

Overwhelming Hazards

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Abstract

We show that a standard result of the career concerns literature (e.g., Holmström, 1982/1999) is not robust to the case of promotions. Specifically, we demonstrate that effort can increase when output is garbled by more extraneous noise. We investigate the implication of this result for the functioning of the firm. We show that managers whose ability is thought to be high can rationally choose projects which are highly informative about ability. We also show that employers and employees play a cat and mouse game in which employers may chose a job or organization design which is little informative about employees' ability.

Keywords: implicit incentives, output accuracy, reputation, promotion, career concerns.

JEL Codes: D83, L14, M51, M52.

1 Introduction

Fama (1980) introduced the notion that building a reputation in the labour market provides economic agents with implicit incentives to exert effort because a better reputation raises future job opportunities, inside and outside the firm. This notion, first formalized by Holmström

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(1982/1999), has given rise to a large body of literature. A standard result of that career concerns literature is that incentives to exert effort decline as the agent's output is garbled by more extraneous noise. The reason is that output is a less accurate measure of ability and thus is used with more caution when revising reputation (see Dewatripont, Jewitt and Tirole, 1999, for a generalization). However, it is frequent to observe agents exhibiting higher diligence when uncertainty regarding the outcome of their actions is higher. The present paper rationalizes the casual observation that agents work hard to overwhelm the potential adverse consequences of hazards they are confronted with when the issue at stake is important. Next the paper studies the consequences for the functioning of a firm.

To illustrate, think of a product manager at a software development firm who wishes to become the head of the marketing department. The product manager, whose reputation gives him a good chance to obtain this lucrative promotion, fears that the outcome of his current action convey negative information about his ability, which would thwart his plans. Specifically, the product manager has to launch a new software on the market to replace the existing one. Technical uncertainty surrounds the new product, weakening the relation between the level of sales realized and the product manager's marketing ability. According to received theory, this feature should reduce the effort exerted. By contrast, we argue in this paper that the product manager will exert more effort to market the new software if it is launched than if the firm sticks to its existing product. Broadly speaking, we show that effort will be high since the manager has much more to lose than to gain if his reputation changes when the new product rather than the existing one is proposed to customers.

The model we use to obtain these results builds on the classic two-period setting of career concerns à la Holmström (1982/1999). In Holmström's (1982/1999, Part II) setting, a risk-

neutral agent performs a task whose output depends on the agent's ability as well as on the agent's unobservable, costly effort. Ability is unknown both to the agent and the potential employers. However, all share the same prior estimate of ability: the agent is endowed with an initial reputation. Because the labour market is competitive, the agent is paid a wage equal to the expected output (in monetary value). Since ability and effort are substitutes, the agent is induced to exert effort in the first period to appear as talented as possible: the better the updated reputation, the larger the second-period wage. We differ from this seminal model along a major dimension: the employer can promote the agent to a higher level in the hierarchy where the agent has a deeper impact on output. Specifically, a higher position in the hierarchy increases output if the agent's ability is high, whereas it decreases output if the agent's ability is low.

In this new framework, a promotion is based on the assessment of an agent's ability after observing the agent's first-period output. We show that a simple rule emerges. The agent is promoted if posterior reputation exceeds the threshold above which *actual* ability would make output larger if the agent were at the higher level in the hierarchy rather than at the lower level. Importantly, the return to reputation (in terms of wage) is non-linear since the return to ability differs according to the position in the hierarchy. If prior reputation is reasonably above the promotion threshold, the agent has more to lose if his reputation deteriorates than to gain if it is improved. Because of the local concavity of the wage function with respect to reputation, the agent exerts effort to avoid reputation deterioration. We show that lower output accuracy makes the wage function more concave. Thus, the agent exerts more effort to compensate for possible adverse realizations of factors which are out of his reach. If the rise in wage associated with the promotion is large, this effect dominates the traditional learning effect à la Holmström, and effort eventually decreases in output accuracy. Thus, the key insight of the career concerns

literature we discussed above is not robust to the case of promotions. Specifically, the fact that incentives to exert effort decline as the agent's output is a less accurate measure of ability is not systematically true in our context.

Yet, the accuracy of the information about the agent's ability which accrues to the market is not exogenous. It has long been argued that managers can modify it by herding (Scharfstein and Stein, 1990), hedging (DeMarzo and Duffie, 1995; Breeden and Viswanathan, 1998), choosing project risk (Holmström, 1982/1999, Part III; Hermalin, 1993), or avoiding to undertake informative projects (Holmström and Ricart I Costa, 1986). Symmetrically, it has been argued that employers can influence how visible their managers are. For instance, granting them power raises visibility (Ortega, 2003) while team production distorts individual visibility (Jeon, 1996; and Bar-Isaac, 2007).

In order to investigate the consequences of our result for the functioning of the firm, we further modify Holmström's (1982/1999, Part II) setting along two dimensions. We allow the agent to influence how learning about ability operates by choosing the project he executes. For simplicity, there are two projects which differ in terms of information they disclose about ability.¹ Symmetrically, we allow the employer to influence the learning process by choosing one of two job (or organization) designs which also differ in terms of information they disclose about ability.²

To illustrate, we let the product manager decide which of the new and the existing software will be proposed to customers. And we let the employer decide whether to add to the product manager's responsibility an existing product which is known to require marketing ability or

¹In Part III of his paper, Holmström (1982/1999) also considers project selection by agents as a way to manipulate information about their ability. However, the agent under consideration has no effort to exert, then.

²Using job design to optimize incentives is reminiscent of, among others, Holmström and Milgrom (1991). See also Schottnner (2007).

create a distinct profit center for the new software.

The following results obtain. First, employers and employees play a cat and mouse game. For instance, when effort increases in output accuracy in our model, the employer facing an agent who chooses a project which provides the market with little information to minimize execution effort will restore implicit incentives to be diligent by designing the agent's job or the organization so as to make it more informative about the agent's ability. Typically, this case occurs when the agent's initial reputation is so good that the agent is almost sure to be promoted (or initial reputation is so bad that the agent is almost sure not be promoted). Indeed, the agent is indifferent between the two projects in terms of revision of reputation, but exhibits a clear preference for the less informative project in terms of cost of execution effort saved. To come back to our example, the product manager would launch the new software and exert little marketing effort in the absence of any reaction from the employer. Thus, the employer's optimal design strategy would be to add to the product manager's responsibility an existing product which is known to require marketing ability. The employer's optimal organization design strategy would be to create a distinct profit center for each product to filter out noise.

Second, we show that some agents whose prior reputation is reasonably above the promotion threshold (i.e., the case we discussed in the opening discussion) choose the more informative project. At first sight, this is surprising since such a project increases the extent to which reputation is revised at the end of the first period, which is not in the agent's interest. The reason for choosing this project is that it increases the agent's wage in case of promotion and reduces the effort to be exerted in equilibrium. Then, the employer's optimal reaction is to design the job so that output is less informative about ability. By doing so, the employer maximizes the execution effort exerted. In terms of our example, the employer's best job design strategy would be to let the product manager launch the new software without adding to his responsibilities

any other product which is known to require marketing ability.

Finally, we show that in equilibrium there exists cases in which the opportunistic agent makes project choices that raise social welfare by reducing the gap between the execution effort exerted and the first-best level, whereas the employer makes design choices that increase this gap. For instance, this occurs when (i) the agent exerts a level of effort strictly higher than the first-best whatever project is chosen because the pay rise associated with a promotion is substantial, (ii) the agent's prior reputation is reasonably above the promotion threshold so that the agent chooses the more informative project to decrease the effort exerted (see above our discussion), and (iii) the employer chooses the less informative job or organization design to over-stimulate the agent's diligence, increasing the risk of a "burnout".

Recently, promotions received considerable attention in the career concerns literature. The present paper offers several contributions to that literature. First, the wage profile which drives our results is endogenous. By contrast, earlier contributions posit an exogenous return-to-reputation (e.g., Foerster and Martinez, 2006). Second, we obtain a wage profile alternatively exhibiting concavity and convexity properties, which allows the strength of incentives to differ according to the agent's initial reputation. By contrast, earlier contributions (e.g., Casas-Arce, 2010) focus on the impact of the convexity of the wage function on incentives. Third, we study how the variation in the precision of the shocks which affect output influences incentives. This complements the analyses which focus on the impact of the precision of reputation (Martinez, 2009; Miklos-Thal and Ullrich, 2014). In our context, promotions have no signalling role. This complements the analyses of papers in which information is asymmetric (Gibbs, 1995; Zabojnik and Bernhardt, 2001; Gosh and Waldman, 2010; Zabojnik, 2012; or Miklos-Thal and Ullrich, forthcoming) who all build on Waldman (1984). Our paper is most closely related to Kovrijnykh

(2007) in the sense that he does not consider incentives facing managers as given. Kovrijnykh (2007) analyzes how the timing of the information released about the manager's performance is manipulated by the employer to control the strength of incentives. In the present paper, we alternatively analyze how job or organization design is utilized for a similar purpose, and study the cat and mouse game between firms and managers since we allow managers to impact on how information flows to the market.

Our paper is also related to the recent stream of the career concerns literature which studies how incentives are affected by organizational characteristics. Organizations themselves can differ in terms of the visibility they offer to their agents (Acemoglu et al. 2008; Casas-Arce, 2010). We endogenize visibility through the choice of job or organization design in our paper. It is argued that granting power to managers positively impacts how visible they are (Ortega, 2003) and that individual visibility can be distorted through team production (Jeon, 1996; and Bar-Isaac, 2007). A contribution of the present paper is to allow managers (and not only firms) to influence visibility through the choice of a project. In that, the paper is related to the original strand of the career concerns literature which examined how managers modify the accuracy of the information which accrues to the market (Holmström, 1982/1999 part III; Holmström and Ricart I Costa, 1986; Scharfstein and Stein, 1990; Hermalin, 1993; DeMarzo and Duffie, 1995; Breeden and Viswanathan, 1998). Our contribution to that strand of literature is twofold. First, allowing firms to influence the visibility of managerial ability through job or organization design allows us to model the cat and mouse game employees and employers play. Second, we combine two job dimensions (i.e., selecting and executing a project) which have been studied separately by the seminal papers of the career concerns literature (see Holmström, 1982/1999; and Dewatripont, Jewitt and Tirole, 1999).

The rest of the paper proceeds as follows. In section 2, we present the model and discuss its assumptions. In section 3, we analyze the promotion rule and the second-period wage. In section 4, we examine an agent's choices of project and effort as well as an employer's choice of job or organization design. In section 5, we conduct a social welfare analysis.

2 The Model

We consider an economy which lasts two periods. There is a continuum of agents of mass 1, indexed by $i \in [0; 1]$ and a continuum of firms (or employers) of mass 1, indexed by $j \in [0; 1]$. At the beginning of each period, agents can switch firms costlessly while firms can hire or fire agents costlessly. Firms and agents are risk-neutral and do not discount the future.

Agents

Agents differ in ability. Agent i 's ability, θ_i , is unobservable to all parties. However, all share the same beliefs about θ_i . Specifically, at the beginning of the first period, it is common knowledge that θ_i is normally distributed with mean $E\theta_i$ and variance σ_θ^2 . It is convenient to think of $E\theta_i$ as the agent's initial reputation. This reputation is reassessed at the end of the first period.

To make the problem interesting, we assume in the rest of the paper that there is much to learn about the agent's ability at the beginning of the first period, which amounts to assuming that σ_θ^2 is not trivial (the lower bound is given in the Appendix).

Job content

Jobs have two dimensions. First, an agent makes a decision, i.e., selects one of two projects. Projects differ in terms of information they convey about the agent's ability. The less informative project (p_l) is characterized by a higher dispersion of outcomes, i.e., it is a mean preserving spread

of the more informative project (p_m). Specifically, project $p \in \{p_l; p_m\}$ adds ε_p to the agent's output, where the random shock ε_p is distributed according to $N(0; \sigma_p^2)$, with $\sigma_{p_m}^2 < \sigma_{p_l}^2$. The choice of project is observable but not verifiable.

Second, the agent executes the decision. Execution requires effort, the level of which is at the discretion of the agent. Agent i 's unobservable level of effort $e_i \in [0; +\infty[$ increases the agent's output by e_i and costs the agent $\psi(e_i) = \frac{e_i^2}{2}$.

Firms

Firms are identical. Each of them hires an agent, matches the agent and a level in the hierarchy, and designs the agent's job or the organization. Job (or organization) design allows firms to manipulate the inference of the agents' ability from output. Naturally, there exist real life cases in which firms make job or organization design decisions before agents choose projects. Here, we make the assumption that the firm's decision is posterior to the agent's decision. It is meant to reflect that the employer has an advantage over the employee, that is, the employer has the last word. Design d_l is less informative than design d_m since the random shock ε_d of distribution $N(0; \sigma_d^2)$ which affects output is characterized by $\sigma_{d_m}^2 < \sigma_{d_l}^2$. The choice of design by the firm is observable but not verifiable. The random shocks which characterize the firm's choice of design (ε_d) and the agent's choice of project (ε_p) are independently distributed.

Hierarchy

Promoting an agent of sufficient ability (i.e., $\theta_i \geq \underline{\theta}$) to a higher level in the hierarchy increases output by Δ (which takes finite values). If $\theta_i < \underline{\theta}$ and the agent is promoted, output is reduced by Δ .³ Experience (i.e., working during the first period at a lower level in the hierarchy) is a

³In the literature on promotions, matching higher level jobs with higher ability is reminiscent of, e.g., Sattinger (1975), Calvo and Wellisz (1979) and Carmichael (1983).

prerequisite for being promoted.⁴ The threshold $\underline{\theta}$ is exogenous, which amounts to assuming that there are no constraints on the number of promotions firms can overall grant.

Output

Output is observable. At the lower level in the hierarchy, output is:

$$y_i = \theta_i + \varepsilon_p + \varepsilon_d + e_i, \quad (1)$$

which is the sum of agent i 's ability θ_i , the realization ε_p of the shock resulting from the choice of project by the agent, the realization ε_d of the shock resulting from the choice of design by the firm, and the agent's effort e_i . At the higher level in the hierarchy, output is:⁵

$$Y_i = \begin{cases} (\theta_i + \Delta) + \varepsilon_p + \varepsilon_d + e_i & \text{if } \theta_i \geq \underline{\theta}, \\ (\theta_i - \Delta) + \varepsilon_p + \varepsilon_d + e_i & \text{if } \theta_i < \underline{\theta}. \end{cases} \quad (2)$$

Objective functions

An agent's utility is the difference between the wage the agent receives and the cost of effort the agent incurs, when the agent is employed by a firm. When the agent is unemployed, the agent's utility is $-\infty$. The objective of an agent is to maximize his (expected) utility.

The objective of a firm is to maximize output net of the wage paid to the agent.

Contracts

⁴A managerial hierarchy can be considered as a special case of a more general setting in which (i) there exist a standard technology and a sophisticated technology, and (ii) the sophisticated technology increases output by Δ if utilized by a sufficiently talented agent, whereas it decreases output by Δ if utilized by an insufficiently talented agent.

⁵The production function we opt for is the simplest way to allow the agent's marginal productivity to be not linear in ability and strictly increasing around some threshold. Any other production function such that the marginal productivity with respect to the agent's ability is constant (or weakly increasing) until a first threshold is reached, then is strictly increasing up to a second threshold, above which it is again constant (or weakly increasing), would not qualitatively modify the results, but complicate computations.

Agent i is paid a fixed wage W_i at the end of each period. This wage, which is determined at the beginning of the period, is equal to the agent's *expected* output since the labour market is competitive.

We do not analyze explicit contracts which would relate wages to *realized* output for the following reasons. First and foremost, our objective is to examine the incentives naturally offered by the labour market. This is an important step to understand what explicit incentives (if any) can be offered to agents to complement implicit incentives. Second, in practice, the difficulty of verifying the output of each agent (or some dimensions of the output) often makes it difficult or counterproductive to write explicit contracts (Holmström and Milgrom, 1991; Dewatripont, Jewitt and Tirole, 1999). For instance, explicit incentives facing CEOs in large firms are overall weak (Jensen and Murphy, 1990), which suggests the same pattern for lower-level agents.⁶ Finally, indexing wages to output would help aligning agents' and firms' interests. Thus, in principle, an agent could be induced to act so as to maximize a weighted average of the firm's expected profits and his future compensation (see Gibbons and Murphy, 1992). However, this more general formulation would lead to the same qualitative results that obtain if an agent cares only about the labour market's assessment of his ability- although naturally, inefficiencies are reduced (Scharfstein and Stein, 1990; Prendergast and Stole, 1996; Breeden and Viswanathan, 1998).

No contract is made contingent on the choice of project by the agent to capture that this choice is not verifiable by third parties, e.g., a court of justice. For the same reason, no contract is made contingent on the choice of design by the firm.

Timing of events

⁶See Prendergast (1999) for a survey on the provision of incentives in firms.

Let the superscript t (with $t = 1, 2$) denote the period under consideration. The timing of events is the following:

First Period

1. At the beginning of the first period, firm j hires agent i at the bottom of the hierarchy.
The wage W_i^1 is fixed.
2. Agent i chooses project p_l or p_m .
3. Firm j chooses design d_l or d_m .
4. Agent i exerts effort e_i^1 .
5. Output y_i^1 is realized and agent i is paid W_i^1 .
6. Priors about agent i 's ability are updated by using y_i^1 , $p \in \{p_l; p_m\}$, $d \in \{d_l; d_m\}$, and the first-period equilibrium level of effort e_i^{1*} .

Second Period

1. Firm j decides if agent i is promoted.
2. Then, the timing of events is identical to the first-period one.

In what follows, we focus on perfect Bayesian equilibria (see Fudenberg and Tirole, 1991). In equilibrium, the level of effort exerted by an agent must be optimal given beliefs, and all parties must rationally anticipate the equilibrium level of effort. Working backward, we first look at the condition under which an agent is promoted at the beginning of the second-period and then study the second-period wage.

3 Promotion Rule, Second-Period Wage, and Effort

To help the reader develop an intuition of the results, we first consider that the noise which impacts on output is exogenous. In Section 4, we endogenize noise and study the cat and mouse game the employer and the employee play.

3.1 Promotion Rule

The following Lemma details the promotion rule:

Lemma 1 *At the beginning of the second period, agent i is promoted if and only if: $E(\theta_i | y_i^1, p, d, e_i^{1*}) \geq \underline{\theta}$.*

The intuition for the first part of Lemma 1 is the following. In a competitive labour market, a firm promotes an agent if and only if the agent's expected output if the agent is promoted is higher than the agent's expected output if the agent is not promoted:

$$E(Y_i | y_i^1, p, d, e_i^{1*}) \geq E(y_i^2 | y_i^1, p, d, e_i^{1*}). \quad (3)$$

Career concerns being absent in the second-period, the agent (is indifferent between projects and) exerts no effort. Thus, (3) boils down to $\Pr(\theta_i | y_i^1, p, d, e_i^{1*} < \underline{\theta}) \times (-\Delta) + \Pr(\theta_i | y_i^1, p, d, e_i^{1*} \geq \underline{\theta}) \times \Delta \geq 0$, or $\Pr(\theta_i | y_i^1, p, d, e_i^{1*} \geq \underline{\theta}) \geq \frac{1}{2}$. Since variables are normally distributed⁷ the previous inequality reduces to:

$$E(\theta_i | y_i^1, p, d, e_i^{1*}) \geq \underline{\theta}. \quad (4)$$

A simple and intuitive promotion rule obtains: agent i is promoted at the beginning of the second-period if and only if the agent's posterior reputation (which may differ from the agent's

⁷The random variable $(\theta_i | y_i^1, p, d, e_i^{1*})$ follows $N[E(\theta_i | y_i^1, p, d, e_i^{1*}); Var(\theta_i | y_i^1, p, d, e_i^{1*})]$.

actual ability) is higher than the threshold $\underline{\theta}$.

3.2 Second-period Wage

The following Lemma details the second-period wage:

Lemma 2 *When the agent is promoted, the wage is:*

$$W_i^2 = E(\theta_i | y_i^1, p, d, e_i^{1*}) + \left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \times \Delta. \quad (5)$$

When agent i is not promoted, the wage is: $W_i^2 = E(\theta_i | y_i^1, p, d, e_i^{1*})$.

Figure 1 below represents the agent's second-period wage as a function of the posterior reputation $E(\theta_i | y_i^1, p, d, e_i^{1*})$.

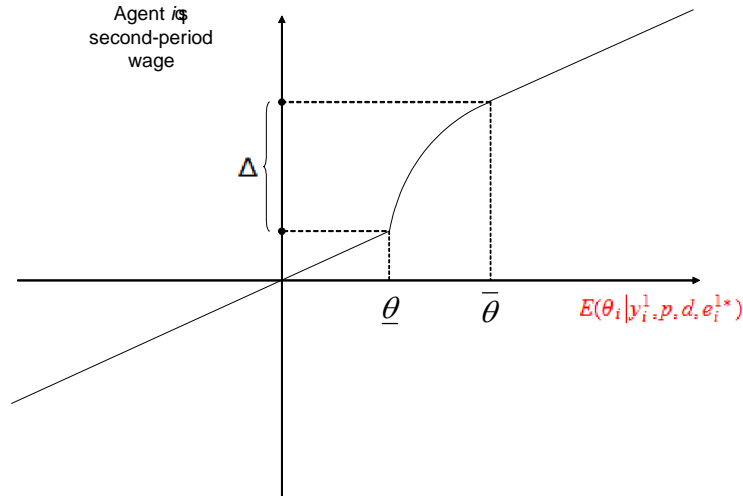


Figure 1

First observe that the wage function is continuous in $E(\theta_i | y_i^1, p, d, e_i^{1*})$. This is because the labour market is competitive. Also observe that when the agent is not promoted, the wage linearly increases with $E(\theta_i | y_i^1, p, d, e_i^{1*})$. When the agent is promoted, the wage still increases

with $E(\theta | y_i^1, p, d, e_i^{1*})$ but not linearly because of the second term in (5). Specifically, denote $\bar{\theta}$ the threshold such that:

$$\Pr(\theta_i | y_i^1, p, d, e_i^{1*} \geq \underline{\theta}) = 1 \text{ when } E(\theta_i | y_i^1, p, d, e_i^{1*}) \geq \bar{\theta}. \quad (6)$$

When $\underline{\theta} \leq E(\theta_i | y_i^1, p, d, e_i^{1*}) < \bar{\theta}$, the second-period wage increases in the agent's posterior reputation and is locally concave (see the Appendix). When $E(\theta_i | y_i^1, p, d, e_i^{1*}) \geq \bar{\theta}$, the second-period wage again linearly increases with posterior reputation.

Working backward, we now determine the effort agent i exerts in the first period.

3.3 Equilibrium Effort

Motivation to exert effort derives from the desire to influence favorably the learning process regarding ability. The eventual objective is to earn a higher wage (whether promotion occurs or not). In order to decide how much effort to supply, an agent calculates the impact of the first-period output on the second-period expected wage. As discussed above, when the agent is not promoted, the second-period expected wage reduces to the posterior estimate of ability conditional on the latter being lower than $\underline{\theta}$ since no effort is exerted in the second period:

$$\int_{-\infty}^{\underline{\theta}} E(\theta_i | y_i^1, p, d, e_i^{1*}) \times f(E(\theta_i | y_i^1, p, d, e_i^{1*})) dE(\theta_i | y_i^1, p, d, e_i^{1*}), \quad (7)$$

where $f(E(\theta_i | y_i^1, p, d, e_i^{1*}))$ denotes the density of $E(\theta_i | y_i^1, p, d, e_i^{1*})$. For the same reason, the second-period expected wage when the agent is promoted reduces to the posterior estimate of ability conditional on the latter being higher than $\underline{\theta}$, plus the expected extra output associated

with the higher position in the hierarchy:

$$\int_{\underline{\theta}}^{+\infty} \left[\begin{array}{c} E(\theta_i | y_i^1, p, d, e_i^{1*}) \\ + \left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \times \Delta \end{array} \right] f(E(\theta_i | y_i^1, p, d, e_i^{1*})) dE(\theta_i | y_i^1, p, d, e_i^{1*}). \quad (8)$$

Thus, agent i 's second-period expected wage can be rewritten as:

$$\begin{aligned} & E[E(\theta_i | y_i^1, p, d, e_i^{1*})] \\ & + \Delta \times \int_{\underline{\theta}}^{+\infty} \left[1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] f(E(\theta_i | y_i^1, p, d, e_i^{1*})) dE(\theta_i | y_i^1, p, d, e_i^{1*}). \end{aligned} \quad (9)$$

Effort and ability being substitutes, the agent is willing to bias the inference process in his favour by increasing effort. Yet, the labour market is not fooled in equilibrium since it anticipates the effort exerted and adjusts inference accordingly. However, the agent must exert the effort expected of him because a lower effort would bias inference against him. The agent exerts the effort which maximizes the second-period expected wage, given by (9), minus the cost of effort $\psi(e_i) = \frac{e_i^2}{2}$. Computing the first-order condition, agent i 's equilibrium level of effort is:

$$e_i^{1*} = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} + 2\Delta \frac{\sqrt{\sigma_\theta^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} \frac{(\underline{\theta} - E\theta_i)^2}{\sigma_\theta^2} \right] \left[1 - \Phi \left(\frac{(\underline{\theta} - E\theta_i) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right]. \quad (10)$$

Equation (10) equalizes agent i 's marginal cost of effort (left-hand side) to the marginal gain of effort (right-hand side). The first term in the right-hand side of (10) represents the ‘‘standard’’ marginal gain from effort derived from the rise in wage when the posterior estimate of ability increases (see Holmström 1982/1999). The second term shows the marginal gain from effort derived from the expected additional wage agent i earns when he is promoted.

Naturally, the larger Δ , the higher-powered the incentives to exert effort. More interestingly,

the larger the distance between the prior estimate of ability $E\theta_i$ and the threshold $\underline{\theta}$ above which the agent is promoted, the lower-powered these incentives. Indeed, as $|\underline{\theta} - E\theta_i|$ increases the impact that effort has on the probability to end up above $\underline{\theta}$ decreases. To see this, consider the following two extreme cases: when $E\theta_i$ is far below $\underline{\theta}$, this probability is close to 0 whatever the effort exerted by the agent; when $E\theta_i$ is far above $\underline{\theta}$, this probability is close to 1 whatever the effort exerted by the agent.

A standard result in the career concerns literature is that a greater precision of output (i.e., a lower σ_p^2 or σ_d^2) fosters incentives to exert effort (e.g., Holmström, 1982/1999; Dewatripont, Jewitt and Tirole, 1999). We show below that this general result no longer holds in a setting in which the agent can be promoted. We first state the formal results and then discuss the intuition.

Proposition 1 *When $E\theta_i \geq \underline{\theta}$, $|\underline{\theta} - E\theta_i|$ takes intermediate values and $\Delta \geq \bar{\Delta}(\underline{\theta} - E\theta_i)$, e_i^{1*} increases in σ_p^2 and σ_d^2 . When otherwise, Holmström's (1982/1999) result holds: e_i^{1*} decreases in σ_p^2 and σ_d^2 .*

The accuracy of output about the agent's ability has two effects on incentives to exert effort. First, greater accuracy leads to the traditional learning effect à la Holmström: the market uses output to a larger extent to revise initial reputation, which naturally strengthens incentives to exert effort.

Second, since greater accuracy implies that the posterior reputation of an agent is less likely to be similar to the agent's initial reputation, the position of the agent's initial reputation with respect to the threshold above which promotion occurs plays a crucial role in strengthening or lowering the agent's incentives to exert effort. Suppose that although (i) the agent's initial reputation is such that if the agent keeps that reputation the agent will be promoted in the

second period (i.e., $E\theta_i > \underline{\theta}$), (ii) there remains some *reasonable* uncertainty (i.e., $|\underline{\theta} - E\theta_i|$ takes intermediate values), which implies that posterior reputation can end up below $\underline{\theta}$. The shape of the agent's second-period wage given in Figure 1 is locally concave in $E(\theta_i | y_i^1, p, d, e_i^{1*})$. This explains part of the agent's motivation to exert effort: gains if the agent's reputation increases over time being lower than losses if reputation decreases, the agent exerts effort to compensate for the expected consequences of out-of-his-reach, potentially adverse shocks. Increasing the accuracy of output (i.e., lowering σ_p^2 or σ_d^2) raises the probability that $E(\theta_i | y_i^1, p, d, e_i^{1*})$ moves away from initial reputation and takes a value for which the agent's second-period wage is no longer (locally) concave in posterior reputation (see Figure 1). In other words, the extent of losses if his reputation deteriorates marginally decreases, which lowers the expected marginal gain of effort. If promotions lead to a large pay rise (because $\Delta \geq \bar{\Delta}(\underline{\theta} - E\theta_i)$), this effect dominates the traditional effect à la Holmström.⁸ Otherwise, increasing σ_p^2 (or σ_d^2) decreases effort.

To complete the analysis, suppose that the agent's initial reputation is around $\underline{\theta}$. Then, the wage function exhibits convexity properties with respect to posterior reputation (see Figure 1): what the agent gains if posterior reputation increases with respect to initial reputation is (in expectation) greater than what the agent loses if reputation decreases over time. By raising the probability that $E(\theta_i | y_i^1, p, d, e_i^{1*})$ moves away from initial reputation, greater accuracy of output increases the impact of the convexity property of the wage function, which strengthens incentives. For the same reason, decreasing σ_p^2 (or σ_d^2) raises effort when the agent's initial reputation is clearly below $\underline{\theta}$ (but does not take extreme values). Thus, the traditional effect and the new effect highlighted here go in the same direction. When the agent's initial reputation

⁸To make the problem interesting, σ_θ^2 needs to be non trivial. This ensures that the extent of updating is non trivial, which leaves room for the effect we highlight.

is sufficiently far below (respectively, above) $\underline{\theta}$, the probability that posterior reputation exceeds the promotion threshold is close to 0 (respectively, 1). The wage function is linear in posterior reputation (see Figure 1), just as in Holmström's (1982/1999) model (with the difference that the wage is raised by Δ in the latter case when the agent is promoted). Thus, Holmström's (1982/1999) result holds.

4 The Cat and Mouse Game

Working backward, we first examine job or organization design by the firm, and then turn to the agent's choice of project.

4.1 Job or Organization Design

The firm opts for the job or organization design which maximizes

$$\max_{\sigma_d^2} y_i^1 - W_i.$$

The agent's wage is already fixed when the firm makes its own choice. Thus, the firm is only concerned about maximizing the effort the agent exerts. The definition of y_i^1 (see (1)) shows that ε_p and ε_d play a symmetric role: opting for the less informative project or the less informative design increases the variance of y_i^1 . Symmetrically, opting for the more informative project or the more informative design decreases the variance of y_i^1 . Thus, we use Proposition 1 to infer the firm's choice.

Proposition 2 *When $E\theta_i \geq \underline{\theta}$, $|\underline{\theta} - E\theta_i|$ takes intermediate values and $\Delta \geq \overline{\Delta}(\underline{\theta} - E\theta_i)$, the firm chooses the less informative job or organization design. Otherwise, the firm chooses the more informative job or organization design.*

The first part of Proposition 2 is of particular interest. It implies that even if obtaining better-quality information about the agent's ability is free, it is not always in the firm's interest. Costly changes of design to improve the gathering of information would a fortiori be a dominated solution. The fact that better-quality information is not always beneficial to the firm bears some resemblance with the result of Crémer (1995). However, the type of information involved and the underlying mechanism are quite different. In Crémer's (1995) paper, the principal sometimes chooses to avoid observing θ directly. This induces higher effort from the agent since the re-hiring/firing decision in the second period then depends on the output realized during the first period. In other words, not observing θ creates reputation incentives. Here, none of the design solutions allow the principal to observe θ directly. And the less informative design has an ambiguous impact on incentives. On the one hand, the fact that the market uses output with more caution when revising reputation lowers the strength of reputation incentives for all agents. On the other hand, by raising reputation risk (recall the local concavity of the wage function), it overall provides agents characterized by $E\theta_i \geq \underline{\theta}$ and $|\underline{\theta} - E\theta_i|$ that takes intermediate values with higher-powered incentives to exert effort if $\Delta \geq \bar{\Delta}(\underline{\theta} - E\theta_i)$.

To come back to our example of a product manager at a software development firm, improving visibility about the manager's ability could lead the employer to add to the manager's responsibility an existing product which is known to demand the marketing ability the employer wants to assess. If possible this job design solution could be complemented by an organization design solution: creating a distinct profit center for each product, to avoid interference from the revenues obtained by the new software when estimating the product manager's marketing ability. To reduce visibility, the employer's best job design strategy would be to let the product manager launch the new software without adding to his responsibilities any other product which is known to require marketing ability.

We now determine the agent's choice of project.

4.2 Choice of Project

At the time the agent makes the decision about the project, the first-period wage is already fixed. Thus, the agent maximizes the second-period expected wage given by (9), minus the first-period cost of effort $\psi(e_i^{1*})$. In equilibrium, the labour market perfectly anticipates e_i^{1*} and observes the choice of project. Therefore,

$$E[E(\theta_i | y_i^1, p, d, e_i^{1*})] = E[E(\theta_i | y_i^1(e_i^{1*}), p, d, e_i^{1*})] = E\theta_i. \quad (11)$$

Hence, the agent focuses on the cost of effort and the expected extra revenue if he is promoted (i.e., the second term in (9)).

Overall, three effects are at work. Two effects relate to the expected extra revenue when the agent is promoted. This is due to the fact that the agent's choice of project both affects $Var(\theta_i | y_i^1, p, d, e_i^{1*})$ and $Var(E(\theta_i | y_i^1, p, d, e_i^{1*}))$ and thus the distributions of the random variables $(\theta_i | y_i^1, p, d, e_i^{1*})$ and $E(\theta_i | y_i^1, p, d, e_i^{1*})$, respectively. To distinguish between these two effects, first consider the distribution of $E(\theta_i | y_i^1, p, d, e_i^{1*})$ as given, and focus on the wage if the agent is promoted. The agent's choice of project affects the shape of the wage when being promoted. According to Lemma 1, agent i maximizes $1 - 2\Phi\left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{Var(\theta_i | y_i^1, p, d, e_i^{1*})}}\right)$, where $E(\theta_i | y_i^1, p, d, e_i^{1*}) \geq \underline{\theta}$. This amounts to minimizing $Var(\theta_i | y_i^1, p, d, e_i^{1*})$, or σ_p^2 since $Var(\theta_i | y_i^1, p, d, e_i^{1*}) = \sigma_\theta^2 - \frac{\sigma_{\theta_i}^4}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}$, and thus calls for the choice of the more informative project. The intuition is the following for this “precision-of-the-revision” effect. Once promoted, the agent prefers the posterior reputation to be as accurate as possible (i.e., minimize $Var(\theta_i | y_i^1, p, d, e_i^{1*})$). Indeed, this increases the probability for the agent who obtains the promotion to be the right man at

the right place, which allows the agent to earn a larger share of Δ .⁹

Second, assume that $Var(\theta_i | y_i^1, p, d, e_i^{1*})$ is given. The choice of project influences the distribution of $E(\theta_i | y_i^1, p, d, e_i^{1*})$. Because (11) holds, we focus on $Var[E(\theta_i | y_i^1, p, d, e_i^{1*})]$. Quite naturally, when the agent's initial reputation does *not* allow the agent to be promoted in case of status quo, the agent is better off facilitating the revision of reputation process. It calls for the choice of the more informative project since $Var[E(\theta_i | y_i^1, p, d, e_i^{1*})] = \frac{\sigma_{\theta_i}^4}{\sigma_{\theta}^2 + \sigma_p^2 + \sigma_d^2}$ decreases in σ_p^2 . Conversely, an agent who *clearly* benefits from the status quo in terms of reputation wants to prevent additional learning about ability. Indeed, he has more to lose in expectation, if his reputation deteriorates than to gain if his reputation becomes better. More subtly, when the agent's initial reputation *just* allows the agent to be promoted if the status quo persists, the agent is better off facilitating the revision of reputation process because of the local convexity property of the wage function (See Figure 1). To summarize, the “extent-of-the-revision” effect goes in differing directions, depending on the initial reputation of the agent.

A third, “cost-of-effort”, effect relates to the impact of the choice of project on the effort exerted, hence, on the cost incurred by the agent. As Proposition 1 shows, effort increases in σ_p^2 when $E\theta_i \geq \underline{\theta}$, $|\underline{\theta} - E\theta_i|$ takes intermediate values and $\Delta \geq \bar{\Delta}(\underline{\theta} - E\theta_i)$, and decreases in σ_p^2 otherwise. Thus, it calls for the choice of the more informative project in the former case and of the less informative project in the latter case.

Combining the three effects discussed above, we now determine the eventual choice of project by the agent. First, suppose that the agent faces almost no uncertainty as to whether promotion will occur in the second-period (i.e., $|\underline{\theta} - E\theta_i|$ is large). The extent-of-the-revision effect is nil (i.e., there is no doubt about the promotion outcome whatever the project chosen). So is the

⁹At the extreme, consider that the agent's ability is perfectly known at the end of the first period (i.e., $Var(\theta_i | y_i^1, p, d, e_i^{1*}) = 0$). Then the wage is raised by Δ when the agent is promoted since $\Pr(\theta_i | y_i^1, p, d, e_i^{1*} \geq \underline{\theta}) = 1$.

precision-of-the-revision effect (i.e., the extra revenue if the agent is promoted is close to either zero or Δ , whatever the project chosen). Only the cost-of-effort effect matters. The agent chooses the less informative project.

Next, consider the opposite case in which there is a great deal of uncertainty about the promotion outcome (i.e., $|\underline{\theta} - E\theta_i|$ is small). The precision-of-the-revision effect and the extent-of-the-revision effect call for the choice of the more informative project, whereas the cost-of-effort effect calls for the choice of the less informative project. Since $|\underline{\theta} - E\theta_i|$ is small, the convexity property of the wage function is more salient (see Figure 1). When the pay rise which accompanies the promotion is limited (because $\Delta \leq \Delta_I(\underline{\theta} - E\theta_i)$), it is worth saving on effort: the agent chooses the less informative project. When the pay rise is substantial (because $\Delta > \Delta_{II}(\underline{\theta} - E\theta_i)$), the agent also chooses the less informative project since the more informative one would make him exert too much of a costly effort. When the pay rise is intermediate (because $\Delta \in]\Delta_I(\underline{\theta} - E\theta_i), \Delta_{II}(\underline{\theta} - E\theta_i)[$), the agent chooses the more informative project since the prospect of a promotion more than offsets the related cost of effort.

Finally, assume that uncertainty about the promotion outcome is moderate (i.e., $|\underline{\theta} - E\theta_i|$ is intermediate). Suppose that $E\theta_i < \underline{\theta}$. The trade-off is the same as and the resulting agent's choice identical to the one just discussed above. Suppose that $E\theta_i \geq \underline{\theta}$. On the one hand, the precision-of-the-revision effect which calls for the choice of the more informative project dominates the extent-of-the-revision effect for the following reason. The precision-of-the-revision effect impacts on the shape of the wage when the agent is promoted. Choosing the more informative project reduces the area in which the second-period wage is concave in posterior reputation. Specifically, this reduces $Var(\theta_i | y_i^1, p, d, e_i^{1*})$, increases $\Pr((\theta_i | y_i^1, p, d, e_i^{1*}) \geq \underline{\theta})$ when the agent is promoted, and thus makes $\underline{\theta}$ and $\bar{\theta}$ closer one to another. Therefore, choosing the more informative project reduces the expected marginal loss in wage associated with a decrease

in posterior reputation. On the other hand, the cost-of-effort effect calls for the choice of the more informative project if $\Delta \geq \overline{\Delta}(\underline{\theta} - E\theta_i)$. Thus, the agent eventually chooses the more informative project if $\Delta \geq \overline{\overline{\Delta}}(\underline{\theta} - E\theta_i)$, with $\overline{\overline{\Delta}}(\underline{\theta} - E\theta_i) < \overline{\Delta}(\underline{\theta} - E\theta_i)$. The following proposition summarizes the previous results.

Proposition 3 *The agent chooses the more informative project when:*

- $E\theta_i < \underline{\theta}$ and $|\underline{\theta} - E\theta_i|$ is small or intermediate, if $\Delta \in]\Delta_I(\underline{\theta} - E\theta_i), \Delta_{II}(\underline{\theta} - E\theta_i)[$,
- $E\theta_i > \underline{\theta}$ and $|\underline{\theta} - E\theta_i|$ is small, if $\Delta \in]\Delta_I(\underline{\theta} - E\theta_i), \Delta_{II}(\underline{\theta} - E\theta_i)[$,
- $E\theta_i > \underline{\theta}$ and $|\underline{\theta} - E\theta_i|$ is intermediate, if $\Delta \geq \overline{\overline{\Delta}}(\underline{\theta} - E\theta_i)$.

Otherwise, the agent chooses the less informative project.

To come back to our example, the product manager would decide not to launch the new software in three cases. When being an underdog (but not a lemon), or when enjoying an initial reputation just good enough to be promoted in case of status quo, if the expected pay rise associated to the promotion is moderate. When enjoying an initial reputation reasonably well above the level required to be promoted if the expected pay rise is substantial.

The case in which $E\theta_i > \underline{\theta}$, $|\underline{\theta} - E\theta_i|$ is intermediate, and $\Delta \geq \overline{\overline{\Delta}}(\underline{\theta} - E\theta_i)$ is a clear illustration of the cat and mouse game employers and employees can play: the employee chooses the more informative project whereas the employer chooses the less informative design. In other cases, employers exhibit preferences which are aligned with the employees' preferences. This occurs when, e.g., $E\theta_i < \underline{\theta}$ and $|\underline{\theta} - E\theta_i|$ is small or intermediate, if $\Delta \in]\Delta_I(\underline{\theta} - E\theta_i), \Delta_{II}(\underline{\theta} - E\theta_i)[$: the two prefer the more informative structure.

The next section compares the execution effort exerted to the first-best level.

5 Comparison with the First-Best

Using (10) we compare the equilibrium level of effort exerted to the First-Best level of effort, $e^{FB} = 1$. For any given $\{\sigma_\theta^2, \sigma_p^2, \sigma_d^2\}$, we have $e_i^{1*} > e^{FB}$ if and only if $\Delta > \Delta_T(\underline{\theta} - E\theta_i)$, with

$$\Delta_T(\underline{\theta} - E\theta_i) \stackrel{d}{=} \frac{1 - \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}{\frac{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}{(\sigma_p^2 + \sigma_d^2)^{\frac{3}{2}} (\sigma_\theta^2)^{\frac{3}{2}} \sqrt{2\pi}} \exp\left[-\frac{1}{2} \frac{(\underline{\theta} - E\theta_i)^2}{\sigma_\theta^2}\right] \times \left(1 - \Phi\left(\frac{(\underline{\theta} - E\theta_i) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta}\right)\right)}. \quad (12)$$

Broadly speaking, if the expected pay rise following a promotion is high (since $\Delta > \Delta_T(\underline{\theta} - E\theta_i)$), the agent exerts a level of effort higher than the first best. Conversely, if the expected pay rise following a promotion is small, the agent exerts a level of effort lower than the first-best. However, observe that (12) implies that when $E\theta_i$ is far below (above) $\underline{\theta}$, $\Delta_T(\underline{\theta} - E\theta_i) \rightarrow +\infty$ so that $e_i^{1*} < e^{FB}$ whatever Δ . In that case, the promotion-related incentives are mute because effort has no impact on the promotion outcome. To summarize:

Lemma 3 *When $|\underline{\theta} - E\theta_i|$ is large, $e_i^{1*} < e^{FB}$. Otherwise, $e_i^{1*} \leq e^{FB}$ if $\Delta \leq \Delta_T(\underline{\theta} - E\theta_i)$ and $e_i^{1*} > e^{FB}$ if $\Delta > \Delta_T(\underline{\theta} - E\theta_i)$.*

How does it relate to the agent's choice of project and the firm's job or organization design analyzed in the previous sections? Comparing the value taken by Δ on the one hand either to $\Delta_I(\underline{\theta} - E\theta_i)$ and $\Delta_{II}(\underline{\theta} - E\theta_i)$ or to $\overline{\Delta}(\underline{\theta} - E\theta_i)$, which determine the agent's choice of a project (see Proposition 3), and on the other hand to $\Delta_T(\underline{\theta} - E\theta_i)$ allows us to relate effort inefficiencies and the agent's choice of project, since we know that $\Delta_T(\underline{\theta} - E\theta_i) < \min\left\{\Delta_I(\underline{\theta} - E\theta_i); \overline{\Delta}(\underline{\theta} - E\theta_i)\right\}$ (see the proof of Proposition 4). The firm's choice of job or organization design is always dictated by the desire to increase the agent's effort. Thus, it increases inefficiency would the effort performed by the agent be higher than the first best in the

absence of the employer's intervention. Otherwise, it increases efficiency. The next Proposition summarizes the results obtained.

Proposition 4 *Since $\Delta_T(\underline{\theta} - E\theta_i) < \min \left\{ \Delta_I(\underline{\theta} - E\theta_i); \overline{\Delta}(\underline{\theta} - E\theta_i) \right\}$, the efficiency in terms of effort is*

- *raised by the agent's choice of project when (i) $|\underline{\theta} - E\theta_i|$ is small if $\Delta_T < \Delta < \Delta_I(\underline{\theta} - E\theta_i)$ or $\Delta \geq \Delta_{II}(\underline{\theta} - E\theta_i)$ and (ii) $|\underline{\theta} - E\theta_i|$ is intermediate, $E\theta_i > \underline{\theta}$ if $\Delta \geq \overline{\Delta}(\underline{\theta} - E\theta_i)$ or $E\theta_i < \underline{\theta}$ if $\Delta_T < \Delta < \Delta_I(\underline{\theta} - E\theta_i)$ or $\Delta \geq \Delta_{II}(\underline{\theta} - E\theta_i)$. Otherwise, it is reduced by the agent's choice of project.*
- *reduced by the firm's job or organization design when $|\underline{\theta} - E\theta_i|$ is small or intermediate if $\Delta \geq \Delta_T$. Otherwise, it is raised by the firm's choice of design.*

An interesting result emerges: in equilibrium there exists cases in which the opportunistic agent makes project choices which reduce effort inefficiency whereas the employer makes design choices which increase inefficiency. For instance, this occurs when (i) the agent exerts a level of effort strictly higher than the first-best whatever project is chosen because the pay rise associated with a promotion is substantial, (ii) the agent's prior reputation is reasonably above the promotion threshold so that the agent chooses the more informative project to decrease the effort exerted (see above our discussion), and (iii) the employer chooses the less informative job or organization design to stimulate the agent's diligence. The fact that social welfare increases in output accuracy occurs for the opposite reason that Holmström (1982/1999) develops in his Proposition 1 (page 174). In Holmström's paper, the equilibrium level of effort is lower than the first-best level of effort, and effort increases when output becomes more informative (see also Dewatripont et al. (1999), the single-task additive-normal model, page 187). In the case

discussed above, the equilibrium level of effort is higher than the first best and effort decreases in output accuracy.

Naturally, there also exist cases in which the agent's choices raise effort inefficiency while employers' choices increase efficiency. For instance, this occurs when (i) the pay rise which accompanies promotion is small so that the agent's execution effort is below the first-best whatever the project chosen, (ii) the agent opts for the project which is less demanding in terms of effort, and (iii) the employer designs the job or the organization so as to over-stimulate the agent's diligence.

6 Conclusion

In this article, we use a career concerns model to study the impact that the prospect of a promotion has on a manager's incentives to let the labour market learn about his managerial ability, through the choice of a project, and to "properly" execute the project selected (i.e., to exert more or less effort). We show that the manager's project choice takes into account the impact on the probability of being promoted, the wage obtained in case of promotion and the effort exerted in equilibrium. The manager's eventual decision depends on the difference between his initial reputation and the expected ability required to be promoted. We obtain the following original results. First, the fact that incentives to exert effort decline as the manager's output is a less accurate measure of ability- a key insight of the career concerns literature - is not systematically true in our context. Indeed, if prior reputation is reasonably above the promotion threshold, the manager has more to lose if his reputation deteriorates than to gain if it is improved since the wage function is then locally concave in reputation. Lower output accuracy increases concavity. Thus, the manager exerts more effort to compensate for adverse realizations of factors which are out of his reach. If the rise in wage associated with the promotion

is large, this effect dominates the traditional learning effect à la Holmström, and effort eventually decreases in output accuracy. Second, this result implies that some of these good-reputation managers choose the more informative project, which at first sight is surprising since such a project increases the extent to which reputation is revised at the end of the first period, and thus raises the chances that they will eventually appear as not being talented enough to deserve a promotion. A reason for choosing this more informative project is that it reduces the effort to be exerted in equilibrium. Third, a consequence of the former result is that the employer's optimal reaction is to design the job or the organization such that output is less informative about ability in order to restore incentives to exert execution effort. In other words, the employer and the employee play a cat and mouse game. Fourth, if the pay rise associated with the promotion is large enough to induce excessive effort from a social point of view, an opportunistic manager who chooses a project which lowers the execution effort to be exerted reduces inefficiency. We believe our model can shed light on other situations as well. For instance, a supplier willing to sign a procurement contract with a private firm or the Government will exhibit varying diligence before the firm's or the Government decision is made, depending on its reputation, the profitability of the contract, and the type of action to be performed.

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7 Appendix

7.1 Proof of Lemmas 1 and 2

Promotion Rule

Agent i is promoted at the beginning of the second-period if and only if:

$$E(Y_i | y_i^1, p, d, e_i^{1*}) \geq E(y_i^S | y_i^1, p, d, e_i^{1*}).$$

Since agent i exerts no effort during the second (and last) period the previous inequality is equivalent to:

$$\begin{aligned} & \left(\int_{-\infty}^{\underline{\theta}} (\theta_i - \Delta) f(\theta_i | y_i^1, p, d, e_i^{1*}) d\theta_i + \int_{\underline{\theta}}^{+\infty} (\theta_i + \Delta) f(\theta_i | y_i^1, p, d, e_i^{1*}) d\theta_i \right) \geq \int_{-\infty}^{+\infty} \theta_i f(\theta_i | y_i^1, p, d, e_i^{1*}) d\theta_i \\ & \Leftrightarrow \Pr(\theta_i | y_i^1, p, d, e_i^{1*} < \underline{\theta}) \times (-\Delta) + \Pr(\theta_i | y_i^1, p, d, e_i^{1*} \geq \underline{\theta}) \times \Delta \geq 0 \\ & \Leftrightarrow \Pr(\theta_i | y_i^1, p, d, e_i^{1*} \geq \underline{\theta}) \geq \frac{1}{2}. \end{aligned} \tag{13}$$

Moreover, since the random variable $(\theta_i | y_i^1, p, d, e_i^{1*})$ follows a $N(E(\theta_i | y_i^1, p, d, e_i^{1*}); Var(\theta_i | y_i^1, p, d, e_i^{1*}))$

(13) is equivalent to:

$$\begin{aligned} & \left[1 - \Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{Var(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] \geq \frac{1}{2} \\ & \Leftrightarrow \Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{Var(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \leq \frac{1}{2} \\ & \Leftrightarrow E(\theta_i | y_i^1, p, d, e_i^{1*}) \geq \underline{\theta} \text{ since } \Phi^{-1} \left(\frac{1}{2} \right) = 0. \end{aligned}$$

Agent i is thus promoted at the beginning of the second-period if and only if $E(\theta_i | y_i^1, p, d, e_i^{1*}) \geq \underline{\theta}$ ($\Leftrightarrow y_i^1 \geq \underline{y}$).

Second-Period Wages

If $E(\theta_i | y_i^1, p, d, e_i^{1*}) < \underline{\theta}$ the agent is not promoted and $W_i^2 = E(y_i | y_i^1, p, d, e_i^{1*}) = E(\theta_i | y_i^1, p, d, e_i^{1*})$.

If $E(\theta_i | y_i^1, p, d, e_i^{1*}) \geq \underline{\theta}$ the agent is promoted and

$$\begin{aligned} W_i^2 &= E(Y_i | y_i^1, p, d, e_i^{1*}) \\ &= E(\theta_i | y_i^1, p, d, e_i^{1*}) + \left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \times \Delta. \end{aligned} \quad (14)$$

Let $\bar{\theta}$ be such that:

$$\text{For } E(\theta_i | y_i^1, p, d, e_i^{1*}) \geq \bar{\theta}, \Pr(\theta_i | y_i^1, p, d, e_i^{1*} \geq \underline{\theta}) = 1.$$

For $\underline{\theta} \leq E(\theta_i | y_i^1, p, d, e_i^{1*}) < \bar{\theta}$, the second term in the RHS of (14) is different from Δ . Moreover,

$$\begin{aligned} &\frac{\partial}{\partial E(\theta_i | y_i^1, p, d, e_i^{1*})} \left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \\ &= \frac{2}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \varphi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) > 0, \\ &\text{and } \frac{\partial^2}{\partial E(\theta_i | y_i^1, p, d, e_i^{1*})^2} \left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \\ &= \frac{2[\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})]}{(\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*}))^{\frac{3}{2}}} \varphi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \leq 0, \\ &\text{since } E(\theta_i | y_i^1, p, d, e_i^{1*}) \geq \underline{\theta} \text{ upon promotion.} \end{aligned}$$

W_i^2 is thus an increasing and concave function of $E(\theta_i | y_i^1, p, d, e_i^{1*})$, when $\underline{\theta} \leq E(\theta_i | y_i^1, p, d, e_i^{1*}) < \bar{\theta}$. For $E(\theta_i | y_i^1, p, d, e_i^{1*}) \geq \bar{\theta}$, the second term in the RHS of (14) just equals Δ and $W_i^2 =$

$E(\theta_i | y_i^1, p, d, e_i^{1*}) + \Delta$. W_i^2 is thus linearly increasing in $E(\theta_i | y_i^1, p, d, e_i^{1*})$.

7.2 Proof of Proposition 1

We first characterize agent i 's second-period expected wage. Then, we determine agent i 's first-period effort. Finally, we study the evolution of agent i 's first-period effort with respect to σ_p^2 .

Agent i 's second-period expected wage

During the first-period, agent i 's chooses to exert an effort which maximizes the following objective function:

$$\int_{-\infty}^{\underline{y}} \left[\int_{-\infty}^{+\infty} \theta_i f(\theta_i | y_i^1, p, d, e_i^{1*}) d\theta_i \right] g(y_i^1) dy_i^1$$

$$+ \int_{\underline{y}}^{+\infty} \left[\int_{-\infty}^{\underline{\theta}} (\theta_i - \Delta) f(\theta_i | y_i^1, p, d, e_i^{1*}) d\theta_i \right. \\ \left. + \int_{\underline{\theta}}^{+\infty} (\theta_i + \Delta) f(\theta_i | y_i^1, p, d, e_i^{1*}) d\theta_i \right] g(y_i^1) dy_i^1$$

$$- \psi(e_i^1),$$

where \underline{y} is the value of y_i^1 which is such that $E(\theta_i | y_i^1, p, d, e_i^{1*}) = \underline{\theta}$. Agent i 's objective function

could be rewritten as:

$$\begin{aligned}
& \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} \theta_i f(\theta_i | y_i^1, p, d, e_i^{1*}) d\theta_i \right] g(y_i^1) dy_i^1 \\
& + \Delta \left[\int_{\underline{y}}^{+\infty} \left[\begin{array}{c} - \int_{-\infty}^{\underline{\theta}} f(\theta_i | y_i^1, p, d, e_i^{1*}) d\theta_i \\ + \int_{\underline{\theta}}^{+\infty} f(\theta_i | y_i^1, p, d, e_i^{1*}) d\theta_i \end{array} \right] g(y_i^1) dy_i^1 \right] \\
& - \psi(e_i^1),
\end{aligned}$$

which is equal to

$$\begin{aligned}
& E_{y_i^1} [E(\theta_i | y_i^1, p, d, e_i^{1*})] \\
& + \Delta \left[\int_{\underline{y}}^{+\infty} [1 - 2 \Pr((\theta_i | y_i^1, p, d, e_i^{1*}) < \underline{\theta})] g(y_i^1) dy_i^1 \right] \\
& - \psi(e_i^1).
\end{aligned}$$

Consider now the term associated with Δ . We have:

$$\begin{aligned}
& \int_{\underline{y}}^{+\infty} [1 - 2 \Pr((\theta_i | y_i^1, p, d, e_i^{1*}) < \underline{\theta})] g(y_i^1) dy_i^1 \\
& = \int_{\underline{y}}^{+\infty} \left[1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] g(y_i^1) dy_i^1 \\
& = \frac{1}{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} \frac{1}{\sqrt{2\pi}} \int_{\underline{y}}^{+\infty} \left[1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] \exp \left(-\frac{1}{2} \frac{(y_i^1 - E y_i^1)^2}{\text{Var} y_i^1} \right) dy_i^1 \\
& = \frac{1}{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} \int_{\underline{y}}^{+\infty} \left[1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] \varphi \left(\frac{y_i^1 - E y_i^1}{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} \right) dy_i^1.
\end{aligned}$$

Moreover, $E(\theta_i | y_i^1, p, d, e_i^{1*}) - E\theta_i = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} (y_i^1 - Ey_i^1)$ and

$Var[E(\theta_i | y_i^1, p, d, e_i^{1*})] = \frac{\sigma_\theta^4}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}$. Thus, $\frac{E(\theta_i | y_i^1, p, d, e_i^{1*}) - E\theta_i}{\sqrt{Var[E(\theta_i | y_i^1, p, d, e_i^{1*})]}} = \frac{y_i^1 - Ey_i^1}{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}$. This implies that

$$\begin{aligned} & \frac{1}{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} \int_{\underline{y}}^{+\infty} \left[1 - 2\Phi \left(\frac{\theta - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{Var(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] \varphi \left(\frac{y_i^1 - Ey_i^1}{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} \right) dy_i^1 \\ &= \frac{1}{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} \int_{\underline{y}}^{+\infty} \left[1 - 2\Phi \left(\frac{\theta - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{Var(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] \varphi \left(\frac{E(\theta_i | y_i^1, p, d, e_i^{1*}) - E\theta_i}{\sqrt{Var[E(\theta_i | y_i^1, p, d, e_i^{1*})]}} \right) dy_i^1. \end{aligned}$$

$E(\theta_i | y_i^1, p, d, e_i^{1*}) = E\theta_i + \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} (y_i^1 - E(y_i^1))$ implies that

$$\begin{aligned} dE(\theta_i | y_i^1, p, d, e_i^{1*}) &= \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} dy_i^1 \\ \Leftrightarrow dy_i^1 &= \frac{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}{\sigma_\theta^2} dE(\theta_i | y_i^1, p, d, e_i^{1*}) \end{aligned}$$

and

$$y_i^1 = \underline{y} \Leftrightarrow E(\theta_i | y_i^1, p, d, e_i^{1*}) = \underline{\theta} \text{ (by definition of } \underline{y}\text{).}$$

We thus have:

$$\begin{aligned}
& \frac{1}{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} \int_{\underline{y}}^{+\infty} \left[1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] \varphi \left(\frac{E(\theta_i | y_i^1, p, d, e_i^{1*}) - E\theta_i}{\sqrt{\text{Var}[E(\theta_i | y_i^1, p, d, e_i^{1*})]}} \right) dy_i^1 \\
&= \frac{1}{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} \frac{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}{\sigma_\theta^2} \int_{\underline{\theta}}^{+\infty} \left[\left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \right. \\
&\quad \left. \times \varphi \left(\frac{E(\theta_i | y_i^1, p, d, e_i^{1*}) - E\theta_i}{\sqrt{\text{Var}[E(\theta_i | y_i^1, p, d, e_i^{1*})]}} \right) \right] dE(\theta_i | y_i^1, p, e_i^{1*}) \\
&= \frac{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \int_{\underline{\theta}}^{+\infty} \left[\left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \right. \\
&\quad \left. \times \varphi \left(\frac{E(\theta_i | y_i^1, p, d, e_i^{1*}) - E\theta_i}{\sqrt{\text{Var}[E(\theta_i | y_i^1, p, d, e_i^{1*})]}} \right) \right] dE(\theta_i | y_i^1, p, e_i^{1*}) \\
&= \frac{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \frac{1}{\sqrt{2\pi}} \int_{\underline{\theta}}^{+\infty} \left[\left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \right. \\
&\quad \left. \times \exp \left(-\frac{1}{2} \frac{(E(\theta_i | y_i^1, p, d, e_i^{1*}) - E\theta_i)^2}{\text{Var}[E(\theta_i | y_i^1, p, d, e_i^{1*})]} \right) \right] dE(\theta_i | y_i^1, p, e_i^{1*}) \\
&= \int_{\underline{\theta}}^{+\infty} \left[1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] f(E(\theta_i | y_i^1, p, d, e_i^{1*})) dE(\theta_i | y_i^1, p, d, e_i^{1*}).
\end{aligned}$$

Agent i 's First-Period Effort

The agent exerts an effort in order to increase his second-period expected wage, since his first-period wage is fixed and does not depend on the first-period output. Agent i 's thus chooses e_i^1 so as to maximize

$$\begin{aligned}
& E[E(\theta_i | y_i^1, p, d, e_i^{1*})] \\
& + \Delta \int_{\underline{\theta}}^{+\infty} \left[1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] f(E(\theta_i | y_i^1, p, d, e_i)) dE(\theta_i | y_i^1, p, d, e_i) \\
& - \psi(e_i^1).
\end{aligned}$$

The first order condition with respect to e_i^1 is

$$\int \int \theta_i \frac{g_{e_i}(y_i^1 | p, d, e_i^{1*})}{g(y_i^1 | p, d, e_i^{1*})} h(\theta_i, y_i^1 | p, d, e_i^{1*}) dy_i^1 d\theta_i + \frac{\partial}{\partial e_i} \left(\int_{\underline{\theta}}^{+\infty} \left[\left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \times f(E(\theta_i | y_i^1, p, d, e_i^{1*})) \right] dE(\theta_i | y_i^1, p, d, e_i^{1*}) \right) \times \Delta \Big|_{e_i=e_i^{1*}} = e_i^{1*} \quad (15)$$

where $g(y_i^1 | p, d, e_i^{1*}) = \int h(\theta_i, y_i^1 | p, d, e_i^{1*}) d\theta_i$ and $h(\theta_i, y_i^1 | p, d, e_i^{1*})$ denote respectively the marginal density of the observables, and the joint density of agent i 's ability and of the observables, given the equilibrium level of effort e_i^{1*} , the choice of project p , and the firm's job or organization design d . g_{e_i} denotes the derivative of the marginal distribution with respect to agent i 's effort.

Consider the first term in the left-hand side of (15). Since the likelihood ratio has zero mean, i.e. $E \left(\frac{g_{e_i}(y_i^1 | p, d, e_i^{1*})}{g(y_i^1 | p, d, e_i^{1*})} \right) = 0$,

$$\int \int \theta_i \frac{g_{e_i}(y_i^1 | p, d, e_i^{1*})}{g(y_i^1 | p, d, e_i^{1*})} h(\theta_i, y_i^1 | p, d, e_i^{1*}) dy_i^1 d\theta_i = \text{cov} \left(\theta_i, \frac{g_{e_i}(y_i^1 | p, d, e_i^{1*})}{g(y_i^1 | p, d, e_i^{1*})} \right). \quad (16)$$

The marginal density $g(y_i^1 | p, d, e_i^{1*})$ is proportional to $\exp \left(-\frac{1}{2} \frac{(y_i^1 - (E\theta_i + e_i))}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \right)$ and (cf. Dewatripont, Jewitt and Tirole, 1999)

$$\frac{g_{e_i}(\cdot)}{g(\cdot)} = \frac{y_i^1 - E(y_i^1)}{\text{Var}(y_i^1)} = \frac{(\theta_i - E\theta_i) + \varepsilon_p}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}.$$

Thus,

$$\text{cov} \left(\theta_i, \frac{g_{e_i}(y_i^1 | p, d, e_i^{1*})}{g(y_i^1 | p, d, e_i^{1*})} \right) = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}. \quad (17)$$

Now turn to the second part in the left-hand side of (15).

$$\begin{aligned}
& \frac{\partial}{\partial e_i} \left(\int_{\underline{\theta}}^{+\infty} \left[\left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \right. \right. \\
& \quad \left. \left. \times f(E(\theta_i | y_i^1, p, d, e_i)) \right] dE(\theta_i | y_i^1, p, d, e_i) \right) \times \Delta \Big|_{e_i=e_i^{1*}} \\
&= \Delta \int_{\underline{\theta}}^{+\infty} \left[\left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \times f_{e_i}(E(\theta_i | y_i^1, p, d, e_i^{1*})) \right] dE(\theta_i | y_i^1, p, d, e_i^{1*}).
\end{aligned} \tag{18}$$

Making computations (See Additional Technical Appendix 1), we obtain that

$$\begin{aligned}
& \Delta \int_{\underline{\theta}}^{+\infty} \left[\left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \times f_{e_i}(E(\theta_i | y_i^1, p, d, e_i^{1*})) \right] dE(\theta_i | y_i^1, p, d, e_i^{1*}) \\
&= \Delta \int_{\underline{\theta}}^{+\infty} \left[\begin{aligned} & \left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \\ & \times \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \times \frac{E(\theta_i | y_i^1, p, d, e_i^{1*}) - E\theta_i}{\text{Var}(E(\theta_i | y_i^1, p, d, e_i^{1*}))} \times f(E(\theta_i | y_i^1, p, d, e_i^{1*})) \end{aligned} \right] dE(\theta_i | y_i^1, p, d, e_i^{1*}).
\end{aligned}$$

Thus, (18) reduces to

$$\begin{aligned}
& \Delta \int_{\underline{\theta}}^{+\infty} \left[\begin{aligned} & \left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \\ & \times \frac{-\sigma_\theta^2}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \times f'(E(\theta_i | y_i^1, p, d, e_i^{1*})) \end{aligned} \right] dE(\theta_i | y_i^1, p, d, e_i^{1*}) \\
&= \Delta \frac{-\sigma_\theta^2}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \int_{\underline{\theta}}^{+\infty} \left[\begin{aligned} & \left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \\ & \times f'(E(\theta_i | y_i^1, p, d, e_i^{1*})) \end{aligned} \right] dE(\theta_i | y_i^1, p, d, e_i^{1*}).
\end{aligned}$$

Integrating by part (See Additional Technical Appendix 2), we obtain:

$$\Delta \frac{-\sigma_\theta^2}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \int_{\underline{\theta}}^{+\infty} \left[\left(1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right) \right. \\ \left. \times f' (E(\theta_i | y_i^1, p, d, e_i^{1*})) \right] dE(\theta_i | y_i^1, p, d, e_i^{1*}) \\ = \left[\frac{2\Delta}{\sqrt{\sigma_\theta^2(\sigma_p^2 + \sigma_d^2)}} \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} \frac{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2}{\sigma_{\theta_i}^4 (\sigma_p^2 + \sigma_d^2)} \left(\frac{\underline{\theta}^2 \sigma_\theta^2 + E(\theta_i)^2 (\sigma_p^2 + \sigma_d^2)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} - \left(\frac{\underline{\theta} \sigma_\theta^2 + E\theta_i \times (\sigma_p^2 + \sigma_d^2)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \right)^2 \right) \right] \right] \\ \times \int_{\underline{\theta}}^{+\infty} \varphi \left(\frac{E(\theta_i | y_i^1, p, d, e_i^{1*}) - \frac{\underline{\theta} \sigma_\theta^2 + E\theta_i \times (\sigma_p^2 + \sigma_d^2)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}{\frac{\sigma_\theta^2 \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} \right) dE(\theta_i | y_i^1, p, d, e_i^{1*}) \right].$$

Consider now the term $\int_{\underline{\theta}}^{+\infty} \varphi \left(\frac{E(\theta_i | y_i^1, p, d, e_i^{1*}) - \frac{\underline{\theta} \sigma_\theta^2 + E\theta_i \times (\sigma_p^2 + \sigma_d^2)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}{\frac{\sigma_\theta^2 \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} \right) dE(\theta_i | y_i^1, p, d, e_i^{1*})$.

Let $z \equiv \frac{E(\theta_i | y_i^1, p, d, e_i^{1*}) - \frac{\underline{\theta} \sigma_\theta^2 + E\theta_i \times (\sigma_p^2 + \sigma_d^2)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}{\frac{\sigma_\theta^2 \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}$

$$\Rightarrow \begin{cases} dz = \frac{dE(\theta_i | y_i^1, p, d, e_i^{1*})}{\frac{\sigma_\theta^2 \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} \Leftrightarrow \frac{\sigma_\theta^2 \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} dz = dE(\theta_i | y_i^1, p, d, e_i^{1*}) \\ \text{and when } E(\theta_i | y_i^1, p, d, e_i^{1*}) = \underline{\theta} \Leftrightarrow z = \frac{\underline{\theta} - \frac{\underline{\theta} \sigma_\theta^2 + E\theta_i \times (\sigma_p^2 + \sigma_d^2)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}{\frac{\sigma_\theta^2 \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} = \frac{(\underline{\theta} - E(\theta_i)) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2}. \end{cases}$$

Thus, we obtain:

$$\int_{\underline{\theta}}^{+\infty} \varphi \left(\frac{E(\theta_i | y_i^1, p, d, e_i^{1*}) - \frac{\underline{\theta} \sigma_\theta^2 + E\theta_i \times (\sigma_p^2 + \sigma_d^2)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}{\frac{\sigma_\theta^2 \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} \right) dE(\theta_i | y_i^1, p, d, e_i^{1*}) \\ = \frac{\sigma_\theta^2 \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \int_{\frac{(\underline{\theta} - E(\theta_i)) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2}}^{+\infty} \varphi(z) dz = \frac{\sigma_\theta^2 \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \left(1 - \Phi \left(\frac{(\underline{\theta} - E(\theta_i)) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right).$$

Finally,

$$\begin{aligned}
& \left[\frac{2\Delta}{\sqrt{\sigma_\theta^2(\sigma_p^2+\sigma_d^2)}} \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} \frac{(\sigma_\theta^2+\sigma_p^2+\sigma_d^2)^2}{\sigma_{\theta_i}^4(\sigma_p^2+\sigma_d^2)} \left(\frac{\varrho^2 \sigma_\theta^2 + E(\theta_i)^2 (\sigma_p^2 + \sigma_d^2)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} - \left(\frac{\varrho \sigma_\theta^2 + E\theta_i \times (\sigma_p^2 + \sigma_d^2)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \right)^2 \right) \right] \right. \\
& \quad \left. \times \int_{\varrho}^{+\infty} \varphi \left(\frac{E(\theta_i | y_i^1, p, d, e_i^{1*}) - \frac{\varrho \sigma_\theta^2 + E\theta_i \times (\sigma_p^2 + \sigma_d^2)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}{\frac{\sigma_\theta^2 \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} \right) dE(\theta_i | y_i^1, p, d, e_i^{1*}) \right] \\
= & \left[\frac{2\Delta}{\sqrt{\sigma_\theta^2(\sigma_p^2+\sigma_d^2)}} \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} \frac{(\sigma_\theta^2+\sigma_p^2+\sigma_d^2)^2}{\sigma_{\theta_i}^4(\sigma_p^2+\sigma_d^2)} \left(\frac{\varrho^2 \sigma_\theta^2 + E(\theta_i)^2 (\sigma_p^2 + \sigma_d^2)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} - \left(\frac{\varrho \sigma_\theta^2 + E\theta_i \times (\sigma_p^2 + \sigma_d^2)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \right)^2 \right) \right] \right. \\
& \quad \left. \times \frac{\sigma_\theta^2 \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \left(1 - \Phi \left(\frac{(\varrho - E(\theta_i)) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right) \right] \\
= & \left[\frac{\sqrt{\sigma_\theta^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \frac{2\Delta}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} \frac{(\sigma_\theta^2+\sigma_p^2+\sigma_d^2)^2}{\sigma_{\theta_i}^4(\sigma_p^2+\sigma_d^2)} \left(\frac{\varrho^2 \sigma_\theta^2 + E(\theta_i)^2 (\sigma_p^2 + \sigma_d^2)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} - \left(\frac{\varrho \sigma_\theta^2 + E\theta_i \times (\sigma_p^2 + \sigma_d^2)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \right)^2 \right) \right] \right. \\
& \quad \left. \times \left(1 - \Phi \left(\frac{(\varrho - E(\theta_i)) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right) \right] \\
= & \left[\frac{\sqrt{\sigma_\theta^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \frac{2\Delta}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} \frac{(\sigma_\theta^2+\sigma_p^2+\sigma_d^2)^2}{\sigma_{\theta_i}^4(\sigma_p^2+\sigma_d^2)} \frac{(\varrho^2 \sigma_\theta^2 + E(\theta_i)^2 (\sigma_p^2 + \sigma_d^2)) (\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2) - (\varrho \sigma_\theta^2 + E\theta_i \times (\sigma_p^2 + \sigma_d^2))^2}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2} \right] \right. \\
& \quad \left. \times \left(1 - \Phi \left(\frac{(\varrho - E(\theta_i)) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right) \right] \\
= & \frac{\sqrt{\sigma_\theta^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \frac{2\Delta}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} \frac{\sigma_\theta^2 (\sigma_p^2 + \sigma_d^2)}{\sigma_{\theta_i}^4 (\sigma_p^2 + \sigma_d^2)} (\varrho - E\theta_i)^2 \right] \times \left(1 - \Phi \left(\frac{(\varrho - E(\theta_i)) \sqrt{(\sigma_p^2 + \sigma_d^2)}}{\sigma_\theta^2} \right) \right) \\
= & \frac{\sqrt{\sigma_\theta^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \frac{2\Delta}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} \frac{(\varrho - E\theta_i)^2}{\sigma_\theta^2} \right] \times \left(1 - \Phi \left(\frac{(\varrho - E(\theta_i)) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right). \tag{19}
\end{aligned}$$

Using both (17) and (19) the first order condition (cf. (15)) reduces to:

$$\begin{aligned}
& \frac{\partial}{\partial e_i} \left[\int \left(\int \theta_i \frac{f(\theta_i, y_i^1 | p, d, e_i^{1*})}{\hat{f}(y_i^1 | p, d, e_i^{1*})} d\theta_i \right) d\hat{F}(y_i^1 | p, d, e_i) \right. \\
& \quad \left. + \Delta \int_{\varrho}^{+\infty} \left[1 - 2\Phi \left(\frac{\varrho - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] f(E(\theta_i | y_i^1, p, d, e_i)) dE(\theta_i | y_i^1, p, d, e_i) \right] \Big|_{e_i=e_i^{1*}} = e_i^{1*} \\
\Leftrightarrow & \left[\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \right. \\
& \quad \left. + \frac{\sqrt{\sigma_\theta^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \frac{2\Delta}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} \frac{(\varrho - E\theta_i)^2}{\sigma_\theta^2} \right] \times \left(1 - \Phi \left(\frac{(\varrho - E(\theta_i)) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right) \right] = e_i^{1*}. \tag{20}
\end{aligned}$$

Evolution of agent i 's first-period effort with respect to σ_p^2

Using (20) we define $Term\ 1 \equiv \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}$ and

$$Term\ 2 \equiv \frac{\sqrt{\sigma_\theta^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \frac{2\Delta}{\sqrt{2\pi}} \exp\left[-\frac{1}{2} \frac{(\underline{\theta} - E\theta_i)^2}{\sigma_\theta^2}\right] \times \left(1 - \Phi\left(\frac{(\underline{\theta} - E\theta_i) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2}\right)\right).$$

Thus, we have:

$$\frac{\partial Term\ 1}{\partial \sigma_p^2} = -\frac{\sigma_\theta^2}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2} < 0. \quad (21)$$

Moreover:

$$\frac{\partial Term\ 2}{\partial \sigma_p^2} = \left[\begin{array}{c} \frac{-\frac{2\Delta}{\sqrt{2\pi}} \frac{\sqrt{\sigma_\theta^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \exp\left[-\frac{1}{2} \frac{(\underline{\theta} - E\theta_i)^2}{\sigma_\theta^2}\right]}{\left(\frac{1}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \left(1 - \Phi\left(\frac{(\underline{\theta} - E\theta_i) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2}\right)\right)\right)} \\ \times \left(\underbrace{\frac{(\underline{\theta} - E\theta_i)}{2\sigma_{\theta_i}^2 \sqrt{\sigma_p^2 + \sigma_d^2}} \times \varphi\left(\frac{(\underline{\theta} - E\theta_i) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2}\right)}_{Term\ B} \right) \end{array} \right]. \quad (22)$$

$Term\ A$ is always positive. However, the sign of $Term\ B$ depends on $E\theta_i$.

Case 1: $E\theta_i < \underline{\theta}$: Terms A and B are both positive, which implies that $\frac{\partial Term\ 2}{\partial \sigma_p^2}$ is strictly negative. Thus $\frac{\partial Term\ 1}{\partial \sigma_p^2}$ and $\frac{\partial Term\ 2}{\partial \sigma_p^2}$ are both strictly negative and e_i^{1*} is strictly decreasing in σ_p^2 .

Case 2: $E\theta_i \geq \underline{\theta}$: $Term\ B$ is then negative while $Term\ A$ is positive. Let $X \stackrel{d}{=} (\underline{\theta} - E\theta_i) \times \frac{\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2}$ (note that $X \leq 0$). This implies that

$$Term\ A + Term\ B = \frac{1}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} (1 - \Phi(X)) + \frac{1}{2(\sigma_p^2 + \sigma_d^2)} \times X \times \varphi(X).$$

Thus, $Term\ A + Term\ B < 0$ if and only if:

$$\frac{\sigma_\theta^2}{2(\sigma_p^2 + \sigma_d^2)} + \frac{1}{2} > -\frac{1 - \Phi(X)}{X\varphi(X)}. \quad (23)$$

The RHS of (23) tends to infinity, both when $X \rightarrow 0^-$ and $X \rightarrow -\infty$. However, it takes finite values when X takes intermediate values and $\min_{X < 0} \left(-\frac{1 - \Phi(X)}{X\varphi(X)} \right) < 3.4$. This implies that, for any finite value $A > 3.4$, there exists an interval $[\underline{X}, \overline{X}]$, with $\overline{X} < 0$ such that:

$$-\frac{1 - \Phi(X)}{X\varphi(X)} \leq A, \text{ for } X \in [\underline{X}, \overline{X}].$$

Therefore, for any finite value that takes $(\sigma_p^2 + \sigma_d^2)$, (23) is necessarily satisfied for $\sigma_\theta^2 > 2 \times (3.4 - 0.5) \times (\sigma_p^2 + \sigma_d^2)$ (σ_θ^2 high enough) and $|\underline{\theta} - E\theta_i| \in \left[\underline{X} \times \frac{\sigma_\theta^2}{\sqrt{\sigma_p^2 + \sigma_d^2}}, \overline{X} \times \frac{\sigma_\theta^2}{\sqrt{\sigma_p^2 + \sigma_d^2}} \right]$, that is what we call $|\underline{\theta} - E\theta_i|$ taking intermediate values.

Using (21) and (22) $\left[\frac{\partial Term\ 1}{\partial \sigma_p^2} + \frac{\partial Term\ 2}{\partial \sigma_p^2} \right] > 0$ if and only if:

$$\Delta \geq \overline{\Delta}(\underline{\theta} - E\theta_i) \equiv \frac{\frac{\sqrt{\sigma_\theta^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}{-2\varphi\left(\frac{(\underline{\theta} - E\theta_i)}{\sqrt{\sigma_\theta^2}}\right) \left[\frac{1}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \left(1 - \Phi\left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2}\right) \right) + \frac{(\underline{\theta} - E\theta_i)}{2\sigma_\theta^2 \sqrt{\sigma_p^2 + \sigma_d^2}} \times \varphi\left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2}\right) \right]} \quad (24)$$

Note that when $|\underline{\theta} - E\theta_i|$ takes intermediate values, $\frac{2}{\sqrt{2\pi}} \frac{\sqrt{\sigma_\theta^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \exp\left[-\frac{1}{2} \frac{(\underline{\theta} - E\theta_i)^2}{\sigma_\theta^2}\right] \times \left[\frac{1}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \left(1 - \Phi\left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2}\right) \right) + \frac{(\underline{\theta} - E\theta_i)}{\sigma_\theta^2} \times \frac{1}{2\sqrt{\sigma_p^2 + \sigma_d^2}} \times \varphi\left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2}\right) \right] \neq 0$, which implies that $\overline{\Delta}(\underline{\theta} - E\theta_i)$ is finite.

7.3 Proof of Proposition 3

Consider the agent's choice of project at the beginning of the first-period. The agent maximizes his expected second-period wage since his first-period wage is already fixed. Moreover, since

markets correctly anticipate e_i^{1*} , posterior reputation is in expectation equal to initial reputation, i.e., $E_{y_i^1} [E(\theta_i | y_i^1, p, d, e_i^{1*})] = E\theta_i$. Hence, the agent makes his choice of project by considering his expected promotion extra revenue and the cost of effort implied by the choice of project. He thus chooses the project p so as to maximize

$$E\theta_i + \Delta \int_{\underline{\theta}}^{+\infty} \left[1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] dF [E(\theta_i | y_i^1, p, d, e_i^{1*})] - \psi(e_i^{1*}).$$

Let us compute

$$\frac{\partial}{\partial \sigma_p^2} \left[E\theta_i + \Delta \int_{\underline{\theta}}^{+\infty} \left[1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] dF [E(\theta_i | y_i^1, p, d, e_i^{1*})] - \psi(e_i^{1*}) \right]. \quad (25)$$

Let us define

$$\text{Term 3} \equiv \left[1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right],$$

and $\text{Term 4} \equiv f(E(\theta_i | y_i^1, p, d, e_i^{1*}))$.

We have:

$$\frac{\partial \text{Term 3}}{\partial \sigma_p^2} = \frac{\sqrt{\sigma_\theta^2}}{(\sigma_p^2 + \sigma_d^2)^{3/2}} \frac{(\underline{\theta} - E(\theta_i | \cdot))}{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} \varphi \left(\left(\frac{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}{\sqrt{\sigma_\theta^2 (\sigma_p^2 + \sigma_d^2)}} \times (\underline{\theta} - E(\theta_i | \cdot)) \right) \right),$$

since $Var(\theta_i | y_i^1, p, d, e_i^{1*}) = \sigma_\theta^2 - \frac{\sigma_{\theta_i}^4}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} = \frac{\sigma_\theta^2(\sigma_p^2 + \sigma_d^2)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}$. We don't have to differentiate $E(\theta_i | y_i^1, p, d, e_i^{1*})$ with respect to σ_p^2 since it corresponds to the variable of integration. Besides,

$$\frac{\partial Term\ 4}{\partial \sigma_p^2} = \left[\begin{aligned} & \frac{1}{2\sqrt{2\pi}} \frac{1}{\sigma_\theta^2} \left(\frac{1}{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} - \frac{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}{\sigma_{\theta_i}^4} (E(\theta_i |) - E\theta_i)^2 \right) \\ & \times \exp \left(-\frac{1}{2} \frac{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}{\sigma_{\theta_i}^4} (E(\theta_i |) - E\theta_i)^2 \right) \end{aligned} \right],$$

since $f(E(\theta_i | y_i^1, p, d, e_i^{1*})) = \frac{1}{\sqrt{2\pi}} \frac{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \exp \left(-\frac{1}{2} \frac{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}{\sigma_{\theta_i}^4} (E(\theta_i |) - E\theta_i)^2 \right)$. Thus

$$\begin{aligned} & \frac{\partial}{\partial \sigma_p^2} \left(\Delta \int_{\underline{\theta}}^{+\infty} \left[1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{Var(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] dF[E(\theta_i | y_i^1, p, d, e_i^{1*})] \right) \\ & = \Delta \left(\int_{\underline{\theta}}^{+\infty} \left(\frac{\partial Term\ 3}{\partial \sigma_p^2} \times f(E(\theta_i | y_i^1, p, d, e_i^{1*})) \right) dE(\theta_i | y_i^1, p, d, e_i^{1*}) + \right. \\ & \left. \int_{\underline{\theta}}^{+\infty} \left(\left[1 - 2\Phi \left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{Var(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] \times \frac{\partial Term\ 4}{\partial \sigma_p^2} \right) dE(\theta_i | y_i^1, p, d, e_i^{1*}) \right). \end{aligned} \quad (26)$$

Consider the first term in (26). We have:

$$\begin{aligned} & \Delta \int_{\underline{\theta}}^{+\infty} \left(\frac{\partial Term\ 3}{\partial \sigma_p^2} \times f(E(\theta_i | y_i^1, p, d, e_i^{1*})) \right) dE(\theta_i | y_i^1, p, d, e_i^{1*}) \\ & = \int_{\underline{\theta}}^{+\infty} \left[\frac{\sqrt{\sigma_\theta^2}}{(\sigma_p^2 + \sigma_d^2)^{3/2}} \frac{(\underline{\theta} - E(\theta_i |))}{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}} \varphi \left(\left(\frac{\sqrt{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}}{\sqrt{\sigma_\theta^2(\sigma_p^2 + \sigma_d^2)}} \times (\underline{\theta} - E(\theta_i |)) \right) \right) \right] \\ & \quad \times f(E(\theta_i | y_i^1, p, d, e_i^{1*})) \Big] dE(\theta_i | y_i^1, p, d, e_i^{1*}) \leq 0 \end{aligned}$$

Finally, after computations which are detailed in Additional Technical Appendix 3, we obtain:

$$\begin{aligned}
& \int_{\underline{\theta}}^{+\infty} \left(\frac{\partial \text{Term 3}}{\partial \sigma_p^2} \times f(E(\theta_i | y_i^1, p, d, e_i^{1*})) \right) dE(\theta_i | y_i^1, p, d, e_i^{1*}) \\
&= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \frac{(\underline{\theta} - E\theta_i)^2}{\sigma_\theta^2}\right) \left[\begin{aligned} & -\frac{1}{\sqrt{2\pi}} \sqrt{\frac{\sigma_\theta^2}{\sigma_p^2 + \sigma_d^2}} \frac{\sigma_\theta^2}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2} \exp\left(-\frac{1}{2} \frac{\sigma_p^2 + \sigma_d^2}{\sigma_\theta^4} (\underline{\theta} - E\theta_i)^2\right) \\ & + \frac{(\underline{\theta} - E\theta_i)}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2) \sqrt{\sigma_\theta^2 (\sigma_p^2 + \sigma_d^2)}} \left[1 - \Phi\left(\frac{\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} (\underline{\theta} - E\theta_i)\right) \right] \end{aligned} \right] \leq 0. \tag{27}
\end{aligned}$$

Consider now the second term in (26). After computations which are detailed in Additional Technical Appendix 4, we obtain:

$$\begin{aligned}
& \int_{\underline{\theta}}^{+\infty} \left(\left[1 - 2\Phi\left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}}\right) \right] \times \frac{\partial \text{Term 4}}{\partial \sigma_p^2} \right) dE(\theta_i | y_i^1, p, d, e_i^{1*}) \\
&= -\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \frac{(\underline{\theta} - E\theta_i)^2}{\sigma_\theta^2}\right) \left[\begin{aligned} & \frac{1}{\sqrt{2\pi}} \frac{\sqrt{\sigma_\theta^2 (\sigma_p^2 + \sigma_d^2)}}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2} \exp\left(-\frac{1}{2} \frac{\sigma_p^2 + \sigma_d^2}{\sigma_\theta^4} (\underline{\theta} - E\theta_i)^2\right) \\ & + \frac{(\underline{\theta} - E\theta_i)}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2) \sqrt{\sigma_\theta^2 (\sigma_p^2 + \sigma_d^2)}} \left(1 - \Phi\left(\frac{\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} (\underline{\theta} - E\theta_i)\right) \right) \end{aligned} \right] \tag{28}
\end{aligned}$$

Using (27) and (28) we finally have:

$$\begin{aligned}
& \frac{\partial}{\partial \sigma_p^2} \left(\Delta \int_{\underline{\theta}}^{+\infty} \left[1 - 2\Phi\left(\frac{\underline{\theta} - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}}\right) \right] dF[E(\theta_i | y_i^1, p, d, e_i^{1*})] \right) \\
&= -\frac{\Delta}{2\pi} \left(\sqrt{\frac{\sigma_\theta^2}{\sigma_p^2 + \sigma_d^2}} \frac{\sigma_\theta^2}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2} + \frac{\sqrt{\sigma_\theta^2 (\sigma_p^2 + \sigma_d^2)}}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2} \right) \exp\left(-\frac{1}{2} \frac{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}{\sigma_\theta^4} (\underline{\theta} - E\theta_i)^2\right) \\
&= -\frac{\Delta}{2\pi} \frac{\sigma_\theta^2}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2} \left(\sqrt{\frac{\sigma_\theta^2}{\sigma_p^2 + \sigma_d^2}} + \sqrt{\frac{\sigma_p^2 + \sigma_d^2}{\sigma_\theta^2}} \right) \exp\left(-\frac{1}{2} \frac{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}{\sigma_\theta^4} (\underline{\theta} - E\theta_i)^2\right) \\
&= -\frac{\Delta}{2\pi} \sqrt{\frac{\sigma_\theta^2}{\sigma_p^2 + \sigma_d^2}} \frac{1}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \exp\left(-\frac{1}{2} \frac{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}{\sigma_\theta^4} (\underline{\theta} - E\theta_i)^2\right) < 0. \tag{29}
\end{aligned}$$

Consider now

$$-\frac{\partial \psi(e_i^{1*})}{\partial \sigma_p^2} = -e_i^{1*} \times \frac{\partial e_i^{1*}}{\partial \sigma_p^2}.$$

Using (20), we have:

$$\frac{\partial e_i^{1*}}{\partial \sigma_p^2} = \left[\begin{array}{c} -\frac{\sigma_\theta^2}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2} \\ -\frac{2\Delta}{\sqrt{2\pi}} \frac{\sqrt{\sigma_\theta^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \exp \left[-\frac{1}{2} \frac{(\theta - E\theta_i)^2}{\sigma_\theta^2} \right] \left(\begin{array}{c} \frac{1}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \left(1 - \Phi \left(\frac{(\theta - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta} \right) \right) \\ + \frac{(\theta - E\theta_i)}{2\sigma_{\theta_i}^2} \frac{1}{\sqrt{\sigma_p^2 + \sigma_d^2}} \times \varphi \left(\frac{(\theta - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta} \right) \end{array} \right) \end{array} \right],$$

and

$$\begin{aligned} & -e_i^{1*} \times \frac{\partial e_i^{1*}}{\partial \sigma_p^2} \\ & = e_i^{1*} \times \left[\begin{array}{c} \frac{\sigma_\theta^2}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2} \\ + \frac{2\Delta}{\sqrt{2\pi}} \frac{\sqrt{\sigma_\theta^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \exp \left[-\frac{1}{2} \frac{(\theta - E\theta_i)^2}{\sigma_\theta^2} \right] \left(\begin{array}{c} \frac{1}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \left(1 - \Phi \left(\frac{(\theta - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta} \right) \right) \\ + \frac{(\theta - E\theta_i)}{\sigma_\theta^2} \times \frac{1}{2\sqrt{\sigma_p^2 + \sigma_d^2}} \times \varphi \left(\frac{(\theta - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta} \right) \end{array} \right) \end{array} \right] \quad (30) \end{aligned}$$

Finally, using (29) and (30) we obtain:

$$\begin{aligned} & \frac{\partial}{\partial \sigma_p^2} \left[E\theta_i + \Delta \int_{\underline{\theta}}^{+\infty} \left[1 - 2\Phi \left(\frac{\theta - E(\theta_i | y_i^1, p, d, e_i^{1*})}{\sqrt{\text{Var}(\theta_i | y_i^1, p, d, e_i^{1*})}} \right) \right] dF [E(\theta_i | y_i^1, p, d, e_i^{1*})] - \psi(e_i^{1*}) \right] \\ & = \underbrace{-\frac{\Delta}{2\pi} \sqrt{\frac{\sigma_\theta^2}{\sigma_p^2 + \sigma_d^2}} \frac{1}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \exp \left(-\frac{1}{2} \frac{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2}{\sigma_{\theta_i}^4} (\theta - E\theta_i)^2 \right)}_{\text{Expression 1} < 0.} \end{aligned}$$

$$+ \underbrace{\left[\left(\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} + \frac{2\Delta}{\sqrt{2\pi}} \frac{\sqrt{\sigma_\theta^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \exp \left[-\frac{1}{2} \frac{(\underline{\theta} - E\theta_i)^2}{\sigma_\theta^2} \right] \times \left(1 - \Phi \left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right) \right)}_{\text{Expression 2}} \right] \times \left(\frac{\sigma_\theta^2}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2} + \frac{\frac{2\Delta}{\sqrt{2\pi}} \frac{\sqrt{\sigma_\theta^2}}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \exp \left[-\frac{1}{2} \frac{(\underline{\theta} - E\theta_i)^2}{\sigma_\theta^2} \right]}{\left(\frac{1}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \left(1 - \Phi \left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right) + \frac{(\underline{\theta} - E\theta_i)}{\sigma_\theta^2} \times \frac{1}{2\sqrt{\sigma_p^2 + \sigma_d^2}} \times \varphi \left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right)} \right) \right]$$

Expression 1 is always negative. Expression 2 is always positive for $E\theta_i < \underline{\theta}$, while it could be negative for $E\theta_i \geq \underline{\theta}$, since $\frac{\partial e_i^{1*}}{\partial \sigma_p^2} < 0$ when $|\underline{\theta} - E\theta_i|$ takes intermediate values, for σ_θ^2 high enough and $\Delta \geq \bar{\Delta}(\underline{\theta} - E\theta_i)$ (see, *Proof of Proposition 1, Evolution of agent i's first-period effort with respect to σ_p^2 , Case 2: $E\theta_i \geq \underline{\theta}$*). Finally, after computations, we find that

Expression 1 + Expression 2 =

$$\Delta^2 \times \left[\frac{2}{\pi} \frac{\sigma_\theta^2}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2} \exp \left[-\frac{(\underline{\theta} - E\theta_i)^2}{\sigma_\theta^2} \right] \left(1 - \Phi \left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right) \right. \\ \left. \times \left(\frac{1}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \left(1 - \Phi \left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right) + \frac{(\underline{\theta} - E\theta_i)}{\sigma_\theta^2} \times \frac{1}{2\sqrt{\sigma_p^2 + \sigma_d^2}} \times \varphi \left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right) \right] \\ + \frac{\Delta}{\sqrt{2\pi}} \times \frac{1}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \exp \left[-\frac{1}{2} \frac{(\underline{\theta} - E\theta_i)^2}{\sigma_\theta^2} \right] \left[\frac{4\sigma_\theta^2 \sqrt{\sigma_\theta^2}}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2} \left(1 - \Phi \left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right) \right. \\ \left. + \frac{(\underline{\theta} - E\theta_i)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \times \sqrt{\frac{\sigma_\theta^2}{\sigma_p^2 + \sigma_d^2}} \times \varphi \left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) - \sqrt{\frac{\sigma_\theta^2}{\sigma_p^2 + \sigma_d^2}} \varphi \left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right] \\ + \frac{\sigma_{\theta_i}^4}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^3}.$$

Expression 1 + Expression 2 is thus a function of Δ , which we denote $H(\Delta)$. We have

$$H(\Delta = 0) > 0.$$

Let us first consider either that $|\underline{\theta} - E\theta_i|$ takes small values or that $|\underline{\theta} - E\theta_i|$ intermediate

values and $E\theta_i < \underline{\theta}$. The term which is proportional to Δ^2 is strictly positive. Thus, $H(\Delta)$ is a convex function and $H'(\Delta) = 0$ will give us the value of Δ for which $H(\Delta)$ is minimum. We show in Additional Technical Appendix 5 that $H'(\Delta) = 0 \Leftrightarrow \Delta = \Delta_{\min}$, with $\Delta_{\min} > 0$ for σ_θ^2 high enough. Moreover, we find that $H(\Delta_{\min})$ is strictly negative for σ_θ^2 high enough.

Therefore, $H(\Delta)$ convex, $H(\Delta = 0) > 0$ and $H(\Delta_{\min}) < 0$ with $\Delta_{\min} > 0$ imply that $H(\Delta)$, which is a polynomial degree 2 function, admits two positive roots. The determinant, denoted

D is

$$D = \left[\begin{array}{c} \frac{1}{2\pi} \frac{1}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2} \exp\left[-\frac{(\underline{\theta} - E\theta_i)^2}{\sigma_\theta^2}\right] \varphi\left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta}\right) \\ \left(\frac{(\underline{\theta} - E\theta_i)^2}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2} \times \frac{\sigma_\theta^2}{\sigma_p^2 + \sigma_d^2} \times \varphi\left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta}\right) \right) \\ - \frac{8}{\sqrt{\sigma_p^2 + \sigma_d^2}} \frac{1}{\left(1 + \frac{\sigma_p^2 + \sigma_d^2}{\sigma_\theta^2}\right)^2} \left(1 - \Phi\left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta}\right)\right) \\ - \frac{2(\underline{\theta} - E\theta_i)}{\sigma_p^2 + \sigma_d^2} \frac{1}{1 + \frac{\sigma_p^2 + \sigma_d^2}{\sigma_\theta^2}} \varphi\left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta}\right) \\ + \frac{\sigma_\theta^2}{\sigma_p^2 + \sigma_d^2} \varphi\left(\frac{(\underline{\theta} - E\theta_i)\sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta}\right) \end{array} \right] \quad (32)$$

D is strictly positive for σ_θ^2 high enough. We thus have two different roots:

$$\Delta_I(\underline{\theta} - E\theta_i) = \frac{\left[\begin{aligned} & -\frac{4\sigma_{\theta_i}^2\sqrt{\sigma_\theta^2}}{(\sigma_\theta^2+\sigma_p^2+\sigma_d^2)} \left(1 - \Phi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_\theta} \right) \right) \\ & - (\underline{\theta} - E\theta_i) \times \sqrt{\frac{\sigma_\theta^2}{\sigma_p^2+\sigma_d^2}} \times \varphi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_\theta} \right) \\ & + (\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2) \sqrt{\frac{\sigma_\theta^2}{\sigma_p^2+\sigma_d^2}} \varphi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_\theta} \right) \end{aligned} \right] \left(\begin{aligned} & (\underline{\theta} - E\theta_i)^2 \times \frac{\sigma_\theta^2}{\sigma_p^2+\sigma_d^2} \times \varphi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_\theta} \right) \\ & - \frac{8\sigma_{\theta_i}^4}{\sqrt{\sigma_p^2+\sigma_d^2}} \left(1 - \Phi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_\theta} \right) \right) \\ & - 2(\underline{\theta} - E\theta_i) (\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2) \frac{\sigma_\theta^2}{\sigma_p^2+\sigma_d^2} \varphi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_\theta} \right) \\ & + \frac{\sigma_\theta^2}{\sigma_p^2+\sigma_d^2} (\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2 \varphi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_\theta} \right) \end{aligned} \right)^{1/2}}{\left[\begin{aligned} & \frac{4}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} \frac{(\underline{\theta}-E\theta_i)^2}{\sigma_\theta^2} \right] \left(1 - \Phi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_\theta} \right) \right) \\ & \times \left(\begin{aligned} & \frac{2\sigma_{\theta_i}^2}{\sigma_\theta^2+\sigma_p^2+\sigma_d^2} \left(1 - \Phi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_\theta} \right) \right) \\ & + (\underline{\theta} - E\theta_i) \times \frac{1}{\sqrt{\sigma_p^2+\sigma_d^2}} \times \varphi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_\theta} \right) \end{aligned} \right) \end{aligned} \right]} \quad (33)$$

and

$$\Delta_{II}(\underline{\theta} - E\theta_i) = \frac{\left[\begin{aligned} & -\frac{4\sigma_{\theta_i}^2\sqrt{\sigma_{\theta}^2}}{(\sigma_{\theta}^2+\sigma_p^2+\sigma_d^2)} \left(1 - \Phi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_{\theta}^2} \right) \right) \\ & -(\underline{\theta} - E\theta_i) \times \sqrt{\frac{\sigma_{\theta}^2}{\sigma_p^2+\sigma_d^2}} \times \varphi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_{\theta}^2} \right) \\ & +(\sigma_{\theta}^2 + \sigma_p^2 + \sigma_d^2) \sqrt{\frac{\sigma_{\theta}^2}{\sigma_p^2+\sigma_d^2}} \varphi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_{\theta}^2} \right) \\ & +\varphi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_{\theta}^2} \right) \times \left(\begin{aligned} & (\underline{\theta} - E\theta_i)^2 \times \frac{\sigma_{\theta}^2}{\sigma_p^2+\sigma_d^2} \times \varphi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_{\theta}^2} \right) \\ & -\frac{8\sigma_{\theta_i}^4}{\sqrt{\sigma_p^2+\sigma_d^2}} \left(1 - \Phi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_{\theta}^2} \right) \right) \\ & -2(\underline{\theta} - E\theta_i) (\sigma_{\theta}^2 + \sigma_p^2 + \sigma_d^2) \frac{\sigma_{\theta}^2}{\sigma_p^2+\sigma_d^2} \varphi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_{\theta}^2} \right) \\ & +\frac{\sigma_{\theta}^2}{\sigma_p^2+\sigma_d^2} (\sigma_{\theta}^2 + \sigma_p^2 + \sigma_d^2)^2 \varphi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_{\theta}^2} \right) \end{aligned} \right)^{1/2} \right]}{\left[\begin{aligned} & \frac{4}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} \frac{(\underline{\theta}-E\theta_i)^2}{\sigma_{\theta}^2} \right] \left(1 - \Phi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_{\theta}^2} \right) \right) \\ & \times \left(\begin{aligned} & \frac{2\sigma_{\theta_i}^2}{\sigma_{\theta}^2+\sigma_p^2+\sigma_d^2} \left(1 - \Phi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_{\theta}^2} \right) \right) \\ & +(\underline{\theta} - E\theta_i) \times \frac{1}{\sqrt{\sigma_p^2+\sigma_d^2}} \times \varphi \left(\frac{(\underline{\theta}-E\theta_i)\sqrt{\sigma_p^2+\sigma_d^2}}{\sigma_{\theta}^2} \right) \end{aligned} \right) \right]} \quad (34)$$

with $\Delta_{II}(\underline{\theta} - E\theta_i) > \Delta_I(\underline{\theta} - E\theta_i)$ (Note $\Delta_I(\underline{\theta} - E\theta_i)$ and $\Delta_{II}(\underline{\theta} - E\theta_i)$ are both positive for σ_{θ}^2 high enough). Therefore, $H(\Delta)$ is strictly negative for $\Delta \in]\Delta_I(\underline{\theta} - E\theta_i), \Delta_{II}(\underline{\theta} - E\theta_i)[$. This implies that the agent chooses a more informative project when $\Delta \in]\Delta_I(\underline{\theta} - E\theta_i), \Delta_{II}(\underline{\theta} - E\theta_i)[$. However, for $\Delta \in [0, \Delta_I(\underline{\theta} - E\theta_i)[$ and $\Delta \geq \Delta_{II}(\underline{\theta} - E\theta_i)$, $H(\Delta)$ is strictly positive and the agent chooses a less informative project.

Consider now that $|\underline{\theta} - E\theta_i|$ takes intermediate values and $E\theta_i > \underline{\theta}$. Proposition 1 implies that the term which is proportional to Δ^2 is strictly negative for σ_{θ}^2 high enough. Thus, $H(\Delta)$ is a concave function. Moreover, the term which is proportional to Δ is also strictly negative for σ_{θ}^2 high enough. Therefore, $H(\Delta)$ becomes strictly decreasing for any $\Delta > 0$. This implies that the polynomial function $H(\Delta)$ admits one strictly negative root, and one strictly positive root, since

$H(\Delta = 0) > 0$. The positive root, which we denote $\overline{\overline{\Delta}}(\underline{\theta} - E\theta_i)$ corresponds to $\Delta_I(\underline{\theta} - E\theta_i)$ (computed for $(\underline{\theta} - E\theta_i) < 0$ and σ_θ^2 high enough such that the denominator in $\Delta_I(\underline{\theta} - E\theta_i)$ is strictly negative). This implies that $H(\Delta)$ is strictly positive for $\Delta \in]0, \overline{\overline{\Delta}}(\underline{\theta} - E\theta_i)[$ while $H(\Delta)$ is strictly negative for $\Delta > \overline{\overline{\Delta}}(\underline{\theta} - E\theta_i)$. The agent thus chooses a less informative project if $\Delta \in]0, \overline{\overline{\Delta}}(\underline{\theta} - E\theta_i)[$ while he chooses a more informative project if $\Delta \geq \overline{\overline{\Delta}}(\underline{\theta} - E\theta_i)$.

Finally, consider that $|\underline{\theta} - E\theta_i|$ takes high values. Then, for $E\theta_i < \underline{\theta}$ or for $E\theta_i > \underline{\theta}$,

$$\text{Expression 1} + \text{Expression 2} = \frac{\sigma_{\theta_i}^4}{(\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^3} > 0.$$

Thus, the agent chooses a less informative project when $|\underline{\theta} - E\theta_i|$ takes high values.

7.4 Proof of Proposition 4

We want to show that $\Delta_T(\underline{\theta} - E\theta_i) < \Delta_I(\underline{\theta} - E\theta_i)$ when $|\underline{\theta} - E\theta_i|$ takes either small or intermediate values and σ_θ^2 is high enough. After computations (See Additional Technical Appendix 6), we find that

$$\Delta_T(\underline{\theta} - E\theta_i) \leq \Delta_I(\underline{\theta} - E\theta_i) \Leftrightarrow$$

$$\begin{aligned}
& \Leftrightarrow \frac{4}{1 + \frac{\sigma_p^2 + \sigma_d^2}{\sigma_\theta^2}} \left(\frac{(\sigma_p^2 + \sigma_d^2)^2}{1 + \frac{\sigma_p^2 + \sigma_d^2}{\sigma_\theta^2}} \frac{1}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} + 1 \right) \left(1 - \Phi \left(\frac{(\underline{\theta} - E\theta_i) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right) \\
& < \frac{1}{\sqrt{\sigma_p^2 + \sigma_d^2}} \varphi \left(\frac{(\underline{\theta} - E\theta_i) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \left[\begin{array}{c} (\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2) - (\underline{\theta} - E\theta_i) \\ \left((\underline{\theta} - E\theta_i)^2 \times \varphi \left(\frac{(\underline{\theta} - E\theta_i) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right. \\ \left. - 8\sigma_{\theta_i}^2 \sqrt{\sigma_p^2 + \sigma_d^2} \left(1 - \Phi \left(\frac{(\underline{\theta} - E\theta_i) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right) \right. \\ \left. - 2(\underline{\theta} - E\theta_i) (\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2) \varphi \left(\frac{(\underline{\theta} - E\theta_i) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right. \\ \left. + (\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2)^2 \varphi \left(\frac{(\underline{\theta} - E\theta_i) \sqrt{\sigma_p^2 + \sigma_d^2}}{\sigma_\theta^2} \right) \right. \\ \left. - \frac{(\sigma_p^2 + \sigma_d^2)^2}{1 + \frac{\sigma_p^2 + \sigma_d^2}{\sigma_\theta^2}} \frac{2(\underline{\theta} - E\theta_i)}{\sigma_\theta^2 + \sigma_p^2 + \sigma_d^2} \right]^{1/2}
\end{array}
\right.
\end{aligned}$$

The previous inequality is satisfied for σ_θ^2 high enough, since for $(\underline{\theta} - E\theta_i)$ that takes small or intermediate values we have: $\lim_{\sigma_\theta^2 \rightarrow +\infty} LHS = 2 < \lim_{\sigma_\theta^2 \rightarrow +\infty} RHS = +\infty$.

Consider first that $|\underline{\theta} - E\theta_i|$ takes either small values or intermediate values and $E\theta_i < \underline{\theta}$. Proposition 3 indicates that the agent chooses the more informative project when $\Delta \in]\Delta_I(\underline{\theta} - E\theta_i), \Delta_{II}(\underline{\theta} - E\theta_i)[$. Moreover, Proposition 1 indicates that the agent exerts a higher effort when he chooses the more informative project. Therefore, Proposition 3 and Proposition 1 imply that for $\Delta \in]\Delta_I(\underline{\theta} - E\theta_i), \Delta_{II}(\underline{\theta} - E\theta_i)[$ the agent chooses the more informative project, which increases effort inefficiencies since $\Delta_T(\underline{\theta} - E\theta_i) < \Delta_I(\underline{\theta} - E\theta_i)$ for σ_θ^2 high enough. However, for $\Delta \geq \Delta_{II}(\underline{\theta} - E\theta_i)$ Proposition 3 indicates that the agent chooses the less informative project and Proposition 1 implies that this choice decreases effort inefficiencies, since $\Delta_T(\underline{\theta} - E\theta_i) < \Delta_I(\underline{\theta} - E\theta_i)$ implies that $\Delta_T(\underline{\theta} - E\theta_i) < \Delta_{II}(\underline{\theta} - E\theta_i)$.

Consider now that $|\underline{\theta} - E\theta_i|$ takes intermediate values and $E\theta_i \geq \underline{\theta}$. Proposition 3 indicates that the agent chooses the more informative project when $\Delta \geq \overline{\overline{\Delta}}(\underline{\theta} - E\theta_i)$. Moreover, Proposition 1 indicates that the agent exerts a lower effort when he chooses the more informative

project. Therefore, for $\Delta \geq \overline{\overline{\Delta}}(\underline{\theta} - E\theta_i)$ the agent's choice of project reduces effort inefficiencies since $\Delta_T(\underline{\theta} - E\theta_i) < \overline{\overline{\Delta}}(\underline{\theta} - E\theta_i) = \Delta_I(\underline{\theta} - E\theta_i)$.