 = 0$.

7.1 Junior vs non-incentivized senior

In that case we do not need to distinguish between different values of b, since both contracts are available for any value of b, and profits are given by

$$\Pi_J = \alpha y - \bar{u}^2$$
$$\Pi_{SN} = \left[\alpha(2p-1)^2 + 2p(1-p)\right] y - (\bar{u} - b)^2 .$$

The junior worker is preferred to the senior worker when

$$\Pi_J > \Pi_{SN} \Leftrightarrow \alpha y - \bar{u}^2 > \left[\alpha(2p-1)^2 + 2p(1-p)\right] y - (\bar{u} - b)^2$$
$$\Leftrightarrow 2(2\alpha - 1)p(1-p)y + b^2 - 2\bar{u}b > 0.$$

Let $f(b) = 2(2\alpha - 1)p(1-p)y + b^2 - 2\bar{u}b$. Then we have $f(0) = 2(2\alpha - 1)p(1-p)y > 0$, so that for low values of b a junior is always preferred.

We note that $f'(b) = 2(b - \bar{u}) < 0$, so that the function is decreasing, and that $f(\bar{u}) = 2(2\alpha - 1)p(1-p)y + -\bar{u}^2$. If $f(\bar{u}) > 0$ then the junior is always preferred, but if $f(\bar{u}) < 0$ then there is a unique threshold $\widetilde{b}$ such that for $b \leq \widetilde{b}$ the junior is preferred and for $b > \widetilde{b}$ the non-incentivized senior is preferred.

The condition $f(\bar{u}) < 0$ is equivalent to $y < \frac{\bar{u}^2}{2(2\alpha-1)p(1-p)}$, in which case the threshold is given by $f(\widetilde{b}) = 0$, which gives

$$\widetilde{b} = \bar{u} - \sqrt{\bar{u}^2 - 2(2\alpha - 1)p(1-p)y} \in (0, \bar{u}).$$

To summarize: If y is high, then the junior is always preferred to the non-incentivized senior. This is because the junior is more accurate, which matters when stakes are high. For a low value of y, then if the bonus is low the junior is preferred, but if the bonus is above the threshold $\widetilde{b}$ then the non-incentivized senior is preferred to the junior, as his efficiency gains can be captured in the form of a lower wage. Note that the threshold can take any value in the interval $(0, \bar{u})$ depending on the value of y.

7.2 Junior vs incentivized senior

For the incentivized senior, we need to distinguish two cases: $b > \bar{b}$, in which case there is limited liability, and $b < \bar{b}$, in which case there is not.

7.2.1 Limited liability: $b \geq \bar{b}$

In that case, profits are given by

$$\begin{aligned} \Pi_J &= \alpha y - \bar{u}^2, \\ \Pi_{SI} &= \alpha y - \alpha \left[\frac{p(1-\alpha) + (1-p)\alpha}{\alpha - p} b \right]^2, \end{aligned}$$

and the junior is preferred when

$$\begin{aligned} \Pi_J > \Pi_{SI} \Leftrightarrow \alpha y - \bar{u}^2 &> \alpha y - \alpha \left[\frac{p(1-\alpha) + (1-p)\alpha}{\alpha - p} b \right]^2 \\ b &> \frac{\bar{u} \times (\alpha - p)}{\sqrt{\alpha} \times [p(1-\alpha) + (1-p)\alpha]} := b^* \end{aligned}$$

We now check whether the threshold b^* is indeed in the interval $\left(\bar{b}, \bar{u}\right)$. First note that

$$\begin{aligned} b^* > \bar{b} \Leftrightarrow & \frac{\bar{u} \times (\alpha - p)}{\sqrt{\alpha} \times [p(1-\alpha) + (1-p)\alpha]} > \frac{\bar{u}}{\left(1 + \frac{\alpha}{\alpha - p}\right)(p(1-\alpha) + (1-p)\alpha)} \\ & \Leftrightarrow 2\alpha - p > \sqrt{\alpha} \\ & \Leftrightarrow 4\alpha^2 - 4\alpha p + p^2 > \alpha \\ & \Leftrightarrow 4\alpha^2 - \alpha(4p+1) + p^2 > 0. \end{aligned}$$

The function $g(\alpha) = 4\alpha^2 - \alpha(4p+1) + p^2$ is increasing in α and such that $g(p) = -p(1-p) < 0$ and $g(1) = 3 + p^2 - 4p > 0$. Therefore, when α is close to p, the threshold is below $\bar{b}$, and hence in that case the junior will always be preferred to the incentivized senior under limited liability. The idea is that providing incentives is too costly when α and p are close.

If on the other hand α is sufficiently large, so that $4\alpha^2 - \alpha(4p+1) + p^2 > 0$, which is equivalent to having $\alpha > \frac{4p+1+\sqrt{8p+1}}{8}$ then the threshold b^* is strictly greater than $\bar{b}$, so that for values of $b \in [\bar{b}, b^*]$ then the incentivized senior is preferred to the junior, while for values of b higher than b^* the junior is preferred.

Note that it may seem counter-intuitive that the junior is preferred when b is large, since b can partly be captured by the principle in the form of lower wages for the senior. However in that case, because of limited liability, not enough of the bonus can be captured and it becomes too costly to provide incentives. Note that we can also check that the threshold b^* is lower than $\bar{u}$.

To summarize: When α is close to p, that is when $\alpha < \frac{4p+1+\sqrt{8p+1}}{8}$, it is always preferable to hire a junior. When α is sufficiently large, then the senior is preferred if $b < b^*$ and the junior is preferred if $b > b^*$.

7.2.2 No limited liability: $b < \bar{b}$

In that case profits are given by

$$\begin{aligned}
\Pi_J &= \alpha y - \bar{u}^2 \\
\Pi_{SI} &= \alpha y - \alpha\Big[\bar{u} + \frac{1-2\alpha+p}{\alpha-p}[p(1-\alpha)+(1-p)\alpha]b\Big]^2 \\
&\quad - (1-\alpha)\Big[\bar{u} - \frac{2\alpha-p}{\alpha-p}[p(1-\alpha)+(1-p)\alpha]b\Big]^2 \\
&= \alpha y - \alpha\left(\bar{u}+\kappa_1\nu b\right)^2 - (1-\alpha)\left(\bar{u}-\kappa_0\nu b\right)^2,
\end{aligned}$$

where

$$\begin{aligned}
\kappa_0 &= \frac{2\alpha-p}{\alpha-p}; \\
\kappa_1 &= \frac{1-2\alpha+p}{\alpha-p} = \frac{1}{\alpha-p} - \kappa_0; \\
\nu &= p(1-\alpha)+(1-p)\alpha.
\end{aligned}$$

The Principal chooses a junior over an incentivized senior if

$$\begin{aligned}
\Pi_J > \Pi_{SI} &\Leftrightarrow \alpha y - \bar{u}^2 > \alpha y - \alpha\left(\bar{u}+\kappa_1\nu b\right)^2 - (1-\alpha)\left(\bar{u}-\kappa_0\nu b\right)^2 \\
&\Leftrightarrow b > \frac{2\bar{u}[(1-\alpha)\kappa_0 - \alpha\kappa_1]}{\nu[\alpha\kappa_1^2 + (1-\alpha)\kappa_0^2]} \\
&\Leftrightarrow b > \frac{2\bar{u}}{p(1-\alpha)+(1-p)\alpha} \times \frac{(\alpha-p)^2}{\alpha-2\alpha p+p^2} := \hat{b}
\end{aligned}$$

Note that when $\kappa_1 \leq 0$ then this inequality cannot be satisfied. This is because in that case the wages offered to the incentivized senior are uniformly lower than the wage offered to the junior. This can be seen easily from the first line. Alternatively, it would imply for the last line that the expression on the right-hand side is greater than one.

Therefore for the junior to be preferred to the incentivized senior, it must be the case that $\alpha < (1+p)/2$. When this is the case, the junior is preferred to the senior when b is high. To understand why, note that as b increases, the low utility of the senior decreases while the high utility increases, but faster. So overall, the expected wage paid to the senior will be increasing in b.

Note that we assumed we are in the case were $b \leq \bar{b}$. We therefore must compare $\bar{b}$ with $\hat{b}$, namely, do we have $\hat{b} < \bar{b}$. It turns out this is not always the case:

$$\hat{b} < \bar{b} \Leftrightarrow \frac{2\bar{u}}{p(1-\alpha)+(1-p)\alpha} \times \frac{(\alpha-p)^2}{\alpha-2\alpha p+p^2} < \frac{\bar{u}}{\left(1+\frac{\alpha}{\alpha-p}\right)(p(1-\alpha)+(1-p)\alpha)}$$
$$\Leftrightarrow 4\alpha^2 - \alpha(4p+1) + p^2 < 0.$$

We have studied this inequality in the case with limited liability, it is equivalent to $\alpha < \frac{4p+1+\sqrt{8p+1}}{8}$.

Hence for low values of α, namely $\alpha < \frac{4p+1+\sqrt{8p+1}}{8}$, the threshold $\hat{b}$ is indeed smaller than $\bar{b}$. In that case, for low values of b, that is $b < \hat{b}$, the senior is preferred to the junior. And for high values of b, that is $b \geq \hat{b}$, it is the junior who is preferred.

If on the other hand $\alpha \in (\frac{4p+1+\sqrt{8p+1}}{8}, \frac{1+p}{2})$ then it is the senior who is preferred to the junior.

We then check whether or not $\hat{b} < \underline{b}$:

$$\hat{b} < \underline{b} \Leftrightarrow \alpha^3[4-8p] + \alpha^2[8p^2+10p-5] + \alpha[-2p^3-11p^2-p+2] + p^3 + 4p^2 - 2p < 0$$

It turns out that the function of α is increasing and is positive when $\alpha = \frac{4p+1+\sqrt{8p+1}}{8}$. Therefore there is a value $\alpha^* < \frac{4p+1+\sqrt{8p+1}}{8}$ such that for $\alpha < \alpha^*$ we have $\hat{b} < \underline{b}$ and for $\alpha > \alpha*$ we have $\hat{b} > \underline{b}$.

To summarize: When α is higher than $\frac{4p+1+\sqrt{8p+1}}{8}$ then the senior is always preferred to the junior. When α is lower than $\frac{4p+1+\sqrt{8p+1}}{8}$ then there is a threshold $\hat{b}$ such that for $b < \hat{b}$ the senior is preferred and for $b > \hat{b}$ the junior is preferred.

7.3 Junior vs screened incentivized senior

It is possible to screen seniors who performed a_0 in the past only when $b < \underline{b}$. In that case profits are given by

$$\begin{aligned}
\Pi_J &= \alpha y - \bar{u}^2 \\
\Pi_{SIa_0} &= \alpha y - \alpha\Big[\bar{u} - b(2\alpha-1)(2p-1) + \frac{1-2\alpha+p}{\alpha-p}[p(1-\alpha)+(1-p)\alpha]b\Big]^2 \\
&\quad - (1-\alpha)\Big[\bar{u} - b(2\alpha-1)(2p-1) - \frac{2\alpha-p}{\alpha-p}[p(1-\alpha)+(1-p)\alpha]b\Big]^2 \\
&= \alpha y - \alpha\left(\bar{u} + (\kappa_1\nu - \kappa)b\right)^2 - (1-\alpha)\left(\bar{u} - (\kappa_0\nu + \kappa)b\right)^2,
\end{aligned}$$

where

$$\kappa = (2\alpha - 1)(2p - 1)$$
$$\kappa_0 = \frac{2\alpha - p}{\alpha - p};$$
$$\kappa_1 = \frac{1 - 2\alpha + p}{\alpha - p};$$
$$\nu = p(1-\alpha) + (1-p)\alpha.$$

The Principal chooses a junior over a screened incentivized senior if

$$\Pi_J > \Pi_{SIa_0} \Leftrightarrow \alpha y - \bar{u}^2 > \alpha y - \alpha\left(\bar{u} + (\kappa_1\nu - \kappa)b\right)^2 - (1-\alpha)\left(\bar{u} - (\kappa_0\nu + \kappa)b\right)^2$$

Note that when $\kappa_1\nu - \kappa \leq 0$ then this inequality cannot be satisfied. This is because in that case the wages offered to the screened incentivized senior are uniformly lower than the wage offered to the junior. This can be seen easily from the first line. That case corresponds to

$$\alpha \geq \frac{2-p}{3-2p}.$$

Note that this is a lower threshold than with the non-screened incentivized senior, as $\frac{2-p}{3-2p} < \frac{1+p}{2}$. Therefore for the junior to be preferred to the incentivized senior, it must be the case that $\alpha < \frac{2-p}{3-2p}$.

When $\alpha < \frac{2-p}{3-2p}$, then the junior is preferred to the senior when

$$\Pi_J > \Pi_{SIa_0} \Leftrightarrow \alpha y - \bar{u}^2 > \alpha y - \alpha\left(\bar{u} + (\kappa_1\nu - \kappa)b\right)^2 - (1-\alpha)\left(\bar{u} - (\kappa_0\nu + \kappa)b\right)^2$$
$$\Leftrightarrow b > \frac{2\bar{u}[(1-\alpha)\kappa_0\nu - \alpha\kappa_1\nu + \kappa]}{\alpha(\kappa_1\nu - \kappa)^2 + (1-\alpha)(\kappa_0\nu + \kappa)^2} := b^\dagger$$

We need to check that the the threshold $b^\dagger$ is consistent with being able to screen senior workers, that is that $b^\dagger < \underline{b}$. First note that we can rewrite $\underline{b}$ as

$$\underline{b} = \frac{\bar{u}}{\kappa_0\nu + \kappa}$$

We then have

$$b^\dagger < \underline{b} \Leftrightarrow \frac{2\bar{u}[(1-\alpha)\kappa_0\nu - \alpha\kappa_1\nu + \kappa]}{\alpha(\kappa_1\nu - \kappa)^2 + (1-\alpha)(\kappa_0\nu + \kappa)^2} < \frac{\bar{u}}{\kappa_0\nu + \kappa}$$
$$\Leftrightarrow 2(\kappa_0\nu + \kappa)[(\kappa_0\nu + \kappa) - \alpha(\kappa_0 + \kappa_1)\nu] < \alpha(\kappa_1\nu - \kappa)^2 + (1-\alpha)(\kappa_0\nu + \kappa)^2$$
$$\Leftrightarrow 2(\kappa_0\nu + \kappa)^2 - 2\alpha(\kappa_0\nu + \kappa)(\kappa_0 + \kappa_1)\nu < (\kappa_0\nu + \kappa)^2 + \alpha[(\kappa_1\nu - \kappa)^2 - (\kappa_0\nu + \kappa)^2]$$
$$\Leftrightarrow 2(\kappa_0\nu + \kappa)^2 - 2\alpha(\kappa_0\nu + \kappa)(\kappa_0 + \kappa_1)\nu < (\kappa_0\nu + \kappa)^2 + \alpha(\kappa_1\nu - \kappa + \kappa_0\nu + \kappa)(\kappa_1\nu - \kappa - \kappa_0\nu - \kappa)$$
$$\Leftrightarrow 2(\kappa_0\nu + \kappa)^2 - 2\alpha(\kappa_0\nu + \kappa)(\kappa_0 + \kappa_1)\nu < (\kappa_0\nu + \kappa)^2 + \alpha(\kappa_1 + \kappa_0)\nu(\kappa_1\nu - \kappa_0\nu - 2\kappa)$$
$$\Leftrightarrow (\kappa_0\nu + \kappa)^2 < \alpha[(\kappa_0 + \kappa_1)\nu]^2$$
$$\Leftrightarrow \alpha > \left[\frac{\kappa_0\nu + \kappa}{(\kappa_0 + \kappa_1)\nu}\right]^2.$$

Given that $\kappa = 1 - 2\nu$, and $\kappa_0 + \kappa_1 = \frac{1}{\alpha - p}$, we have,

$$
\begin{aligned}
\alpha > \left[\frac{\kappa_0\nu + \kappa}{(\kappa_0 + \kappa_1)\nu}\right]^2 &\Leftrightarrow \alpha > \left[1 - (\alpha - p)\kappa_1 + \frac{\alpha - p}{\nu}(1 - 2\nu)\right]^2 \\
&\Leftrightarrow \alpha > \left[1 - (1 - 2\alpha + p) - 2(\alpha - p) + \frac{\alpha - p}{\nu}\right]^2 \\
&\Leftrightarrow \alpha > \left(p + \frac{\alpha - p}{\nu}\right)^2 \\
&\Leftrightarrow \alpha > \left(p + \frac{\alpha - p}{p(1 - \alpha) + (1 - p)\alpha}\right)^2 \\
&\Leftrightarrow \alpha > \left(\frac{\alpha(1 + p - 2p^2) - p(1 - p)}{\alpha(1 - 2p) + p}\right)^2 \\
&\Leftrightarrow \alpha\Big(\alpha(1 - 2p) + p\Big)^2 > \Big(\alpha(1 + p - 2p^2) - p(1 - p)\Big)^2 \\
&\Leftrightarrow \alpha^3(2p - 1)^2 - \alpha^2(1 + p^2(1 - 2p)^2) + \alpha(4p^4 - 6p^3 + p^2 + 2p) - p^2(1 - p)^2 > 0
\end{aligned}
$$

If we study the function $h(\alpha) = \alpha^3(2p - 1)^2 - \alpha^2(1 + p^2(1 - 2p)^2) + \alpha(4p^4 - 6p^3 + p^2 + 2p) - p^2(1 - p)^2$ then it is possible to see that it is decreasing and that $h(p) > 0$. Furthermore we have $h(\frac{2-p}{3-2p}) < 0$, so that the function changes sign in the interval and there is α^{**} such that $f(\alpha^{**}) = 0$.

To summarize: For $\alpha > \alpha^{**}$, the senior is preferred, while for $\alpha < \alpha^{**}$, the junior is preferred if $b > b^{\dagger}$.

8 Competency trap: when incentive schemes are too costly

In this section we offer a brief summary of when incentive schemes to avoid the competency trap are too costly, so that a junior agent will be preferred over a senior agent. There are three cases to distinguish, depending on whether learning-by-doing is low, medium or strong. In each case, junior agents are prefered when learning-by-doing is relatively high (b above a certain threshold), but it is interesting to see that this can occur when the private information of the agent is either weak, intermediate, or strong depending on the case.

8.1 $b < \underline{b}$

When the bonus is low, the junior will be hired if

- y is sufficiently high; and
- $\alpha < \alpha^{**}$; and
- $b > b^{\dagger}$.

8.2 $b \in [\underline{b}, \bar{b}]$

When the bonus is intermediate, the junior will be hired if

- y is high; and
- $\alpha < \alpha*$,

or if

- y is high; and
- $\alpha* < \alpha < \frac{4p+1+\sqrt{8p+1}}{8}$; and
- $b > \hat{b}$.

8.3 $b \in [\bar{b}, \bar{u}]$

When the bonus is high, the junior will be hired if

- y is very high; and
- $\alpha > \frac{4p+1+\sqrt{8p+1}}{8}$; and
- $b > b^*$.

References

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