

Creative Destruction vs Destructive Destruction ? : A Schumpeterian Approach for Adaptation and Mitigation

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Abstract

This article aims to show how a market exposed to catastrophic events finds the equilibrium level of adaptation and mitigation policies through R&D policy, with respect to different levels of Poisson probability of catastrophe. We study the effect of pollution tax on long-run growth rate and the implications of catastrophe probability on this effect. Our results suggest that economy increases its R&D level with a higher catastrophe probability only if penalty rate due to an abrupt event is sufficiently high. We also show that pollution tax could increase the long-run growth. Besides, the catastrophe probability increases the amplitude of this positive effect if penalty rate is high enough. The market makes adaptation much more than mitigation with a higher catastrophe probability if total productivity of R&D is higher than cleanliness of innovations for intermediate goods. Lastly, we show that pollution growth could be higher with less polluting inputs, which we call a Jevons type paradox.

Keywords : Abrupt damage, Occurrence Hazard, Endogenous Technological Change, Adaptation, Mitigation.

JEL Classification : D81, O3, Q54, Q55

1 Introduction

In this paper, we take a step further to answer the following questions : How catastrophic event probability affects the creative destruction process in economy ? What is the effect of pollution tax on growth rate and the implications of catastrophe probability concerning this effect ? How market adjusts the equilibrium level of adaptation and mitigation when it faces a higher catastrophe probability ?

Many recent reports (see European Commission- Road map for Climate Services 2015) started to highlight how important is to create a market economy thorough R&D innovations that handles adaptation and mitigation services to create a low carbon and climate-resilient economy.

Indeed, it is interesting to see the notion of “service” for these two central environmental policies in these reports. Normally, adaptation and mitigation policies are studied in existing literature until now in a social optimum and not in a market economy framework. (See Zemel (2015), Tsur and Zemel 2015, Bréchet et al. 2012)

Climate services aim at providing the climate knowledge to society through informational tools.¹ These services involve very detailed analysis of existing environmental knowledge and R&D activity that inform society about climate impacts. Besides, these services give necessary information to take action against extreme events through vulnerability analysis. To summarize, one can say that the purpose of climate services is to bridge innovation with entrepreneurship that could create new business and market growth. Regarding this recent evolution about adaptation and mitigation, a decentralized market analysis is more than necessary to be able to analyze rigorously the implications of these new policies.

The world faces undesirable extreme events and faces to damages. Our aim in this paper is to see how market could adapt to climate change and decreases its unit emissions (mitigation) by doing R&D in an economy exposed to catastrophic events. To our knowledge, there is no study treating the adaptation and mitigation activity in a decentralized framework with taking into account uncertainty about catastrophic events. Our contribution relies on building a decentralized growth model that analyzes adaptation and mitigation policies. More importantly, existing studies examines these policies on exogenous growth models and endogenous technological progress is a missing component. (See Zemel (2015), Tsur and Zemel 2015, Bréchet et al 2012, Tsur and Withagen 2012, Zemel and Art de Zeeuw 2012), our study is the first one that studies these two policies through an endogenous R&D process.

Firstly, our article builds on the literature on adaptation and mitigation (Bréchet et al. 2012) and also includes the effects of environmental catastrophes on long-term growth. (see Tsur and Zemel (1996), (1998)) Secondly, our model belongs to Schumpeterian growth literature, which started with seminal paper of Aghion and Howitt (1992).

To inform reader about adaptation and mitigation policy mix analysis, Bréchet et al. (2012) , Ayong Le Kama and Pommeret (2014), Kane and Shogren (2014) and Buob and Stephan (2010) are first analytical studies that treat optimal design of these two central environmental policies. However, in these studies focusing on trade-offs between adaptation and mitigation, the uncertainty about abrupt climate events is neglected. At this point, Zemel (2015) and Tsur and Zemel (2016) introduce Poisson uncertainty in Bréchet et al. (2012) framework and shows that a higher catastrophic event probability induces more adaptation capital at long-run.

Now, we return to Schumpeterian growth literature. Very first study that combines environment

¹An example can be a smartphone application that informs farmers about weather and how to proceed in extreme weather events.

and Schumpeterian growth models is Aghion and Howitt (1998). Authors introduce pollution in a Schumpeterian growth and analyze the optimal path. Grimaud (1999) extends this model to a decentralized economy in which he implements the optimum by R&D subsidies and pollution permits.

One of the first attempts to model environmental aspects in a Schumpeterian growth model is Hart (2004). He studies the effects of a pollution tax and finds that environmental policy can be a win-win policy. In the same line, Ricci (2007) shows in a Schumpeterian growth model that long-run growth of the economy is driven by knowledge accumulation. In his model, environmental regulation shifts the production to cleaner vintages. The important difference between these two papers is that Ricci (2007) treats a continuum of different vintages. However, Hart (2004) is offering a model in which there exists only two young vintages on sale. Due to this modeling difference, Ricci (2007) shows that tightening environmental policy does not foster economic growth since the marginal contribution of R&D to economic growth falls. However, uncertainty about abrupt climate events is totally overlooked in these models.

In this article, we build a Schumpeterian growth model in which economy faces abrupt climate events. R&D aims at increasing the total productivity and decreasing pollution intensity of intermediate goods (i.e mitigation) as in Ricci (2007). The benefit of R&D is twofold ; firstly, with the assumption that wealthier countries resist more easily to catastrophic events (see Mendelson, Dinar and Williams (2006)), we show that making R&D increases the wealth of the economy and make it more resilient to catastrophic events. Secondly, R&D increases the total productivity and allows a higher growth rate at balanced growth path.

As in Tsur and Zemel (2015), in the following model, adaptation activity plays only a proactive role. Society can benefit from adaptation knowledge (which stems from R&D) just in case of a catastrophic event but R&D plays an important role also before a catastrophic event, as mentioned above.

It would be necessary to point out some of preliminary results that we will present in the remainder of the paper. We show that there are two opposite effects of catastrophe probability on creative destruction rate. A first channel is straightforward, catastrophic event probability will decrease the value of any R&D and expected value of firms in R&D sectors. This one can be called *discount effect*.

The second channel is more interesting ; when catastrophic event probability increases, the post-value function's role in social planner's utility (or household's) increases. Then, there exists a higher incentive to accumulate adaptation and mitigation knowledge because adaptation and mitigation knowledge decreases penalty rate from catastrophic event. As a result, the stock of knowledge increases post-value function. We show that marginal increase of post value function relative to adaptation and mitigation knowledge is higher if penalty amount is higher. In other words, the opportunity cost of not investing in R&D increases when catastrophic event probability increases. This translates in a decrease of interest rate in market. This channel can be also well understood in stock markets. Investors in market could engage in assets of firms in environmental

innovations sector with a higher catastrophe risk, as they know that environmental innovations responds to climate catastrophes. As a nutshell, higher penalty amount creates a higher incentive to devote resources to R&D, thus promoting creative destruction process. We show that after some threshold of penalty amount, an increase in catastrophe probability can boost creative destruction.

Before coming to pollution tax analysis, it is worthwhile to note that firms mitigates since they face a pollution tax. Hence, they are making R&D to decrease pollution intensity in order to lower the tax burden. Our model shows a positive effect of pollution tax effect on growth as in Ricci (2007). Higher catastrophe probability can increase the positive effect of tax on growth rate of the economy at long run, if penalty rate is sufficiently high. This effect is due to higher marginal benefit of R&D since it helps an economy to better respond to catastrophic events.

Our results indicate that market's allocation between adaptation and mitigation depends on the ratio between pollution intensity and total productivity rate. Besides, market starts to adapt much more rather than mitigate with a higher catastrophic event probability, only if the total productivity parameter is higher than the pollution intensity rate. Indeed, the usual trade-off between adaptation and mitigation (see Bréchet et al. 2012) is present in our model. Since polluting good is an input for production, a lower pollution intensity means a decrease in production. In this case, economy accumulates less wealth at long run with cleaner intermediate goods, which means that economy adapts less to climatic extreme events.

Lastly, despite the decrease of pollution intensity of intermediate goods, we show that pollution growth can be higher if cleanliness of R&D is not that much high. This is so-called Jevons Paradox which states that technological improvements increases energy efficiency but results in a higher pollution in long term. This result is also supported by an empirical study for India. (See Ollivier and Barrows (2016))

2 Model

A decentralized economy is characterized by a final sector that uses a continuum of intermediate goods that are provided by the monopolistic intermediate good sector. Each monopolistic intermediate good producer buys patents from R&D sector, to be able to supply the intermediate good and faces a pollution tax for using polluting intermediate good. Consumption is determined by infinitely lived household, facing a catastrophic event that reduces to consumption level to subsistence level and implies a penalty rate. The negative consequences of catastrophic event can be alleviated by the wealth accumulation, which increases the R&D.

2.1 Household

We write the maximization program of household in a very similar way to Tsur and Zemel (2008) (and we can cite many other papers.) The utility function of household is

$$\max E_T \left\{ \int_0^T u(c(t)) e^{-\rho t} dt + e^{-\rho T} \Gamma(a(T)) \right\} \quad (2.1)$$

where ρ is the pure time preference of household. $\Gamma(a(t))$ is the value function after catastrophic event. After integrating by parts the equation (1), we can write the following maximization program. The household's objective function reduces to

$$\max \int_0^\infty u(c(t) + \bar{\theta} \Gamma(a(t))) e^{-(\rho + \bar{\theta})t} dt \quad (2.2)$$

where $\bar{\theta}$ is the constant hazard rate of catastrophe. The reason behind the use of a constant hazard rate relies on the fact that the paper focuses on balanced growth path analysis. Note that we don't specify a stock variable for pollution. However, even when one specifies an endogenous hazard function depending on pollution stock, it is easy to see that at balanced growth path (BGP), hazard would converge to a constant value.²

A recent IPCC Report (2014) claims that the frequency of tropical cyclones would remain the same unchanged.³ Moreover, not every climate scientist agree on a variable climate induced changes in catastrophic event frequency. (See IPCC Report 2014).

The budget constraint of household

$$\dot{a}(t) = r(t)a(t) + w(t) - c(t) + T(t) + \chi x(t) \quad (2.3)$$

where $w(t)$, $T(t)$ and $\chi x(t)$ stand for wage, tax collected from the use of polluting intermediate good $x(t)$ and constant cost of producing one unit of $x(t)$. Household takes the pollution level as given and does not take it into account in its maximization program. We assume a log utility function for household's utility as $u(c(t)) = \log(c(t))$ for analytical tractability of the model. The familiar Euler equation is written as

$$\frac{\dot{c}(t)}{c(t)} = \left(r(t) - (\rho + \bar{\theta}) + \frac{\bar{\theta} \Gamma_a(a(t))}{\lambda_1(t)} \right) \quad (2.4)$$

where $\lambda_1(t)$ is the marginal utility of consumption per capita. (See Appendix [Household's Maximization Program](#)).

2.1.1 What happens after catastrophic event ?

We firstly define the following penalty function and assumptions similar to Bréchet et al. (2012)

²In order to illustrate this one, we take a hazard function used in Tsur and Zemel (2007), $\theta(S) = \bar{\theta}(1 - e^{-bS})$ where S is supposed to be the stock of pollution. It is easy to remark that $\lim_{S \rightarrow \infty} \theta(S) = \bar{\theta}$. Using an endogenous hazard rate matters only for transitional path but gives the exact same outcome at long run. In this paper, we don't focus on transitional dynamics but make a balanced growth path analysis. Then, the scoop of the paper justifies the use of a constant hazard rate.

³In the future, it is likely that the frequency of tropical cyclones globally will either decrease or remain unchanged, but there will be a likely increase in global mean tropical cyclone precipitation rates and maximum wind speed." (IPCC 2014, p.8)

Assumption 1. $\psi(a(t)) > 0$, $\psi_a(a(t)) < 0$, $\psi_{aa}(a(t)) > 0$

$$\psi(a(t)) = \bar{\psi}(\omega - (1 - \omega) \log(a(t))) \quad (2.5)$$

where $\bar{\psi}$ is the amount of penalty. Wealth accumulation $a(t)$ helps an economy to better respond to catastrophic events. The first term in parenthesis is the part of the damage that can not be recovered by the available adaptation technology. The second expression stands for the part of the damage that can be reduced by the accumulation of wealth, which is accumulated through innovation patents.

Assumption 2. $\omega > \frac{(\rho + \bar{\theta}) \ln(a(0))}{(\rho + \bar{\theta})(1 + \ln(a(0))) - g_Y}$

We assume that whatever the level of the accumulated wealth is, there exists a constant damage that can not be recovered. For this one, in order to ensure that second term does not exceed the first one on equation (2.5), we make the assumption on the parameter ω , which is the part of the unrecoverable damage. (See Appendix [Condition on Penalty Function](#) for details about Assumption 2.)

ω is the part of catastrophic event that is unrecoverable. The post value function can be given as

$$\Gamma(a(t)) = u(c_{min}) - \psi(a(t)) \quad (2.6)$$

where $u(c_{min}) = 0$ is the utility function where the consumption is reduced to subsistence level. Note that the subsistence level consumption does not provide any utility. (See Tsur and Zemel (2015)). In this post-catastrophe value function, we defend the idea that accumulating wealth helps an economy to better respond to negative consequences occurred due to catastrophic event. The empirical evidence suggests as well the higher capability of wealthier countries to adapt to climate change. (See Mendhelson, Dinar and Williams (2006)).

Lemma 1. $a(t) = V(t)$. *Environmental innovation patents ($V(t)$ is the value of an innovation.) are held by households.*

Proof. See Appendix

The lemma shows that household owns the firms in market. Household receives dividend from environmental innovation assets on the market. On the other hand, wealth accumulation helps an economy to decrease its vulnerability against any kind of catastrophic event, as specified above.

With the functional forms defined above and using the resource constraint $Y(t) = c(t) + x(t)$, where $x(t)$ represents spending for intermediate good production. The growth rate of economy can be written

$$g_c = \frac{\dot{c}(t)}{c(t)} = r(t) - (\rho + \bar{\theta}) + \frac{\lambda \bar{\theta} \bar{\psi} (1 - \omega)}{(1 - \alpha)} \frac{\alpha^2 \Omega_2(H)}{\Omega_1(H)} L_Y \quad (2.7)$$

where $\Omega_1(H)$ and $\Omega_2(H)$ are aggregation factors for production and intermediate good. (See Appendix [Aggregate Economy](#)) The solution of the dynamic optimization program should satisfy the no-Ponzi game condition $\lim_{t \rightarrow \infty} e^{-\int_0^t r(s) ds} a(s) = 0$.

2.2 Production of Final Good

The production function of the economy is similar to Stokey (1998) and Ricci (2007);

$$Y(t) = L_Y(t)^{1-\alpha} \int_0^1 \phi(v,t) z(v,t) x(v,t)^\alpha dv \quad (2.8)$$

where $\phi(v,t)$ is the technology level, that can be also interpreted as environmental innovations which increases the productivity of the labor. (see Appendix [Production Function](#)) This one can be loosely interpreted as an adaptation variable. The important feature of the production function in this model is that emission intensity $z(v,t)$ is heterogeneous across firms. There is an infinite number of different technologies used in production, that is indexed by $v \in [0; 1]$. Note that environmental taxation can promote growth only if goods are differentiated in pollution intensity. This justifies the use of the production function. (See Ricci (2007)) We have the following labor market clearing condition ;

$$L(t) = L_Y(t) + L_R(t) = 1 \quad (2.9)$$

where $L_Y(t)$ is labor used in production of final good, $L_R(t)$ holds for labor in research. In the present model, mitigation activity aims to reduce unit emissions of production. This one serves as a cost reduction for a final producing firm because the government puts a tax on the use of polluting intermediate goods. When intermediate goods are more polluting, the cost of producing increases for final good producer. (see Da Costa (2006))

2.3 Final Good Producer's Program

We normalize the price of final good to unity. Moreover, government imposes an environmental tax on pollution $h(t)$ for the use of intermediate good.

$$\max_{x(v,t), L_Y(t)} \psi(t) = Y(t) - \int_0^1 p(v,t) x(v,t) dv - w(t) L_Y(t) \quad (2.10)$$

where $p(v,t)$ and $w(t)$ are price of intermediate good and wage respectively. The final good sector is in perfect competition. From this program, we can find demand for intermediate good and labor in final production sector ;

$$p(v,t) = \alpha \phi(v,t) z(v,t) \left(\frac{L_Y(t)}{x(v,t)} \right)^{1-\alpha} \quad (2.11)$$

$$w(t) = (1 - \alpha) \int_0^1 (\phi(v, t)) \left(\frac{x(v, t)}{L_Y(t)} \right)^\alpha dv = (1 - \alpha) \frac{Y(t)}{L_Y(t)} \quad (2.12)$$

In fact, final good firms are maximizing their instantaneous profit. Then, they take the technology level as given and take their decisions statically for machine and labor demand. In this model, the cost of realizing research comes at expense of not producing a final good. For example, if there is no R&D in economy, there would be more labor allocation in production as the whole active population would work in final good sector.

In this model, household owns all firms and we can say that final good firms owns intermediate good sectors and R&D sectors. For example, a firm produces a final good but he has a demand for intermediate goods and technology, which is not included in his economic activity. So, final good producer only decides for labor in final production L_Y and he demands some amount of machines (intermediate goods) in order to maintain its activity and the firm takes the available technology level as given in its instantaneous profit maximization program.

2.4 Pollution Flow

We define the pollution as a flow variable similar to Stokey (1998) ;

$$P(t) = \int_0^1 P(v, t) = \int_0^1 (z(v, t))^\eta \phi(v, t) x(v, t) \quad (2.13)$$

where $\eta > 0$ is the pollution share in production (See Appendix [Production Function](#)) and $z(v, t)$ stands for the emission/intermediate good ratio. Recall that index v is the index for different sectors. We state that there is a continuum $[0, 1]$ of sectors in the economy. We can make two important remarks. First, pollution is an outcome of the production process, which uses intermediate goods. Second, intermediate good's productivity depends not only on total productivity improvements $\phi(v, t)$ but also on pollution intensity $z(v, t)$. In this framework, a polluting intermediate good is more productive. Then, a decrease in pollution intensity comes at the cost of less productive intermediate goods. Therefore, mitigation activity represents a cost in terms of foregone production.

2.5 Intermediate Good Producer's Program

The intermediate good producer is a monopolist. This one facing a factor demand (2.11), offers an intermediate good to final good sector, by taking the final good as given. The maximization program of intermediate good producer is ;

$$\max_{x(v, t)} \pi(t) = p(v, t) x(v, t) - \chi x(v, t) - h(t) P(v, t) \quad (2.14)$$

where $P(v, t) = (z(v, t))^\eta \phi(v, t) x(v, t)$. As in Verdier (1995), we say that there exists always a Pigouvian tax $h(t)$ on pollution. We make the assumption that marginal cost of using a polluting intermediate good in production process is constant as in Howart et Norgaard (1992) and Michel

(1993). In absence of this pollution tax, market economy will not have any incentives to do R&D for pollution intensity (i.e mitigation). As pollution enters in program of producer as a cost, this one can give the incentive to make more R&D⁴ to reduce this cost.

From this program, we can find the supply of machines and profits of the intermediate good producer :

$$x(v, t) = \left(\frac{\alpha^2 \phi(v, t) z(v, t)}{\chi + h(t) \phi(v, t) z(v, t)^\eta} \right)^{\frac{1}{1-\alpha}} L_Y(t) \quad (2.15)$$

By replacing the supply function of intermediate good producer (2.15) in price function (2.11) found in final good producer's program, we can express the profit and price of intermediate good in a simpler way ;

$$p(v, t) = \frac{\chi + h(t) \phi(v, t) z(v, t)^\eta}{\alpha} \quad (2.16)$$

$$\pi(t) = (1 - \alpha) p(v, t) x(v, t) \quad (2.17)$$

2.6 Aggregate Economy

The aggregate production function can be written

$$Y(t) = \frac{\gamma_1}{1 - \gamma_1} \frac{\alpha^{\frac{2\alpha}{1-\alpha}}}{\chi^{\frac{\alpha}{1-\alpha}}} L_Y \left(\bar{\phi}_{max}(t) \bar{z}_{min}(t) \right)^{\frac{1}{1-\alpha}} \Omega(H) \quad (2.18)$$

where $\Omega(H)$ is the aggregation function which is explained in details in Appendix [Aggregate Economy](#)

$$\Omega(H) = \int_0^1 \frac{a^{\bar{\gamma}}}{\left(1 + \frac{H}{\chi} a^{1 + \frac{\gamma_2}{\gamma_1}} \right)^{\frac{\alpha}{1-\alpha}}} da \quad (2.19)$$

where $H = h(t) \bar{\phi}_{max}(t) \bar{z}_{min}^\eta(t)$, which is the marginal cost of a leading-edge firm. The aggregation factor gives a measure about the relative productivity⁵ between heterogeneous firms in the economy. Aggregation factor depends on marginal cost of pollution because when a government increases its tax, a shift from polluting intermediate goods towards cleaner ones occurs. This term is shown to be a decreasing function because cleaner intermediate goods are less productive relative to polluting intermediate goods. It means that a tightened environmental policy targeted to mitigate the source of pollution has a cost in terms of output.

In order to have a balanced growth path and to ensure that aggregation term is constant over time, we define a policy rule as in Nakada (2004) and Ricci (2007). The policy rule aims to balance

⁴We will discuss the effect of a pollution tax on R&D in details in further sections.

⁵In terms of productivity improvements and pollution intensity.

its tax revenue from pollution tax. This means that tax revenues $T(t) = h(t)P(t)$ are constant over time. We define this policy rule as follows :

$$g_h = -(\eta g_z + g_\phi) \quad (2.20)$$

According to this rule, the government commits to increase its tax rate when innovations aiming at pollution intensity decrease is higher. This rule is a credible when one thinks that tax revenues decreases when pollution intensity decreases (due to decrease of collected tax). Recall that government wants to balance its budget for all date t . Contrarily, government decreases the pollution tax when technological improvements which result of higher pollution level.

Indeed, this rule is also crucial to ensure the balanced growth path in economy. If the pollution tax h is constant, it means that at each date, pollution tax burden relatively to marginal cost of R&D decreases because pollution intensity z decreases. This is not compatible with balanced growth concept.

According to all these elements, it is easy to write down the growth rate of the economy at BGP

$$g_Y = \frac{1}{1 - \alpha} (g_\phi + g_z) \quad (2.21)$$

2.7 R&D Sector

In R&D sector, each firm aims at improving the productivity and pollution of one specific intermediate good. R&D innovations are modeled respecting a Poisson process with instantaneous arrival rate λL_R , which we can denote as the creative destruction rate. Similar to Ricci (2007), to keep things simpler, we adopt only one type of R&D firm, which specializes in both productivity and pollution intensity improvements ϕ and z . However, the reader could consider this feature of modeling R&D unusual. A two-sector R&D model would require that expected profits should be the same. Then, it would be possible that two sectors can make R&D.⁶

Da Costa (2006) proposes a model with two R&D sectors and finds a balance between the allocation of labor between two R&D sectors, which ensures the same expected value of R&D in both sectors. Recall that in his model, when the allocation of labor increases in one R&D sector, the other one sees its labor allocation increasing as well. This modeling is surely more realistic but involves same economic implications. We can write the dynamics for technology and pollution intensity improvements ;

$$\frac{\dot{\phi}_{max}(t)}{\phi_{max}(t)} = \gamma_1 \lambda (1 - L_Y), \quad \gamma_1 > 0 \quad (2.22)$$

$$\frac{\dot{z}_{min}(t)}{z_{min}(t)} = \gamma_2 \lambda (1 - L_Y), \quad \gamma_2 < 0 \quad (2.23)$$

⁶In case of asymmetric profits, there will be corner solutions where only one type of R&D will take place.

where $\bar{\phi}(v, t)$ stands for the best available technological level. This level can be loosely interpreted as an environmental productivity. If economy invests in environmental innovations, its productivity increases. (See Zivin and Neidell (2012).). $\underline{z}(v, t)$ represents the lowest available level of pollution intensity. Making R&D decreases the pollution intensity as well.

The parameter γ_2 shows the direction of the R&D activity. A negative value means that innovations are environmental friendly. When $\gamma_2 = 0$, all goods have the same pollution intensity as in Nakada (2004). In this case, there is no discrimination between different intermediate goods and we can not mention of any skewing effect of production from dirty to clean intermediate goods. Therefore, the pollution tax can not have any effect on long run economic growth.

The free-entry condition ensures that arbitrage condition holds ;

$$w(t) = \lambda V(t) \quad (2.24)$$

where $V(t)$ is the present value of expected profit streams of future. The equation (2.24) states that an agent is indifferent between working in production sector and doing research. This ensures the equilibrium in the model at BGP. If R&D takes place, its marginal cost $w(t)$ is equal to its expected marginal value.

$$V(t) = \int_{\tau}^{\infty} e^{-\int_{\tau}^t (r(s) + \lambda L_R(s)) ds} \pi(\bar{\phi}(t), \underline{z}(t)) dt \quad (2.25)$$

where $\pi(\bar{\phi}(v, t), \underline{z}(v, t))$ denotes the profit at time t . r is the interest rate which is the opportunity cost of savings, which involves catastrophic event probability and λL_R is the *creative destruction rate* of the economy which are constant at balanced growth path. The creative destruction rate shows at which extent the incumbent firm is replaced by an entrant. Basically, it is the survival rate of the incumbent firm as an entrant makes the patent of incumbent firm obsolete.

Differentiating equation (2.25) yields the Hamilton-Jacobi-Bellman equation at BGP

$$(r + \lambda L_R) - \frac{\dot{V}(t)}{V(t)} = \frac{\pi(\bar{\phi}_{max}, \underline{z}_{min})}{V(t)} \quad (2.26)$$

where by free-entry condition, $g_V = g_w = g_Y$. Using equations (2.7), (2.21) and (2.17), we reformulate the expected value of an innovation

$$\underbrace{\frac{1}{1-\alpha}(g_{\phi} + g_Z) + (\rho + \bar{\theta}) - \frac{\bar{\theta}\bar{\psi}(1-\omega)\alpha^2\Omega_2(H)}{\lambda(1-\alpha)\Omega_1(H)}(1-L_R) + \lambda L_R}_{=r+\lambda L_R} - \underbrace{\frac{\lambda L_R}{1-\alpha}(\gamma_1 + \gamma_2)}_{=\frac{\dot{V}(t)}{V(t)}} = \underbrace{\frac{\alpha\gamma_1}{\lambda(1-\gamma_1)} \frac{(\chi + H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)}}_{=\frac{\pi(\bar{\phi}_{max}, \underline{z}_{min})}{V(t)}} (1-L_R) \quad (2.27)$$

From equation (2.27), labor allocation in R&D can be designated

$$L_R = \frac{\frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha} \frac{\Omega_2(H)}{\Omega_1(H)} + \frac{\alpha\lambda\gamma_1}{(1-\gamma_1)} \frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)} - (\rho + \bar{\theta})}{\lambda + \frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha} \frac{\Omega_2(H)}{\Omega_1(H)} + \frac{\alpha\lambda\gamma_1}{(1-\gamma_1)} \frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)}} \quad (2.28)$$

One can easily remark that the level of labor allocated in R&D sector depends on both catastrophic event probability, penalty rate and marginal cost of using a polluting intermediate good.

Proposition 1. *The market allocates much more labor to R&D with a higher catastrophe probability if the amount of penalty to due to catastrophic event is sufficiently high.*

Proof. See Appendix

The first proposition can be better understood when one looks closely the Hamilton-Jacobi-Bellman equation (2.27). In fact, there exists two different opposite effects of catastrophic event on interest rate. The first one can be nominated as a discount effect. One should pay much more to investors as they face a higher catastrophic event.

On the other hand, investing in R&D becomes more valuable when an economy faces a higher catastrophic event because R&D activities helps an economy to respond to catastrophic events. Lemma 1 shows clearly that an economy gets wealthier when there is more R&D. In this manner, higher catastrophic event probability increases the value of R&D in some sense. Then, the interest rate decreases by this second channel.

When the penalty rate $\bar{\psi}$ is sufficiently high, the second channel dominates the discounting effect channel. This result is plausible because higher penalty amount implies increases the marginal benefit of environmental assets $a(t)$.

We illustrate the Proposition 1. graphically

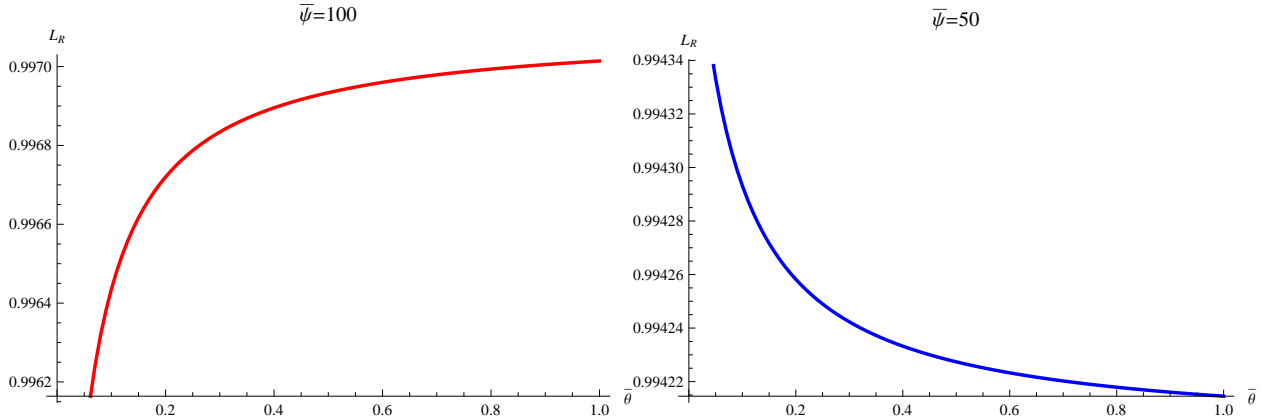


Figure 1. The Effect of Hazard rate on labor allocation in R&D

We can also observe by a simple numerical exercise that labor allocation in R&D increases due to a higher penalty rate. This effect is understandable when one thinks to marginal benefit of R&D, which increases with higher amount of penalty.

Proposition 2 . (i) The effect of pollution tax is positive on growth if the elasticity of aggregation factor is high enough. (ii) This effect increases positively with catastrophic event probability if the amount of penalty is sufficiently high.

Proof. See Appendix [Impact of environmental taxation on labor allocation in R&D Sector](#).

The pollution tax decreases the profits of intermediate good producer, as he produces goods that are polluting. A shift from polluting intermediate goods to cleaner ones occurs. This translates in a decrease in production as clean intermediate goods are less productive. At the same time, a higher marginal cost of pollution H implies a decrease in wage, which case decreases the cost of labor. Hence, market allocates much labor to R&D. When the second effect dominates the first one. One may say that environmental policy encourages R&D activities. Otherwise, pollution tax impedes economic growth.

This explanation can be also understood by looking at distributed dividends to shareholders $\frac{\pi}{V}$ in equation (2.27). When marginal cost of pollution H increases, the dividends decreases directly but on the other hand, aggregation factor $\Omega(H)$ decreases, which makes increase $\frac{\pi}{V}$.

In order to asses this effect more clearly, one may look at how labor allocation reacts to a change in marginal cost of pollution H . As R&D is known to promote growth in economy. Allocating much more labor to R&D will generate more growth.

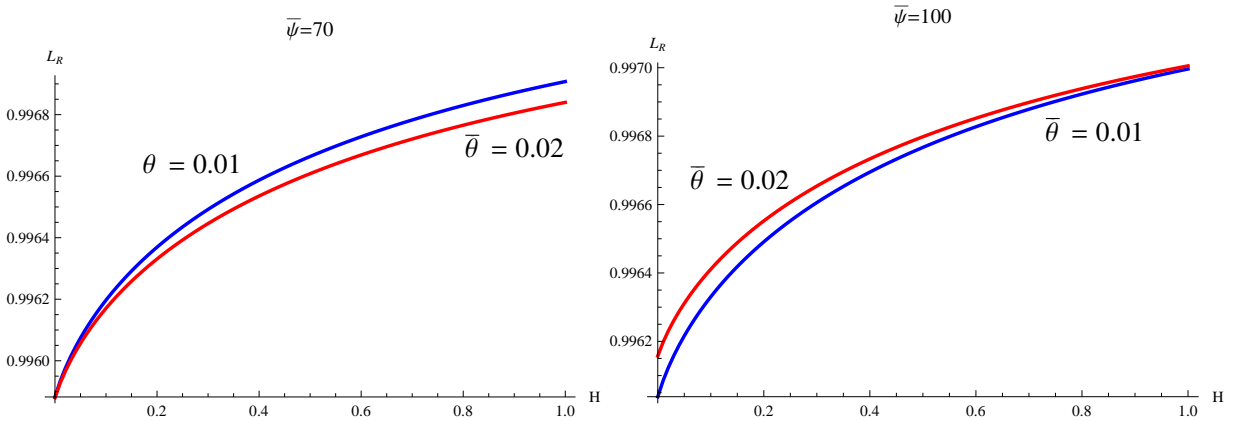


Figure 2. The Effect of Tax on labor allocation in R&D

The elasticity of aggregation factor is essential to understand this result. Intuitively, when aggregation factor is highly elastic, the output decreases relatively much more. This translates in a higher decrease in labor demand and a lower wage rate. Then, market allocates much more labor to R&D activities due to the decrease of labor cost.

As shown in Figure 2. , R&D activity increases with higher pollution tax H with an increasing catastrophe probability when penalty rate is higher. This effects is reverse while penalty rate is lower. Another interesting point to emphasis is how pollution flow changes at BGP with a higher environmental tax. One can easily write the aggregate pollution.

$$P(t) = \frac{\gamma_1}{1 - \gamma_1} \frac{\alpha^{\frac{2\alpha}{1-\alpha}}}{\chi^{\frac{\alpha}{1-\alpha}}} L_Y \left[\bar{\phi}_{max}(t) \right]^{\frac{2-\alpha}{1-\alpha}} \left[z_{min}(t) \right]^{\frac{1+\eta(1-\alpha)}{1-\alpha}} \Omega(H) = \left[\bar{\phi}_{max}(t) \right] \left[z_{min}(t) \right]^\eta Y(t) \quad (2.29)$$

It is easy to remark that pollution $P(t)$ is proportional to aggregate production $Y(t)$.

2.8 Adaptation and Mitigation in a Market Economy

It is interesting to look at how economy adapts and mitigates when it faces a higher catastrophe event probability $\bar{\theta}$. To assess the implications of pollution tax on adaptation capability of the economy, one should observe how production Y changes with respect to catastrophic event probability. Recall that economy adapts more when it becomes wealthier. (see lemma 1.) On the other hand, the mitigation activity can be captured through variable Z , which stands for the pollution intensity. Indeed, it is worthwhile to note that market does not target explicitly to do adaptation and mitigation activities. It is clear in our framework that adaptation and mitigation activities are promoted by the means of R&D activity, which aims primarily to have a monopoly situation in market. Then, it is plausible to say that adaptation and mitigation mix are the natural outcome of the R&D made in market. A proxy indicator can be easily constructed to understand how adaptation and mitigation activities are designated in a market economy.

Variable $M = \frac{1}{Z}$ can be considered as the mitigation activity. As pollution intensity decreases, mitigation increases. The economy starts to adapt more when it gets wealthier. This means that when wealth accumulation a increases, adaptation to a climatic catastrophe increases. Relative ratio between adaptation and mitigation gives ;

$$\frac{A}{M} = \frac{a(t)}{\frac{1}{Z(t)}} = \frac{\gamma_1}{1 - \gamma_1} \frac{\alpha^{\frac{2\alpha}{1-\alpha}}}{\chi^{\frac{\alpha}{1-\alpha}}} \left(\bar{\phi}_{max}(t) \right) \left(z_{min}(t) \right)^{1+\frac{1}{1-\alpha}} \Omega(H)$$

At BGP, one can easily find

$$g_{\frac{A}{M}} = \left(\frac{\gamma_1}{1 - \alpha} + \left(1 + \frac{1}{1 - \alpha} \right) \gamma_2 \right) \lambda L_R$$

Proposition 3. (i) *In a market economy, economy adapts much more when innovations in pollution intensity is not sufficiently high (γ_2). Equivalently, this means that economy mitigates much more relatively to adaptation when R&D activity enables producing with less polluting intermediate goods.*

$$\text{Case 1. } g_{\frac{A}{M}} > 0 \text{ if } - \left(\frac{\gamma_1}{\gamma_2} \right) > \frac{1}{2 - \alpha}$$

$$\text{Case 2. } g_{\frac{A}{M}} < 0 \text{ if } - \left(\frac{\gamma_1}{\gamma_2} \right) < \frac{1}{2 - \alpha}$$

In case 1, cleanliness of R&D γ_2 is not that high relatively to productivity parameter γ_1 . This means that growth rate for adaptation/mitigation ratio $\frac{A}{M}$ is positive. Then, economy adapts always much more than it mitigates at long run. In case 2, economy permits more clean innovations relatively to case 1. Therefore, the growth rate of adaptation/mitigation ratio is negative, which means that mitigation is higher than adaptation.

A straightforward question to ask is to know what would be the effect of a higher catastrophic event $\bar{\theta}$ on the equilibrium level of adaptation and mitigation. Basing on proposition 1, an economy facing a high-level penalty rate would allocate more labor to R&D activities. This one results in more adaptation than mitigation in case 1 and vice versa in case 2.

We illustrate this result numerically :

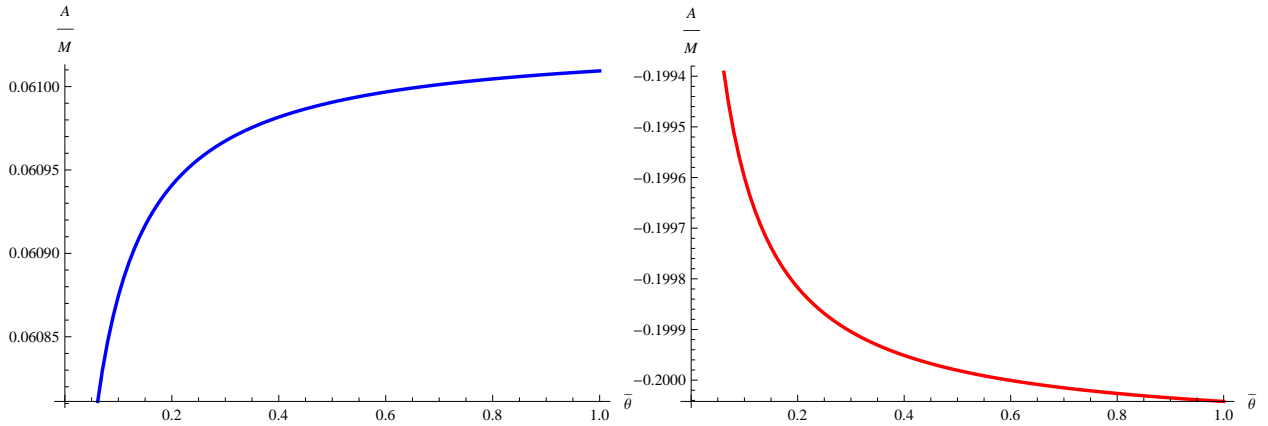


Figure 3. Adaptation and Mitigation in Case 1 (blue) and Case 2 (red).

As one can see, the economy starts to accumulate much more wealth with higher catastrophe probability $\bar{\theta}$ in order to adapt to penalty from catastrophic event. In case where the penalty rate is not high, economy would allocate less labor to R&D. Then, ratio adaptation/mitigation would fall, which means that mitigation increases relatively to adaptation with a higher catastrophe probability.

In case 2, the ratio of total productivity parameter and cleanliness of R&D $\left(\frac{\gamma_1}{\gamma_2}\right)$ is low. Therefore, market mitigates more than it adapts to catastrophic event. From the case 2, we remark an important trade-off between adaptation and mitigation activities. When cleanliness of R&D is higher, economic growth decreases. From production function, we know that a polluting intermediate good is more productive. (see Ricci 2007). This leads to the fact that cleanliness of intermediate good decreases the productivity, which means a decrease in production Y . Then, economy adapts less when it mitigates more.

This trade-off which is also present in Tsur and Zemel (2016) and Bréchet et al. (2012) is also intuitive when we think about the policy rule. When R&D innovations are more environmental friendly, policy maker increases its tax to balance its budget. This is normally supposed to impede economic growth if the pollution tax discourages growth.⁷

⁷This case is possible if the elasticity of aggregation factor is low. (See Appendix (4.9))

2.9 Aggregate Pollution

Differentiating equation (2.29), at BGP, pollution growth can be written

$$g_P = \left(\frac{2 - \alpha}{1 - \alpha} g_\phi + \frac{1 + \eta(1 - \alpha)}{1 - \alpha} g_Z \right) = \frac{1}{1 - \alpha} ((2 - \alpha) \gamma_1 + (1 + \eta(1 - \alpha)) \gamma_2) \lambda L_R \quad (2.30)$$

Proposition 4. *Pollution growth depends on pollution share, cleanliness of R&D and total productivity parameter. This growth rate is positive if the pollution share or cleanliness of R&D are not sufficiently high. In this case, economy faces a Jevons type paradox.*

$$\text{Case 1. } g_P > 0 \text{ if } - \left(\frac{\gamma_1}{\gamma_2} \right) > \frac{(1 + \eta(1 - \alpha))}{2 - \alpha}$$

$$\text{Case 2. } g_P < 0 \text{ if } - \left(\frac{\gamma_1}{\gamma_2} \right) < \frac{(1 + \eta(1 - \alpha))}{2 - \alpha}$$

In market economy, pollution could grow even when economy allocates much more labor to R&D with a decrease in pollution intensity. In fact, total productivity increase, as it increases production, leads to more pollution. If the cleanliness rate of R&D γ_2 is not high, the increase in total productivity can would lead to more pollution growth at long run, as it can be seen on equation (2.30). This is so-called Jevons Paradox.

An illustrative example about this topic could be India's increased aggregate pollution despite the decreased pollution intensity. Barrows and Ollivier (2016) show that pollution intensity decrease in India between 1990-2010. However, the emissions has increased in India between this period.⁸

2.10 Welfare Analysis

In this section, we study the welfare implications of pollution tax and catastrophic event probability. As mentioned before, we are interested in balanced growth path and not in transitional dynamics. Using equation (2.6), the total welfare at BGP is

$$W^* = \int_0^\infty \left[\log(c(t)) + \bar{\theta} \left(u(c_{min}) - \bar{\psi} (\omega - (1 - \omega) \log(a(t))) \right) \right] e^{-(\rho + \bar{\theta})t} dt \quad (2.31)$$

By integrating welfare function (2.31), we have

$$W^* = \frac{\log(Y(0)) (1 + \bar{\psi} \bar{\theta})}{\rho + \bar{\theta} - g} - \frac{\bar{\psi} \bar{\theta} (\omega + (1 - \alpha) \log(1 - L_R(0)) - \log(\lambda(1 - \alpha)))}{\rho + \bar{\theta} - g} \quad (2.32)$$

Proposition 5 . *The effect of pollution tax and catastrophic event probability on welfare is ambiguous and depends on how labor in R&D sector is affected by the presence of catastrophic event and the amount of penalty $\bar{\psi}$.*

⁸See World Bank Database : <http://data.worldbank.org/indicator/EN.ATM.CO2E.PC?end=2010&locations=IN&start=1990>

Proof. See Appendix [Impact of environmental taxation on Welfare](#)

The tax rate has a negative effect on production due to the exit of dirty intermediate goods, which are supposed to be more productive. On the other hand, pollution tax enhances R&D activity and growth if the aggregation factor is sufficiently elastic. Then, the penalty rate due to catastrophic event decreases. As a nutshell, welfare is increasing with pollution tax if latter effect dominates the former.

3 Conclusion

In this paper, we examine the effect of catastrophe probability on R&D decisions of market. R&D activity aims to increase total productivity and decrease the emission intensity and it serves as an intermediary tool to adapt to abrupt events. Indeed, the main idea is that when economy gets richer, it becomes more resilient to abrupt events according to empirical evidence. (see Mendelson, Dinar and Williams (2006)). In this case, a higher probability of catastrophic event increases the marginal benefit of doing R&D since a higher expected value of R&D implies higher accumulation of assets. (see Lemma 1. ($a(t) = V(t)$))

We show that pollution tax increases growth rate at long run if the elasticity of aggregation factor of production and dividends of R&D assets are high enough. A higher catastrophe probability increases the effect of pollution tax if the penalty rate is sufficiently higher. This result relies on the fact that higher penalty rate implies a higher marginal benefit from R&D.

Market economy makes more adaptation activity than mitigation if total productivity of R&D is higher than the cleanliness of innovations. One may understand this by looking the presence of pollution intensity on production function. In our model, a polluting intermediate good is more productive than a clean one. Hence, when environmental innovations are so clean, the production decreases much more. This implies a lower asset accumulation at long run, which makes that adaptation to abrupt events decreases. This shows the traditional trade-off between adaptation and mitigation, as highlighted in many recent articles. (See Bréchet et al. (2012), Zemel (2015), Tsur and Zemel (2016))

Our results show also that pollution growth could be higher at long run if total productivity of R&D is higher than cleanliness of innovations. In this case, even when pollution intensity decreases at long run, growth rate of pollution is positive, which case we denote Jevons-type paradox.

Our contribution to literature is to analyze the adaptation and mitigation policies thorough R&D policies in a decentralized market economy. Extending our model to a model in which transitional dynamics are possible to be studied is desired and planned in our further research agenda.

4 Appendix

4.1 Production Function

As in Ricci (2007), we define the function as

$$Y(t) = \int_0^1 (\phi(v, t) L_Y(t))^{1-\alpha} \left(P(v, t)^\beta x(v, t)^{1-\beta} \right)^\alpha$$

where $P(v, t)$ is the polluting input. From production function, we can define a emissions-intermediate good ratio in order to have simpler form for production function ;

$$z(v, t) = \left(\frac{P(v, t)}{\phi(v, t) x(v, t)} \right)^{\alpha\beta}$$

The production function takes a simpler form

$$Y(t) = L_Y(t)^{1-\alpha} \int_0^1 \phi(v, t) z(v, t) x(v, t)^\alpha dv$$

For sake of simplicity in notation, we put $\eta = \alpha\beta$ for the remainder of paper.

4.2 Household's Maximization Program

The Hamiltonian for the maximization program reads

$$\mathcal{H} = u(c(t)) + \bar{\theta}\Gamma(a(t)) + \mu(r(t)a(t) + w(t) - c(t) + T(t) + \chi x(t)) \quad (4.1)$$

The first-order conditions can be written

$$u_c(c) = \mu \quad (4.2)$$

$$\frac{\dot{\mu}}{\mu} = (\rho + \bar{\theta}) - r - \frac{\bar{\theta}\Gamma_a(a(t))}{\mu} \quad (4.3)$$

With $u(c) = \log(c)$. The Keynes-Ramsey equation yields

$$\frac{\dot{c}}{c} = \left(r - (\rho + \bar{\theta}) \right) + \frac{\bar{\theta}\Gamma_a(a)}{u_c(c)} \quad (4.4)$$

By making trivial algebra, we can reformulate equation (4.4) as (2.7).

4.3 Proof of Lemma 1

We can reformulate the budget constraint in the form

$$\dot{a}(t) = r(t)a(t) + w(t) - c(t) + T(t) + \chi x(t) \quad (4.5)$$

With the perfect competition assumption in final good sector, the profits are equal to zero

$$c(t) = Y(t) = w(t) L_Y(t) + \int_0^1 p(v, t) x(v, t) \quad (4.6)$$

By replacing zero profit condition (4.6) in budget constraint of the household (4.5), the budget constraint becomes

$$\dot{a}(t) = r(t) a(t) + w(t) L_R(t) - \left[\int_0^1 p(v, t) x(v, t) - h(t) P(t) - \chi x(t) \right]$$

From free-entry condition in R&D sector, we know $\lambda L_R(t) V(t) - w(t) L_R(t) = 0$. Recall that the term in brackets is the total profit $\pi(t) = \int_0^1 \pi(v, t)$ in intermediate good sector. Then, the budget constraint becomes

$$\dot{a}(t) = r(t) a(t) + \lambda L_R(t) V(t) - \pi(t)$$

Consequently, the Hamilton-Jacobi-Bellman equation for expressing the expected value of an innovation in R&D sector allows us to conclude that

$$a(t) = V(t)$$

This completes the proof of Lemma 1.

4.4 Cross-Sectoral Distribution

4.4.1 Productivity Distribution

We follow a method similar to Aghion and Howitt (1998) in order to characterize long-run distribution of relative productivity terms, both for technology improvements $\phi(v, t)$ and emission intensity $z(v, t)$. Let $F(., t)$ be the cumulative distribution of technology index ϕ across different sectors at a given date t and write $\Phi(t) \equiv F(\phi, t)$. Then

$$\Phi(0) = 1 \quad (4.7)$$

$$\frac{\dot{\Phi}(t)}{\Phi(t)} = -\lambda L_R(t) \quad (4.8)$$

Integrating this equation yields

$$\Phi(t) = \Phi(0) e^{-\lambda \gamma_1 \int_0^t L_R(s) ds} \quad (4.9)$$

The equation (4.7) holds because it is not possible that a firm has a productivity parameter ϕ larger than the leading firm in the sector. The equation (4.8) means that at each date a mass of λn firm lags behind, due to innovations that take place with Poisson distribution. From equation (2.22), we write

$$\frac{\dot{\bar{\phi}}_{max}(t)}{\bar{\phi}_{max}(t)} = \gamma_1 \lambda L_R \quad (4.10)$$

Integrating equation (4.10), we have ;

$$\bar{\phi}_{max}(t) = \bar{\phi}_{max}(0) e^{\lambda \gamma_1 \int_0^t L_R(s) ds} \quad (4.11)$$

where $\bar{\phi}_{max}(0) \equiv \bar{\phi}$. By using equations (4.9) and (4.11), we write

$$\left(\frac{\bar{\phi}}{\bar{\phi}_{max}} \right)^{\frac{1}{\gamma_1}} = e^{-\lambda \int_0^t L_R(s) ds} = \Phi(t) \quad (4.12)$$

We define a to be the relative productivity $\frac{\bar{\phi}}{\bar{\phi}_{max}}$. Basically, $\Phi(t)$ is the probability density distribution.

4.4.2 Emission Intensity Distribution

By proceeding exactly in same manner, we have

$$\frac{\dot{\bar{z}}_{min}(t)}{\bar{z}_{min}(t)} = \gamma_2 \lambda L_R \quad (4.13)$$

By integrating equation (4.13), we have

$$\bar{z}_{min}(t) = \bar{z}_{min}(0) e^{\lambda \gamma_2 \int_0^t L_R(s) ds} \quad (4.14)$$

We rewrite the equation as

$$\left(\frac{\bar{z}}{\bar{z}_{min}} \right)^{\frac{1}{\gamma_2}} = e^{-\lambda \int_0^t L_R(s) ds} \quad (4.15)$$

We can easily remark that this last equation is the same that we have found in equation (4.12). We write

$$\left(\frac{\bar{\phi}}{\bar{\phi}_{max}} \right)^{\frac{1}{\gamma_1}} = \left(\frac{\bar{z}}{\bar{z}_{min}} \right)^{\frac{1}{\gamma_2}} \quad (4.16)$$

From equation (4.16), We can find the relative distribution for emission intensity across firms

$$\frac{\bar{z}}{\bar{z}_{min}} = \left(\frac{1}{a} \right)^{-\frac{\gamma_2}{\gamma_1}}$$

4.5 Aggregate Economy

We replace equation of supply of machines (2.15) in equation (2.8) and write

$$Y(t) = L_Y(t) \int_0^1 \phi(v,t) z(v,t) \left(\frac{\alpha^2 \phi(v,t) z(v,t)}{\chi + h(t) \phi(v,t) z(v,t)^\eta} \right)^{\frac{\alpha}{1-\alpha}} dv \quad (4.17)$$

We proceed to reformulate the production in a way that it is possible to write productivity and emission intensity gaps. Note that according to Aghion and Howitt (1992), they are constant along time. By dividing and multiplying nominator and denominator by $\bar{\phi}_{max} \bar{z}_{min}$;

$$Y(t) = \alpha^{\frac{2\alpha}{1-\alpha}} L_Y \left(\bar{\phi}_{max} \bar{z}_{min} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{\phi(v,t) z(v,t)}{\bar{\phi}_{max} \bar{z}_{min}} \right)^{\frac{1}{1-\alpha}} \left(\frac{1}{\left(\chi + h(t) \bar{\phi}_{max} \bar{z}_{min}^\eta \frac{z(v,t)^\eta \phi(v,t)}{\bar{z}_{min}^\eta \bar{\phi}_{max}} \right)} \right)^{\frac{\alpha}{1-\alpha}} dv \quad (4.18)$$

By using the productivity and emission intensity distributions in Appendix 2.1, We find the aggregate production function as follows ;

$$Y(t) = \frac{\gamma_1}{1-\gamma_1} \frac{\alpha^{\frac{2\alpha}{1-\alpha}}}{\chi^{\frac{\alpha}{1-\alpha}}} L_Y \left(\bar{\phi}_{max}(t) \bar{z}_{min}(t) \right)^{\frac{1}{1-\alpha}} \Omega_1(H) \quad (4.19)$$

where the aggregation function for production $\Omega_1(H)$;

$$\Omega_1(H) = \int_0^1 \frac{a^{\frac{1}{1-\alpha} \left(1 + \frac{\gamma_2}{\gamma_1} \right)}}{\left(1 + \frac{H}{\chi} a^{1 + \frac{\gamma_2}{\gamma_1}} \right)^{\frac{\alpha}{1-\alpha}}} \nu'(a) da \quad (4.20)$$

where $H = h(t) \bar{\phi}_{max} \bar{z}_{min}^\eta$ which is a constant term along time t by the policy rule and $\nu'(a)$ is the density function for the function $\nu(a) = F(.,t) = a^{\frac{1}{\gamma_1}}$.

The aggregation of intermediate factor $x(t)$ is obtained in same manner.

$$x(t) = \int_0^1 x(v,t) dv = \frac{\gamma_1}{1-\gamma_1} \frac{\alpha^{\frac{2\alpha}{1-\alpha}}}{\chi^{\frac{\alpha}{1-\alpha}}} L_Y \left(\bar{\phi}_{max}(t) \bar{z}_{min}(t) \right)^{\frac{1}{1-\alpha}} \Omega_2(H) \quad (4.21)$$

where the aggregation factor $\Omega_2(H)$ for intermediate good $x(t)$ is

$$\Omega_2(H) = \int_0^1 \frac{a^{\frac{1}{1-\alpha} \left(1 + \frac{\gamma_2}{\gamma_1} \right)}}{\left(1 + \frac{H}{\chi} a^{1 + \frac{\gamma_2}{\gamma_1}} \right)^{\frac{1}{1-\alpha}}} \nu'(a) da \quad (4.22)$$

The final good market equilibrium yields $Y(t) = c(t) + x(t)$, since some part of the final good is used for the production of intermediate good. From equation (2.14), we know that aggregate cost of the production good $x(t)$ is given by $\chi x(t)$.

$$c(t) = Y(t) - \chi x(t) = \alpha^2 \frac{\Omega_2(H)}{\Omega_1(H)} Y(t) \quad (4.23)$$

which gives the consumption $c(t)$ as a function of production function $Y(t)$.

4.6 Aggregation Factor

From production function, in order to solve the integral (4.20),

$$\Omega_1(H) = \int_0^1 \frac{a^{\bar{\gamma}}}{\left(1 + \frac{H}{\chi} a^{1+\frac{\gamma_2}{\gamma_1}}\right)^{\frac{\alpha}{1-\alpha}}} da \quad (4.24)$$

where $\bar{\gamma} = \frac{1}{1-\alpha} \left(1 + \frac{\gamma_2}{\gamma_1}\right) + \frac{1}{\gamma_1} - 1$. We use substitution method. We define

$$y = -\frac{H}{\chi} a^{1+\frac{\gamma_2}{\gamma_1}} \quad \text{and} \quad dy = -\left(1 + \frac{\gamma_2}{\gamma_1}\right) \frac{H}{\chi} a^{1+\frac{\gamma_2}{\gamma_1}-1} da \quad (4.25)$$

We rewrite the aggregation factor,

$$\Omega_1(H) = \int_0^{-\frac{H}{\chi}} y^{\frac{\bar{\gamma}+1-b}{b}} (1-y)^{-\frac{\alpha}{1-\alpha}} dy \quad (4.26)$$

where $b = 1 + \frac{\gamma_2}{\gamma_1}$. It is easy to remark that expression in the integral is the incomplete beta function. Then, we can express this integral by using Gaussian hypergeometric function as follows

$$\Omega_1(H) = \left(\frac{1}{\bar{\gamma}b}\right) {}_2F_1\left(\frac{\bar{\gamma}+1}{b}, \frac{\alpha}{1-\alpha}; \frac{\bar{\gamma}+b+1}{b}; -\frac{H}{\chi}\right) \quad (4.27)$$

In order to see the marginal change of aggregation factor with respect to marginal cost of pollution H ;

$$\frac{\partial \Omega_1(H)}{\partial H} = -\frac{1}{\chi} \left(\frac{\alpha(\bar{\gamma}+1)}{(1-\alpha)(\bar{\gamma}+1+b)}\right) {}_2F_1\left(\frac{\bar{\gamma}+1}{b} + 1, \frac{\alpha}{1-\alpha} + 1; \frac{\bar{\gamma}+b+1}{b} + 1; -\frac{H}{\chi}\right) < 0$$

4.7 Condition on Penalty Function

From the household problem, we define the post-value function as

$$\Gamma(a(t)) = u(c_{min}) - \psi(a(t))$$

$$\psi(a(t)) = \bar{\psi}(\omega - (1-\omega) \log(a(t)))$$

At Balanced Growth Path, the post value function can be written in the following manner ;

$$\Gamma^* = -\int_0^\infty \psi(a(t)) e^{-(\rho+\bar{\theta})t} dt = -\bar{\psi}\left(\frac{\omega}{\rho+\bar{\theta}} - \frac{(1-\omega) \log(a(0))}{\rho+\bar{\theta}-g_Y}\right)$$

$$\omega > \frac{(\rho+\bar{\theta}) \ln(a(0))}{(\rho+\bar{\theta})(1+\ln(a(0))) - g_Y}$$

where $a(0)$ is the level of wealth at initial date.

4.8 Impact of hazard rate on labor allocation in R&D Sector

To assess the impact catastrophe probability on labor in R&D, we take derivative of L_R (equation (2.28)) with respect to hazard rate $\bar{\theta}$;

$$\frac{\partial L_R}{\partial \bar{\theta}} = \frac{\bar{\psi} \left(\frac{\lambda(1-\omega)\alpha^2}{(1-\alpha)} \frac{\Omega_2(H)}{\Omega_1(H)} (\lambda + \rho) \right) - \left(\lambda + \frac{\alpha\gamma_1\lambda}{(1-\gamma_1)} \frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)} \right)}{\left(\lambda + \frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha} \frac{\Omega_2(H)}{\Omega_1(H)} + \frac{\alpha\gamma_1\lambda}{(1-\gamma_1)} \frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)} \right)^2}$$

The impact depends whether the penalty rate $\bar{\psi}$ is sufficiently high or not.

$$\text{sign} \left(\frac{\partial L_R}{\partial \bar{\theta}} \right) > 0 \text{ if } \bar{\psi} > \frac{\lambda + \frac{\alpha\gamma_1\lambda}{(1-\gamma_1)} \frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)}}{\frac{\lambda(1-\omega)\alpha^2}{(1-\alpha)} \frac{\Omega_2(H)}{\Omega_1(H)} (\lambda + \rho)}$$

$$\text{sign} \left(\frac{\partial L_R}{\partial \bar{\theta}} \right) < 0 \text{ if } \bar{\psi} < \frac{\lambda + \frac{\alpha\gamma_1\lambda}{(1-\gamma_1)} \frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)}}{\frac{\lambda(1-\omega)\alpha^2}{(1-\alpha)} \frac{\Omega_2(H)}{\Omega_1(H)} (\lambda + \rho)}$$

4.9 Impact of environmental taxation on labor allocation in R&D Sector

Taking the derivative of L_R (equation (2.28)) with respect to marginal cost of pollution H ;

$$\frac{\partial L_R}{\partial H} = \frac{\left[\frac{\lambda(1-\omega)\alpha^2\bar{\theta}\bar{\psi}}{(1-\alpha)} \left[\frac{\partial\Omega_2(H)}{\partial H} \frac{1}{\Omega_2(H)} - \frac{\Omega_2(H)}{(\Omega_1(H))^2} \frac{\partial\Omega_1(H)}{\partial H} \right] - \frac{\pi}{V} \left[\frac{\alpha}{(1-\alpha)(\chi+H)} + \frac{\partial\Omega_1(H)}{\partial H} \frac{1}{\Omega_1(H)} \right] \right] \left[\lambda + \rho + \bar{\theta} \right]}{\left(\lambda + \frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha} \frac{\Omega_2(H)}{\Omega_1(H)} + \frac{\alpha\gamma_1\lambda}{(1-\gamma_1)} \frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)} \right)^2}$$

We observe that denominator is always positive. The first term in brackets in nominator is always negative.⁹ The impact of pollution tax depends on elasticity of aggregation factor $\Omega(H)$;

$$\text{Case 1. } \text{sign} \left(\frac{\partial L_R}{\partial H} \right) < 0 \text{ if } \frac{\frac{\partial\Omega_1(H)}{\Omega_1(H)}}{\frac{\partial H}{H}} < \frac{\alpha H}{(1-\alpha)(\chi+H)}$$

In this case, the effect of marginal cost of pollution H is without doubt negative on labor allocation in R&D. The second case is more interesting.

$$\text{Case 2. } \text{sign} \left(\frac{\partial L_R}{\partial H} \right) > 0$$

To ensure Case 2. We need two conditions ;

$$\frac{\frac{\partial\Omega_1(H)}{\Omega_1(H)}}{\frac{\partial H}{H}} > \frac{\alpha H}{(1-\alpha)(\chi+H)}$$

and

⁹The mathematical proof of this result is available upon request from author.

$$\frac{\pi}{V} > \frac{\frac{\lambda(1-\omega)\alpha^2\bar{\theta}\bar{\psi}}{(1-\alpha)} \left[\frac{\partial\Omega_2(H)}{\partial H} \frac{1}{\Omega_2(H)} - \frac{\Omega_2(H)}{(\Omega_1(H))^2} \frac{\partial\Omega_1(H)}{\partial H} \right]}{\frac{\alpha}{(1-\alpha)(\chi+H)} + \frac{\partial\Omega_1(H)}{\partial H} \frac{1}{\Omega_1(H)}}$$

To assess the impact of hazard rate on the effect of environmental taxation, we compute

$$\frac{\partial}{\partial\bar{\theta}} \left(\frac{\partial L_R}{\partial H} \right) = \bar{\psi} (\lambda + \rho + \bar{\theta}) \left(\frac{\underbrace{k^2 \frac{\lambda(1-\omega)\alpha^2}{(1-\alpha)} \Upsilon_1}_{<0} - \underbrace{\frac{\lambda\bar{\theta}\bar{\psi}(1-\omega)\alpha^2}{(1-\alpha)} \frac{\Omega_2(H)}{\Omega_1(H)} (\Upsilon_1)}_{<0} - \underbrace{\frac{\lambda(1-\omega)\alpha^2}{(1-\alpha)} \frac{\Omega_2(H)}{\Omega_1(H)} \left(\frac{\pi}{V} \Upsilon_2 \right)}_{<0 \text{ or } >0}}{k^4} \right)$$

where $k = \lambda + \frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha} \frac{\Omega_2(H)}{\Omega_1(H)} + \frac{\alpha\gamma_1\lambda}{(1-\gamma_1)} \frac{(\chi+H)^{-1-\alpha}}{\Omega(H)}$ and $\Upsilon_1 = \frac{\partial\Omega_2(H)}{\partial H} \frac{1}{\Omega_2(H)} - \frac{\Omega_2(H)}{(\Omega_1(H))^2} \frac{\partial\Omega_1(H)}{\partial H} < 0$, $\Upsilon_2 = -\frac{\alpha}{(1-\alpha)(\chi+H)} - \frac{\partial\Omega_1(H)}{\partial H} \frac{1}{\Omega_1(H)} \leq 0$.

Then, we can analyze the implications of hazard rate on the effect of pollution tax H ;

$$\text{sign} \left(\frac{\partial}{\partial\bar{\theta}} \left(\frac{\partial L_R}{\partial H} \right) \right) > 0 \text{ if } \bar{\psi} > \frac{\frac{\lambda(1-\omega)\alpha^2}{(1-\alpha)} \frac{\Omega_2(H)}{\Omega_1(H)} \left(\frac{\pi}{V} \Upsilon_2 \right)}{-\frac{\lambda\bar{\theta}(1-\omega)\alpha^2}{(1-\alpha)} \frac{\Omega_2(H)}{\Omega_1(H)} (\Upsilon_1) - k^2 \frac{\lambda(1-\omega)\alpha^2}{(1-\alpha)} \Upsilon_1}$$

$$\text{sign} \left(\frac{\partial}{\partial\bar{\theta}} \left(\frac{\partial L_R}{\partial H} \right) \right) < 0 \text{ if } \bar{\psi} < \frac{\frac{\lambda(1-\omega)\alpha^2}{(1-\alpha)} \frac{\Omega_2(H)}{\Omega_1(H)} \left(\frac{\pi}{V} \Upsilon_2 \right)}{-\frac{\lambda\bar{\theta}(1-\omega)\alpha^2}{(1-\alpha)} \frac{\Omega_2(H)}{\Omega_1(H)} (\Upsilon_1) - k^2 \frac{\lambda(1-\omega)\alpha^2}{(1-\alpha)} \Upsilon_1}$$

4.10 Impact of environmental taxation on Welfare

The differentiation of (2.32) yields ;

$$\frac{dW^*}{dH} = \underbrace{\frac{(1 + \bar{\psi}\bar{\theta})}{\rho + \bar{\theta} - g} \frac{1}{Y(0)} \frac{dY(0)}{dH}}_{<0} + \underbrace{\frac{(1 + \bar{\psi}\bar{\theta}) \log(Y(0))}{(\rho + \bar{\theta} - g)^2} \frac{dg}{dH}}_{>0} + \underbrace{\frac{\bar{\psi}\bar{\theta}}{\rho + \bar{\theta}} \frac{1}{L_R(0)} \frac{dL_R}{dH}}_{>0}$$

where the sign of $\frac{dL_R}{dH}$ is positive. Then, total effect of pollution tax on growth is ambiguous.

The effect of catastrophe probability on welfare is

$$\frac{dW^*}{d\bar{\theta}} = \underbrace{\frac{\bar{\psi} \log Y(0)}{\rho + \bar{\theta} - g}}_{>0} + \underbrace{\frac{(1 + \bar{\psi}\bar{\theta}) \log(Y(0))}{(\rho + \bar{\theta} - g)^2}}_{<0} - \underbrace{\frac{\bar{\psi} (\omega + (1-\alpha) \lambda \log \left(\frac{1-L_R(0)}{1-\alpha} \right))}{\rho + \bar{\theta} - g}}_{<0} + \underbrace{\frac{\bar{\psi}\bar{\theta} (\omega + (1-\alpha) \lambda \log \left(\frac{1-L_R(0)}{1-\alpha} \right))}{(\rho + \bar{\theta} - g)^2}}_{>0}$$

The total effect of catastrophe probability on welfare is ambiguous as well.

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