

Why Fiscal Regimes Matter for Fiscal Sustainability

Analysis: An Application to France^{*}

Pierre Aldama[†]

Paris School of Economics, Université Paris 1

Jérôme Creel[‡]

OFCE-Sciences Po and ESCP-Europe

This version: February, 2017

First draft: February, 2015

Abstract

This paper introduces a Regime-Switching Model-Based Sustainability test allowing for periodic (or local) violations of Bohn (1998, QJE)'s sustainability condition. We assume a Markov-switching fiscal policy rule whose parameters stochastically switch between sustainable and unsustainable regimes. We demonstrate that long-run fiscal sustainability not only depends on regime-specific feedback coefficients of the fiscal policy rule but also on the average durations of fiscal regimes. Evidence on French data suggests that both the No-Ponzi Game condition and the Debt-stabilizing condition hold in the long-run, when accounting for fiscal regimes, contrary to standard MBS tests.

JEL: E6, H6

Keywords: Fiscal rules, Fiscal regimes, Public debt sustainability, Time-varying parameters, Markov-switching models

^{*}We thank Antonio Afonso, Florin Bilbiie, Nuno Coimbra, Peter Claeys, Nicolas Dromel, Bruno Ducoudré, Jean-Olivier Hairault, Hubert Kempf, Mathieu Plane, Xavier Ragot, Guillaume Roussellet, Gilles Saint-Paul, Raul Sampognaro, Henri Sterdyniak, Vincent Touzé and various participants at EEA-ESEM Annual Congress 2016 (Geneva, Switzerland), 5th UECE Conference on Economic and Financial Adjustments at ISEG (Lisbon, Portugal), PSE Macro Workshop 2015, PSE Macro Retreat 2015 and OFCE seminar for many useful comments. We are also grateful to four anonymous referees for their useful comments on a earlier version of this paper. Any remaining errors are ours.

[†]Corresponding author. E-mail: pierre.aldama@univ-paris1.fr

[‡]E-mail: jerome.creel@sciencespo.fr

1 Introduction

Fiscal policy rules describing the reaction of primary balance to the initial level of public debt have been widely used to analyze fiscal sustainability. According to [Bohn \(1998\)](#)'s seminal contribution, primary public balance must increase after an increase of public debt-to-GDP ratio to ensure the sustainability of public finance, as defined by the government intertemporal budget constraint. This paper is motivated by the numerous empirical evidence of fiscal regimes during which public debt-to-GDP becomes explosive without any improvement in primary public balance. It raises critical questions regarding long-run fiscal sustainability since it appears that fiscal policy periodically violates Bohn's sustainability condition. Is a periodically unsustainable fiscal policy a threat to long-run sustainability of public finance? How long can fiscal policy be periodically unsustainable without violating its sustainability constraints in the long-run?

To our knowledge, only a few papers have addressed these difficulties about a regime-switching (or time-varying) fiscal policy rule and proposed a testing framework for long-run sustainability. In their seminal contribution [Canzoneri et al. \(2001\)](#) consider a time-varying fiscal policy rule and derive a necessary and sufficient condition such that the government intertemporal budget constraint holds in the long-run. [Davig \(2005\)](#) extends [Wilcox \(1989\)](#)'s unit-root testing procedure to a Markov-switching framework in which discounted debt can be periodically expanding. Finally, there is a literature on regime-switching monetary and fiscal policy rules that has successfully identified *local* equilibria in the data where fiscal policy (and monetary policy) is either "active" or "passive", following [Leeper \(1991\)](#). Still, these works cannot test whether fiscal policy *globally* satisfies its intertemporal budget constraint or the debt-stabilizing criterion in the long-run, given estimated regimes' transition probabilities and/or expected durations. Based on a Markov-switching monetary policy rule, [Davig and Leeper \(2007b\)](#) have proposed a Long-Run Taylor Principle such that the price-level is *globally* determined despite periodic violations of the Taylor monetary principle; but there is no equivalent proposition for a *globally* sustainable fiscal policy.

This paper introduces a Regime-Switching Model-Based Sustainability test for fiscal policy, building on Bohn's Model-Based Sustainability (MBS) framework and on the literature on Markov-switching fiscal policy rules. We assume a Markov-switching fiscal policy rule that stochastically switches between sustainable and unsustainable regimes. We define unsustainable regimes by periodic and persistent *negative or null* feedback effect of initial public debt on primary surplus, i.e. violating Bohn's sustainability condition. Consequently, the public debt-to-GDP ratio becomes periodically and persistently explosive during unsustainable regimes, so that fiscal regimes matter for global (in opposition with local) fiscal sustainability analysis.

The paper addresses the two usual concepts of long-run fiscal sustainability: the No-Ponzi game condition (related to the transversality condition) and the debt-stabilizing condition (related to the stationarity of the debt-to-GDP ratio). For each concept of fiscal sustainability, we demonstrate that long-run (or global) fiscal sustainability not only depends on regime-specific feedback coefficients of the Markov-switching fiscal policy rule but also on expected durations (or persistence, equivalently) of fiscal regimes. We derive necessary and sufficient conditions, for both No-Ponzi Game and debt-stabilizing conditions, such that sustainable regimes balance unsustainable regimes in the long-run. Consequently, fiscal policy can be *locally* unsustainable, with a periodically explosive public-debt-to-GDP ratio, and still be *globally* sustainable. The situation of a locally-explosive debt, which does not lead to global unsustainability or default can be related to the recent theoretical result by [Blot et al. \(2016\)](#).

We apply this test to France. As a Euro Area member state, France has neither a domestic monetary

policy nor a lender of last resort. Both features make fiscal sustainability issues very acute. First, the French government cannot expect a domestic accommodative monetary policy when or after it implements a non-Ricardian fiscal policy. Second, sustainability issues cannot be disregarded and left to the management of the lender of last resort. As a result, we focus exclusively on Ricardian equilibria for which the government intertemporal budget constraint must hold for any path of the price-level.¹

Our results are threefold. First, we estimate various specifications of Bohn's constant-parameters fiscal policy rule. These estimates do not allow to reject unsustainability: the feedback coefficient on public debt-to-GDP is rarely positive and never significant, according standard MBS tests. Second, we estimate a Markov-switching fiscal policy rule. We identify two different fiscal regimes over the period: one regime is sustainable, with a strong positive and significant feedback effect of lagged public debt-to-GDP on primary surplus-to-GDP, while the second one is unsustainable with no significant feedback effect. In addition, identified fiscal regimes are found to be strongly persistent. In particular, our findings support the view that the Maastricht Treaty and the Stability and Growth Pact (SGP) actually made France's fiscal policy more sustainable, and notably, despite being under an Excessive Deficit Procedure from 2003 to 2007. Third, we perform RS-MBS tests for No-Ponzi Game and Stationary debt-output ratio. We conclude to reject the null hypothesis of a Ponzi Scheme as well as the null of an explosive public debt-to-GDP ratio, using two measures of the real interest rate paid on public debt.

The rest of the paper is organised as follows. Section 2 briefly reviews the related literature on fiscal sustainability. Section 3 presents the extension of the Model-based approach of sustainability to regime switches and develops a new condition for fiscal sustainability. Section 4 deals with an application of the empirical methodology to French data. Section 5 concludes.

2 Related literature

The starting point for the analysis of fiscal rules is the government budget constraint: government noninterest spending G_t is financed by tax revenues T_t or by issuing debt B_t at price $(1 + r_t)^{-1}$; thus government faces the following flow budget constraint:

$$(1 + r_t)^{-1} B_t = (G_t - T_t) + B_{t-1} \quad (1)$$

where B_t is the end-of-period stock of public debt. Denote the price of a bond that matures j -periods ahead of t by $(1 + r_{t,j})^{-1} = \prod_{i=1}^j (1 + r_{t+i-1})^{-1}$; thus $(1 + r_{t,1})^{-1} = (1 + r_t)^{-1}$. Then, iterating on equation (1), we obtain:

$$B_{t-1} = S_t + \sum_{i=1}^{+\infty} \mathbb{E}_t \left[\frac{S_{t+i}}{1 + r_{t,i}} \right] + \lim_{T \rightarrow +\infty} \mathbb{E}_t \left[\frac{B_{t+T}}{1 + r_{t,T+1}} \right] \quad (2)$$

where $S_t \equiv T_t - G_t$ is the primary budget surplus.

A sustainable fiscal policy must satisfy the standard solvency condition according to which the initial stock of public debt must be backed by future expected present-value primary budget surpluses. The Present-Value Budget Constraint (PVBC henceforth) can be written as:

$$B_{t-1} \leq S_t + \sum_{i=1}^{+\infty} \mathbb{E}_t \left[\frac{S_{t+i}}{1 + r_{t,i}} \right] \quad (3)$$

¹Another consequence of the first feature is methodological: the [Leeper \(1991\)](#) and [Davig and Leeper \(2011\)](#)'s policy interaction framework is not applicable to France.

Following equation (2), the PVBC can be equivalent to a transversality condition on the expected present-value stock of public debt:

$$\lim_{T \rightarrow +\infty} \mathbb{E}_t \left[\frac{B_{t+T}}{1 + r_{t,T+1}} \right] \leq 0 \quad (4)$$

This latest condition is the No-Ponzi Game condition (NPG). At equilibrium, both the PVBC (3) and the Transversality Condition (4) must hold with equality, preventing both lenders and government from playing a Ponzi scheme against each other, see [Bohn \(1995\)](#).

Seminal empirical investigations on fiscal sustainability proposed a testing framework based on the PVBC and the transversality condition, drawing on stationarity or cointegration properties of fiscal data ([Hamilton and Flavin, 1986](#); [Trehan and Walsh, 1988, 1991](#); [Wilcox, 1989](#); [Wickens and Uctum, 1993](#); [Quintos, 1995](#)). Still, the econometric analysis of fiscal sustainability has raised a number of issues and led to important criticisms by [Bohn \(1995, 1998, 2007\)](#). First, it usually involves restrictive assumptions on the real discounting rate. [Bohn \(1995\)](#) shows that in a stochastic economy the PVBC should require the introduction of the stochastic discount factor which is the common pricing kernel for all financial assets, under the complete market hypothesis. Bohn argues that these tests of fiscal sustainability with a constant safe interest rate ([Hamilton and Flavin, 1986](#)) or the actual interest rate ([Wilcox, 1989](#)) rather than the stochastic discount factor are misleading since they assume that lenders are risk-neutral and/or that there is no uncertainty. Second, based on [Barro \(1979\)](#) tax-smoothing model, [Bohn \(1998\)](#) argues that ignoring cyclical components of primary surplus (like output gap and government cyclical spending) induces a bias in unit-root test. Controlling for cyclical components, [Bohn \(1998\)](#) provides evidence that U.S. public debt-to-GDP ratio is actually mean-reverting. Third, [Bohn \(2007\)](#) formulates a general criticism: usual econometric restrictions² are not necessary for the PVBC to hold³ and lead to "absurdly weak" sustainability because debt-to-GDP would violate any upper bound implied by the existence of a fiscal limit. As a result, [Bohn \(2007\)](#) acknowledges that an upper bound on primary surplus requires a stationary public debt-to-GDP for fiscal sustainability to hold; but this condition would stem from additional economic considerations, such as the existence of distortionary taxation, implying a dynamic "Laffer curve" and a fiscal limit, but not from the PVBC *per se*.⁴

As an alternative to the econometric analyses *à la* Hamilton-Flavin, [Bohn \(1998\)](#) suggests analyzing fiscal sustainability through the lens of fiscal policy rules, or fiscal reaction functions, in a simple general equilibrium model. Basically, he assumes the following framework composed of a linear fiscal rule (5), the government one-period budget constraint (6) and the Euler equation (7) derived from the consumer's optimal choice:

$$s_t = \gamma b_{t-1} + \mu_t \quad (5)$$

$$b_t = \frac{1 + r_t}{1 + y_t} b_{t-1} - (1 + r_t) s_t \quad (6)$$

$$(1 + r_t)^{-1} = \beta \mathbb{E}_t \frac{u'(C_{t+1})}{u'(C_t)} \quad (7)$$

²Such that the public debt must be stationary ([Hamilton and Flavin, 1986](#)), or difference-stationary, implying cointegration between revenues and spending ([Trehan and Walsh, 1988, 1991](#)), or at least integrated of order two ([Quintos, 1995](#)).

³[Bohn \(2007\)](#) shows that a m -th order integrated debt-to-GDP process is actually sufficient for the PVBC and TC to hold, for any arbitrary high m . Proof can be summarized as follows: for any discount factor $\rho < 1$, the transversality condition is exponential in the time horizon n and the conditional expectation of a m -th order integrated variable is at most a polynomial of order m . Since exponential growth dominate polynomial growth in the long-run, given $\rho < 1$, then the transversality condition holds, see [Bohn \(2007\)](#).

⁴Research about the upper-bound of primary surplus has been recently explored by [Bi \(2012\)](#); [Bi and Traum \(2012\)](#); [Davig et al. \(2011\)](#); [Daniel and Shiamptanis \(2013\)](#). Daniel and Shiamptanis show that stationarity and cointegration restrictions are necessary for fiscal sustainability when assuming existence of a fiscal limit. Existence of a fiscal limit (i.e. an upper bound on primary balance-to-GDP and on public debt-to-GDP) requires a sustainability criterion ensuring that public debt must be stable around a long-run value compatible with fiscal limit.

where s_t is the primary surplus-to-GDP ratio, b_t is the end-of-period public debt-to-GDP ratio and finally μ_t is a vector including all cyclical components of primary surplus (e.g. output gap, temporary public spending), plus a constant and an error term. The utility function $u(\cdot)$ is assumed to be strictly increasing ($u'(\cdot) > 0$) and concave ($u''(\cdot) < 0$); β is the subjective discount factor and C_t is the level of consumption. Thus, Bohn seeks a condition on the fiscal rule feedback parameter γ such that the No-Ponzi Game (NPG) condition holds. He argues that if there is a strictly positive feedback effect of public debt on primary surplus, that is

$$\gamma > 0 \tag{8}$$

then fiscal policy satisfies the NPG condition (4). If one considers a stricter sustainability concept, such as a debt-stabilizing fiscal policy rule, then this feedback effect should be larger than the growth-adjusted real average interest rate on public debt ⁵ such that

$$\gamma > r - y \tag{9}$$

with r the average real interest rate and y the average growth rate of real GDP. Hence, for a debt-stabilizing fiscal policy, after substituting (5) into (6) and taking the expectation, public debt converges to its unconditional mean:

$$\mathbb{E}[b_t] \simeq \frac{-(1+r)\bar{\mu}}{(1+r)\gamma - \frac{r-y}{1+y}} \tag{10}$$

where $\bar{\mu} < 0$ is the mean value of μ_t . Bohn's framework for fiscal sustainability analysis is often labeled "Model-Based Sustainability" (henceforth MBS) because it relies on general equilibrium conditions (i.e. Euler equation) and on an explicit fiscal policy rule, see [Bohn \(1995, 1998\)](#). MBS analysis has been shown to be empirically powerful in the case of US fiscal policy on long-run data ([Bohn, 1998, 2008](#)). On international panel data, [Mendoza and Ostry \(2008\)](#) find evidence that fiscal policy is "responsible" (i.e. evidence of a strictly positive feedback rule).

Recent empirical research has introduced two types of nonlinear specifications. One strand of the literature specifies fiscal policy rules as polynomial functions of public debt-to-GDP ratio, i.e. including quadratic and cubic terms ([Bohn, 1998](#)). This specification is motivated by the idea that primary surplus may either react more to lagged public debt or on the contrary may become "flatter", at higher public debt levels. This approach has been followed by [Ghosh et al. \(2013a,b\)](#) to account for "fiscal fatigue" where they derive fiscal limits as the maximum level of public debt beyond which primary balance can no longer adjust to stabilize debt. Another strand of empirical research considers time-varying fiscal policy rules. The assumption that simple linear policy rules (either monetary or fiscal) are constant overtime seems not convincing regarding empirical evidence, given multiple evidence of "structural breaks" or "regime changes", thus motivating a regime-switching approach in empirical studies.⁶ In particular, empirical literature on regime-switching fiscal policy rules has produced evidence that fiscal policy rules vary overtime and may be better described by "fiscal regimes", see [Favero and Monacelli \(2005\)](#); [Chung et al. \(2007\)](#); [Davig and Leeper \(2007a, 2011\)](#); [Bianchi \(2012\)](#); [Burger and Marinkov \(2012\)](#); [Afonso and Toffano \(2013\)](#). This literature generally identifies sub-periods during which fiscal policy does not stabilize public debt by increasing the primary surplus after an increase in public debt, and sometimes even displays a negative feedback effect of initial public debt on primary surplus.⁷

⁵If one considers a fiscal rule with variables in absolute level rather than as share of GDP, then this feedback should be larger than the real average interest rate on public debt. This is basically what [Leeper \(1991\)](#) finds when describing the stability conditions of an active monetary/passive fiscal regime.

⁶For monetary policy, see [Clarida et al. \(2000\)](#); [Auerbach \(2002\)](#); [Lubik and Schorfheide \(2004\)](#), among others.

⁷In some specifications, the fiscal rule is modelled as the reaction of the aggregate tax rate, rather than the primary surplus, as in [Davig and Leeper \(2011\)](#) for example.

The former literature on regime-switching monetary and fiscal policy rules builds on [Leeper \(1991\)](#)'s seminal contribution, which developed a set of conditions for *local* equilibrium determinacy stemming from the properties of the monetary and fiscal rules. Fiscal policy is *passive* under the debt-stabilizing condition (9), *active* otherwise⁸. Recent research on fiscal policy ([Bi, 2012](#); [Bi and Leeper, 2013](#)) has explored consequences of regime-switching fiscal policy to derive an endogenous and stochastic fiscal limit. This literature analyzes fiscal sustainability as the sovereign default probability, computed from the fiscal limit distribution⁹, rather than as generalized conditions on the regime-switching fiscal rule. [Davig and Leeper \(2007b\)](#) define the long-run Taylor monetary principle, based on a Markov-switching Taylor rule, allowing for periodic (or local) violations of the Taylor principle. But, to our knowledge, none has proposed and tested analogous conditions on a regime-switching fiscal rule, such that NPG and debt-stabilizing conditions hold in the long-run. In this respect, this paper's motivation is similar to [Davig and Leeper \(2007b\)](#).

Finally, this paper is motivated by two important contributions in the field of fiscal sustainability analysis. [Canzoneri et al. \(2001\)](#) study a particular time-varying fiscal policy rule in which public debt feedback effect on primary surplus is positive or null. They show primary surplus only has to react positively to public debt on an infrequent basis but "infinitely often" in order to satisfy the government intertemporal budget constraint. This analysis is restrictive in at least two respects. First, assuming primary surplus does not react negatively to initial public debt is a critical assumption, at odds with some empirical evidence on regime-switching policy rules ([Favero and Monacelli, 2005](#); [Davig and Leeper, 2007a, 2011](#); [Afonso and Toffano, 2013](#)). Second, the sustainability condition does not ensure a stationary public debt-to-GDP ratio, which is probably the relevant fiscal sustainability condition when the economy faces a fiscal limit. In contrast, we propose a necessary and sufficient condition on the regime-switching fiscal policy rule such that fiscal policy stabilizes public debt-to-GDP ratio in the long-run. Alternatively, [Davig \(2005\)](#) proposes a modified version of [Wilcox \(1989\)](#)'s unit-root testing framework using a Markov-switching model, which takes account for episodes of periodically expanding discounted public debt. Still, this approach is inherently subject to the criticisms addressed by [Bohn \(1995, 2007\)](#) to the econometric analysis of fiscal sustainability. In particular, unit-root testing does not provide any information about fiscal policy behavior since it does not involve an explicit model of fiscal policy.

3 Theory: Regime-Switching Model-Based Sustainability

We assume a stochastic real endowment and cashless economy composed of a representative rational household and a government. By assuming a real cashless economy, we implicitly assume that monetary policy has full control over the price-level and inflation dynamics. Using the terminology of the Fiscal Theory of Price-Level ([Leeper, 1991](#); [Sims, 1994](#); [Woodford, 1995](#); [Cochrane, 2005](#)), we only consider Ricardian equilibria for which the government intertemporal budget constraint must hold for any path of the price-level.

By this assumption, we do not mean in any way there is no interactions between monetary and fiscal authorities nor we states monetary policy (and the price-level) has no role to play in ensuring fiscal sustainability. On the contrary, this analysis should be interpreted in the following sense: suppose fiscal

⁸Condition on monetary policy is the Taylor Principle: monetary policy is labeled "active" when it reacts aggressively to inflation (i.e. the Taylor principle holds) and "passive" otherwise. From these two conditions, [Leeper \(1991\)](#) identify four local regimes: Monetary regime (AM/PF), Fiscal regime (PM/AF), Indeterminacy regime (PM/PF) and Explosive regime (AM/AF).

⁹Fiscal limit distributions are obtained by numerical approximation of the decision rule in calibrated or sometimes estimated RBC models.

policy is the only game in town, i.e. the "worst-case" scenario, what are the fiscal sustainability requirements and can we reject the null hypothesis of unsustainability (i.e. violation of these requirements)? Rejection of unsustainability based on this "worst-case" scenario might be then considered as credible evidence of sustainability.

3.1 Model

Stochastic real endowment. Total output Y_t is following a unit-root with drift:

$$Y_t = Y_{t-1}(1 + \gamma + \varepsilon_t^\gamma) \quad (11)$$

where $\gamma > 0$ is the long-run growth rate of output and ε_t^γ is an i.i.d random shock to the growth rate.

Representative household. Representative household's preferences are represented by the utility function $u(\cdot)$ which is strictly increasing ($u'(\cdot) > 0$) and concave ($u''(\cdot) < 0$) and a subjective discount factor β . At each period, consumer chooses consumption C_t and buys public bond B_t at a price $(1 + r_t)^{-1}$ in order to maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$$

subject to the following budget constraint:

$$C_t + (1 + r_t)^{-1} B_t = B_{t-1} + Y_t - T_t$$

and transversality condition:

$$\lim_{T \rightarrow +\infty} \mathbb{E}_t \frac{B_{t+T}}{1 + r_{t,T+1}} \geq 0$$

with $(1 + r_{t,T+1})^{-1}$ being the T -period ahead real interest rate. First order conditions of the representative consumer's maximization program yield the standard Euler equation:

$$(1 + r_t)^{-1} = \beta \mathbb{E}_t \frac{u'(C_{t+1})}{u'(C_t)} \quad (12)$$

Equation (12) evaluates the stochastic discount factor $Q_{t,1} \equiv \beta \frac{u'(C_{t+1})}{u'(C_t)}$ at the optimal solution of the representative consumer's program, which is the common pricing kernel of any asset in the economy. Hence, a j -period public bond has a price $(1 + r_{t,j})^{-1} = \mathbb{E}_t Q_{t,j}$ with $Q_{t,j} = \beta^j \frac{u'(C_{t+j})}{u'(C_t)}$.

Government. Government spends G_t and collects lump-sum taxes T_t . At each start of period t , government carries one-period public bonds B_{t-1} and it will issue B_t at a price $(1 + r_t)^{-1}$ at end of period. Thus, government faces the following one-period budget constraint:

$$(1 + r_t)^{-1} B_t = (G_t - T_t) + B_{t-1} \quad (13)$$

with $S_t \equiv T_t - G_t$ representing the primary budget balance. Under balanced growth, all variables in level grow at rate γ_t , thus we rewrite the government budget constraint in terms of output ratios:

$$b_t = \frac{1 + r_t}{1 + \gamma_t} b_{t-1} - (1 + r_t) s_t \quad (14)$$

where b_t is the end-of-period debt-output ratio, s_t is the primary surplus-output ratio, r_t and y_t are respectively the real interest rate and the growth rate of real output.

Preventing government from running a Ponzi scheme against its creditor implies the following Present-Value Budget Constraint (PVBC). Following [Bohn \(1995\)](#), we write the PVBC using the stochastic discount factor in order to account for uncertainty and consumer's risk-aversion:

$$B_t = \sum_{i=0}^{+\infty} \mathbb{E}_t [Q_{t,i} S_{t+i}] \quad (15)$$

which is equivalent to the following transversality condition (TC):

$$\lim_{T \rightarrow +\infty} \mathbb{E}_t [Q_{t,T+1} B_{t+T}] = 0 \quad (16)$$

Both the PVBC and TC must hold with equality since the representative consumer cannot run a Ponzi Scheme against government ([Bohn, 1995](#)).

We assume the following Markov-switching fiscal policy rule:

$$s_t = \gamma(z_t) b_{t-1} + \mu_t(z_t) \quad (17)$$

Regime-switching parameter $\gamma(z_t)$ represents the feedback effect of the initial public debt-output ratio b_{t-1} on primary surplus-output ratio *conditional on* fiscal regime z_t . Fiscal regimes are then defined as:

$$\gamma(z_t) = \begin{cases} \gamma_S > 0 & \text{if } z_t = 1 \quad (\text{Sustainable Regime}) \\ \gamma_{NS} \leq 0 & \text{if } z_t = 0 \quad (\text{Unsustainable Regime}) \end{cases} \quad (18)$$

During sustainable regimes ($\gamma_S > 0$) primary balance improves following a debt increase while it does not improve or even worsen during unsustainable regimes ($\gamma_{NS} \leq 0$)¹⁰. Finally, we define $\mu_t(z_t)$ by:

$$\mu_t(z_t) = \alpha(z_t) + \alpha_y(z_t) \hat{y}_t + \alpha_g(z_t) \hat{g}_t + \sigma(z_t) \varepsilon_t^s \quad (19)$$

where \hat{y}_t is the output gap, \hat{g}_t is temporary public spending, $\alpha(z_t)$ is a regime-switching constant, $\sigma(z_t)$ is the regime switching standard-error associated to an i.i.d distributed shock $\varepsilon_t^s \sim \mathcal{N}(0,1)$. We assume regime-switching to be stochastic and exogenous, following a hidden two-state Markov process z_t describing fiscal regimes. The use of a Markov-switching model rather than endogenous or threshold-switching models represents an agnostic way of modelling regime changes of fiscal policy without making any critical assumption about what drives fiscal regime shifts. In addition, given our economy is purely Ricardian, we also assume that fiscal regime z_t is independent of real output's growth rate.

Define $\gamma = (\gamma_S \ \gamma_{NS})$ a row-vector containing regime-specific parameters and $Z_t = (z_t \ 1 - z_t)^T$ a column-vector associated to the Markov process z_t . Hence, we can define the scalar $\gamma(z_t)$ by:

$$\gamma(z_t) \equiv \gamma Z_t = \begin{pmatrix} \gamma_S & \gamma_{NS} \end{pmatrix} \times \begin{pmatrix} z_t \\ 1 - z_t \end{pmatrix} \quad (20)$$

¹⁰ [Canzoneri et al. \(2010, p.959\)](#) discuss empirical results of [Davig and Leeper \(2007a, 2011\)](#) and note that a negative coefficient on lagged debt in the fiscal rule may be difficult to interpret since "regardless of whether the fiscal rule is Ricardian or non-Ricardian, we would expect a positive estimated coefficient". Indeed, [Cochrane \(2001\)](#) shows there exist a positive correlation between primary surplus and initial debt at *equilibrium*, even when fiscal policy is active (with primary surplus following an AR(1) process). Still, empirical research on regime-switching fiscal policy rules provides some evidence of periodic *negative* feedback effect, see [Davig and Leeper \(2011\)](#) and [Afonso and Toffano \(2013\)](#) for instance; these empirical results motivate our specification of unsustainable fiscal regimes by $\gamma_{NS} \leq 0$.

Markov process z_t is associated to a transition matrix P whose elements are $p_{ij} \equiv \mathbb{P}(z_t = i | z_{t-1} = j)$ for all $(i, j) \in \{0, 1\}$ such that:

$$Z_t = PZ_{t-1} + v_t \quad \text{with} \quad v_t \equiv Z_t - \mathbb{E}_{t-1}[Z_t] \quad (21)$$

We assume z_t to be an ergodic Markov process¹¹ implying that $\mathbb{E}_t Z_{t+j} = P^j Z_t$ converges to a unique ergodic distribution π :

$$P^j Z_t \xrightarrow{j \rightarrow +\infty} \pi \quad (22)$$

where $\pi = (\pi_S \ \pi_{NS})^T$ is the column-vector of ergodic probabilities associated to each fiscal regime. Ergodic probabilities are defined by:

$$\pi_i = \frac{1 - p_{jj}}{(1 - p_{ii}) + (1 - p_{jj})} \quad (23)$$

for all $(i, j) \in \{0, 1\}$. Hence, using equations (20) and (22), the conditional expectation at time t of feedback parameter $\gamma(z_t)$ converges toward its *unconditional* expectation, i.e. ergodic (or long-run) value:

$$\mathbb{E}_t[\gamma(z_{t+j})] = \gamma P^j Z_t \xrightarrow{j \rightarrow +\infty} \gamma \pi \quad (24)$$

3.2 No-Ponzi Game condition

Following [Bohn \(1998\)](#), we derive sufficient condition on the sequence $\{\gamma(z_{t+i})\}_{i=0}^{\infty}$ such that Present-Value Budget Constraint (15) and Transversality condition (16) hold. Denoting the j -periods growth-adjusted stochastic discount factor by

$$\tilde{Q}_{t,j} \equiv Q_{t,j} \prod_{i=0}^{j-1} (1 + y_{t+i}) \quad (25)$$

allows us to rewrite Transversality condition (16) in terms of debt-output ratio by:

$$\lim_{T \rightarrow +\infty} \mathbb{E}_t [\tilde{Q}_{t,T+1} b_{t+T}] = 0 \quad (26)$$

Then, using the regime-switching fiscal policy rule (17) and iterating on the flow budget constraint of government (14) up to date $t + T$, we obtain an expression for expected present-value debt-output ratio $\mathbb{E}_t [\tilde{Q}_{t,T+1} b_{t+T}]$ which explicitly depends on $\{\gamma(z_{t+i})\}_{i=0}^{\infty}$. Finally, we find a sufficient condition on the regime-switching fiscal policy rule to satisfy the No-Ponzi Game condition, that allows us to conclude to the following proposition.

Proposition 1 (No-Ponzi Game) *In a dynamically efficient economy, and provided that $\mu_t(z_t)$ is bounded, a sufficient condition such that transversality condition (26) holds is*

$$\gamma \pi > 0 \quad (27)$$

with $\gamma \pi \equiv \gamma_S \pi_S + \gamma_{NS} \pi_{NS}$ being the unconditional expectation of $\gamma(z_t)$. Using the definition of ergodic probabilities (23) and denoting expected duration of regimes by $d_i = \frac{1}{1 - p_{ii}}$, we can express condition (27) by

$$\gamma_S > |\gamma_{NS}| \frac{d_{NS}}{d_S} \quad (28)$$

¹¹ Any Markov process is ergodic as long as $p_{ii} < 1$ and $p_{ii} + p_{jj} > 0$ for all $(i, j) \in \{0, 1\}$ ([Hamilton, 1994](#), Chap. 22), meaning there is no absorbing state.

Proof 1 See appendix A.1.

To understand this condition, let's consider the following approximation of transversality condition when $T \rightarrow +\infty$:

$$\mathbb{E}_t[\tilde{Q}_{t,T+1}b_T] \approx (1 - (1+y)\gamma\pi)^T b_t \quad (29)$$

Following Bohn (2008), consider a Ponzi Scheme such that $\{s_t\}_{t=0}^{\infty} = 0$. This Ponzi Scheme implies debt-output ratio growing at a rate $\frac{r_t - y_t}{1 + y_t}$. As a consequence the limit value of future discounted debt-output ratio is equal to initial debt-output ratio (which violates the transversality condition):

$$\mathbb{E}_t[\tilde{Q}_{t,T+1}b_{t+T}] = b_t \quad (30)$$

Thus, $\gamma\pi > 0$ implies the reduction of $\mathbb{E}_t[\tilde{Q}_{t,T+1}b_{t+T}]$ by a factor $(1 - (1+y)\gamma\pi)^T$ relative to a Ponzi Scheme. Saying it differently: the average growth rate of debt-output ratio is reduced by a factor $(1 - (1+y)\gamma\pi) > 0$.

Condition (27) states that a regime-switching fiscal policy has to satisfy the NPG condition *on average*, that is, sustainable regimes have to be frequent enough to balance unsustainable regimes in the long-run. Ruling out a Ponzi Scheme means that the longer unsustainable regimes are *vis-à-vis* duration of sustainable regimes, the larger primary deficits are during unsustainable regimes, then the larger the required reaction of primary surplus to debt has to be during sustainable regimes. Still, provided (28) holds, fiscal policy can be periodically unsustainable *and* satisfying its PVBC.

3.3 Debt-stabilizing condition

A stronger constraint on fiscal policy would require that debt-output ratio must be stationary at a sufficiently low level, below a "fiscal limit" defined as following. We assume an exogenous upper-bound on the primary surplus-output ratio such that $s_t \leq s^{max}$. This assumption can be justified by tax evasion, following Daniel (2014) or more generally by the political inability and/or unwillingness to reduce public spending and increase taxes, following Daniel and Shiamptanis (2013)¹². This directly implies the existence of a maximum level of debt-output ratio, i.e. a fiscal limit, such that:

$$b^{max} = s^{max} \sum_{i=0}^{+\infty} \mathbb{E}_t[\tilde{Q}_{t,i}] \quad (31)$$

Thus, for $b_t > b^{max}$ fiscal policy would be necessarily running a Ponzi Scheme against creditors. Since Proposition 1 does not rule out explosive path for debt-output ratio, then a necessary and sufficient condition for fiscal sustainability, in presence of a fiscal limit on debt-output ratio, would be a debt-stabilizing fiscal rule around a steady-state level below the fiscal limit.

Therefore, a regime-switching fiscal rule implies that debt-output ratio follows a Markov-switching autoregressive process, defined by equations (17) and (14):

$$b_t = \phi(z_t)b_{t-1} + u_t(z_t) \quad (32)$$

where

$$\phi(z_t) = \frac{1 + r_t}{1 + y_t} (1 - (1 + y_t)\gamma(z_t)) \quad \text{and} \quad u_t(z_t) = -(1 + r_t)\mu_t(z_t).$$

¹²In a framework with distortionary taxation, the fiscal limit would arise endogenously from the existence a dynamic Laffer curve, see Bi (2012); Bi and Leeper (2013). But since we do not intend to derive an endogenous fiscal limit, we only consider lump-sum taxation and assume an exogenous upper-bound on the primary surplus-output ratio.

A sufficient condition for (strict) stationarity of stochastic processes like (32) is given by [Kesten \(1973\)](#), from which we deduce the following proposition.

Proposition 2 (Debt-stabilizing condition) *A sufficient condition for a (strictly) stationary debt-output ratio is*

$$\gamma\pi > \frac{r-y}{1+y} \quad (33)$$

which can be expressed in term of expected durations

$$\gamma_S > |\gamma_{NS}| \frac{d_{NS}}{d_S} + \frac{r-y}{1+y} \frac{d_S + d_{NS}}{d_S} \quad (34)$$

Proof 2 See appendix A.2.

Provided conditions (33) or (34) hold, then public debt-output has an ergodic mean:

$$\mathbb{E}[b_t] = \frac{-\mathbb{E}[(1+r_t)\alpha(z_t)] + \text{Cov}(\phi(z_t), b_{t-1})}{\mathbb{E}[1-\phi(z_t)]} \quad (35)$$

where $\mathbb{E}[\alpha(z_t)] < 0$ is the ergodic value of $\alpha(z_t)$.

As long as growth-adjusted real interest rate is positive, a debt-stabilizing condition is stricter than the NPG condition. During sustainable regimes, the required reaction of primary surplus to initial debt must be large enough to compensate for both primary deficits during unsustainable regimes, weighted by the ratio of expected durations, and the growth-adjusted real interest rate, weighted by the inverse fraction of (expected) time spent in sustainable regimes. On the contrary, when $r < y$, condition (34) could eventually imply government is violating NPG condition (28) which is the minimum requirement for fiscal sustainability. Since history provides numerous examples of $r < y$, this illustrates why testing stationarity of debt-output ratio may sometimes be misleading as a test of fiscal sustainability. As a result, NPG condition and debt-stabilizing condition would be complements rather than substitutes: a stationary public debt-output ratio does not always rule out Ponzi Schemes.

The assumption of the existence of different fiscal regimes may, in general, imply that public debt-output ratio can periodically follow an explosive path. To see why, let us consider the example of [Canzoneri et al. \(2001\)](#) and assume $\gamma_{NS} = 0$. We find exactly the same proposition they made: based on equation (28), any infrequent $\gamma_S > 0$ would be sufficient to rule out Ponzi Schemes. Yet this equilibrium does not ensure a stable debt-output ratio, that is public debt is $I(1)$. For a stable debt-output ratio, assuming $r - y > 0$ and $\gamma_{NS} = 0$, a regime-switching fiscal policy must satisfy the following condition, from equation (34):

$$\gamma_S > \frac{r-y}{1+y} \frac{d_S + d_{NS}}{d_S} \quad (36)$$

For any $\gamma_{NS} < 0$ the condition on γ_S would be stronger. Under a regime-switching debt-stabilizing fiscal policy, debt-output ratio becomes periodically explosive, and explosive regimes can be really frequent without necessarily implying debt-output is globally non-stationary.

Periodic explosive dynamics of public debt, implied by existence of fiscal regimes, has critical consequences on regime-switching policy rules, not only on $\gamma(z_t)$ but also on the constant $\alpha(z_t)$. Rewriting equation (17) in terms of deviations of primary balance and public debt from their respective steady-state values $s^*(z_t) = (s_S^*, s_{NS}^*)$ and $b^*(z_t) = (b_S^*, b_{NS}^*)$ yields:

$$s_t - s^*(z_t) = \gamma(z_t)(b_{t-1} - b^*(z_t)) + \alpha_y(z_t)\hat{y}_t + \alpha_g(z_t)\hat{g}_t + \sigma(z_t)\varepsilon_t^s \quad (37)$$

from what we deduce that $\alpha(z_t)$ is equal to:

$$\alpha(z_t) = s^*(z_t) - \gamma(z_t)b^*(z_t) \quad (38)$$

From the existence of periodic explosive dynamics of public debt, we can deduce that α_{NS} (i.e. the value of the constant during unsustainable regimes) would be expected to be equal to zero, in the case $\gamma_{NS} < 0$. First, unsustainable fiscal regime and implied explosive debt-output ratio dynamics are not compatible with any steady-state debt-output level, hence $b_{NS}^* = 0$. Second, primary balance could either stationary or non-stationary. In the case of $\gamma_{NS} < 0$, it would be necessarily non-stationary during unsustainable regimes from the fact the two variables would be negatively cointegrated with $\{b_t\}$ being non-stationary, implying $s_{NS}^* = 0$. Otherwise if $\gamma_{NS} = 0$, $\{s_t\}$ could be stationary and then s_{NS}^* would be eventually significantly different from zero.

On the contrary, during sustainable regimes, primary surplus-output ratio s_t and debt-output ratio b_t would admit steady-state values. Provided condition (34) holds, we would expect s_S^* to be equal to the debt-stabilizing primary surplus ratio, for a stationary debt-output target ratio b_S^* :

$$s_S^* = \frac{r-y}{1+y} b_S^* \quad (39)$$

which implies:

$$\alpha_S = \left(\frac{r-y}{1+y} - \gamma_S \right) b_S^* < 0 \quad (40)$$

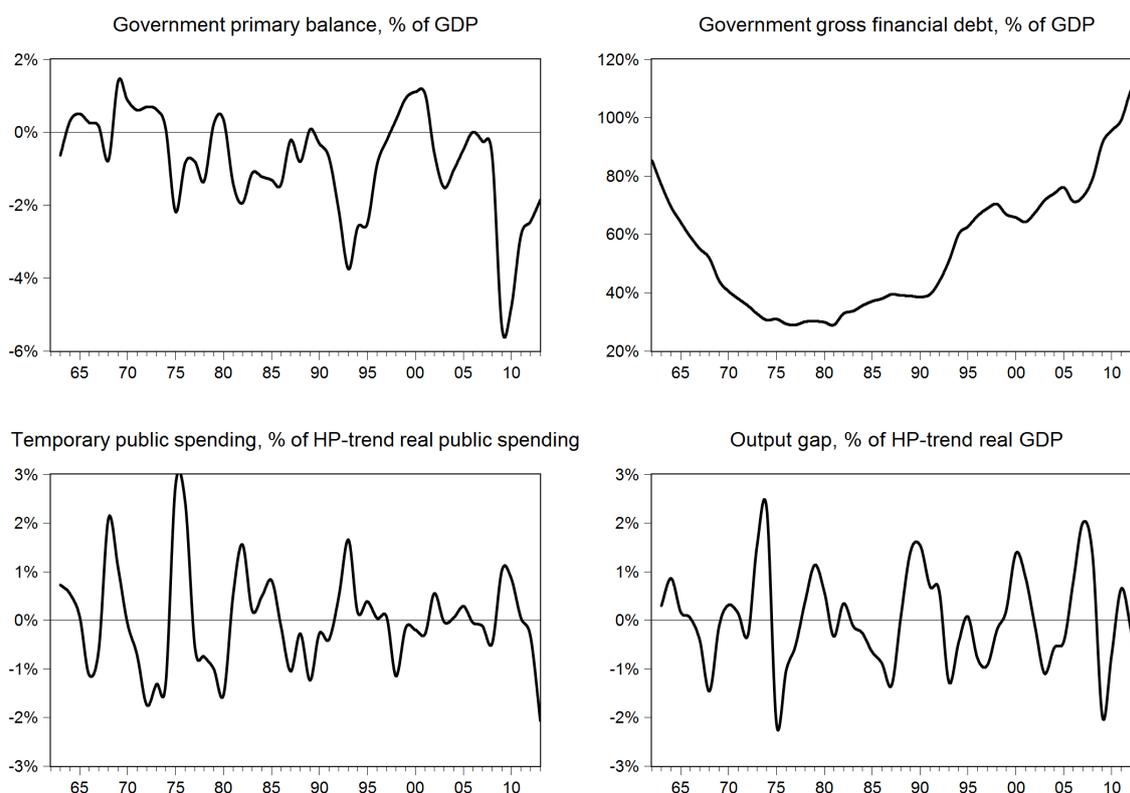
provided that condition (33) holds, which would account for negative estimates of α_S but also $\mathbb{E}[\alpha(z_t)] = \pi_S \alpha_S < 0$ if $\gamma_{NS} < 0$.

4 Empirical analysis

We apply Regime-Switching MBS analysis to French data. Recent empirical investigation about the fiscal sustainability of French public finances has given rise to contradictory outcomes: some recent papers (Afonso, 2005; Lamé et al., 2014; Schoder, 2014) either did not find evidence of a sustainable fiscal policy in France, or reached mixed evidence (Afonso and Jalles, 2016; Chen, 2014; Fincke and Greiner, 2012; Weichenrieder and Zimmer, 2014); an earlier study even found that fiscal policy in France was sustainable (Greiner et al., 2007). The observation that French sovereign interest rates have been historically low during the European sovereign-debt crisis also conveys some information about lenders' seemingly expectations that France's fiscal policy is on a sustainable path.

We argue that this apparent contradiction can be attributed to the lack of account for regime-switching fiscal policies in empirical papers. To assess our argument, we develop a two-stage empirical strategy. First, we estimate various fiscal rules following Bohn's MBS tests. From these tests, we conclude that French public debt is not sustainable. Second, we estimate a Markov-switching fiscal policy rule and perform a Regime-Switching MBS test. The latter outcomes challenge the former results obtained with standard techniques. We conclude that omitting fiscal regime-switches may lead to reject mistakenly French sustainability. A broader, model-based and non-linear, approach, would better fit the reality of fiscal policies in different states of nature and bring two informations: not only overall (un)sustainability but also dated sub-periods of (possible) unsustainability.

Figure 1: Dataset overview, France (1962-2013)



4.1 Dataset

The choice of annual data is guided by two arguments. First, fiscal sustainability can only be appreciated in the long-run since governments are (almost surely) infinitely-lived agents, unlike households, firms or private financial institutions. PVBC or stationary debt-to-GDP ratios might only be satisfied in the long-run – over half a century, or more. Regarding data availability for France, we are forced to renounce using *true* quarterly data which are available from 1995-Q4 only for public debt. It is possible to build a quarterly measure for public debt using interpolation methods and quarterly government budget balance¹³. Still, a second argument prevents us from using quarterly data: fiscal policy decisions are taken on an annual basis, despite some infra-annual adjustments. Thus, using quarterly data may result in spurious results as it may add noise to the *true* response of primary budget balance to initial stock of debt.

This paper uses the longest time series available for French public debt. Indeed, because of changes in national accounts systems, it is relatively hard to find historical data on French public debt. Most of available time series (in particular, those using Maastricht debt definitions) start by 1978. The IMF Historical Public Debt Database (HPDD) proposes a long-run time series for public debt, but still with missing observations for years 1978 and 1979, because of national accounting issues. Thus, regarding public debt, we use the OECD government total gross financial liabilities rather than the Maastricht definition of gross public debt since the OECD series goes back to 1969. **Passage à modifier : expliquer la construction par rétropolation, justifier la négligence des stock-flow adjustments, etc.** Second, we complete this series by backward induction using the government overall budget balance which

¹³Lamé et al. (2014) report the use of recalculated quarterly data of net French public debt, though on a shorter time span (1980Q1-2007Q4) than the one used in this paper.

is approximatively equal to the variation of gross financial liabilities. Regarding time convention in national accounts, public debt stock is the *end-of-period* stock of debt.

Overall budget balance and primary budget balance (budget balance *minus* interests paid) are taken from OECD database for years 1977-2013; observations for years 1963 to 1977 are completed using data collected by [Creel and Le Bihan \(2006\)](#), from French National Institute for Statistics and Economic Studies (INSEE). We build time series for output gap and temporary government spending by detrending and removing the cyclical component of real GDP and real government spending using the HP filter. Regarding the estimation of output gaps, many competing techniques are available and their relative strengths and weaknesses still discussed (see [Cotis et al. \(2005\)](#), for a survey of estimation methods). Our choice of the HP-filtered method has been motivated by its easiness, fastness and recent use by [Fincke and Greiner \(2012\)](#) and, with more sophistication, by [Borio et al. \(2014\)](#). To address the endpoint bias problem, we also add univariate 3-years ahead forecasts for each series, using ARIMA models, prior to filtering and then dropping the last 3 observations (we also drop the first 3 observations at the beginning of the sample); such a "mechanic" correction of the endpoint bias is used, for instance, by the European Commission for its HP-trend GDP estimates. Finally, our dataset covers 51 years of annual data, from 1963 (1962, for gross public debt) to 2013.

4.2 Model-Based Sustainability tests

We estimate various specifications of a standard fiscal policy rule searching for a strictly positive and significant feedback effect between lagged level of public debt and primary surplus, in percent of GDP. We produce OLS estimates as a benchmark for comparison with Regime-Switching estimates. We specify the following fiscal rule, based on equation (5):

$$s_t = \gamma b_{t-1} + \beta X_t + \varepsilon_t \quad (41)$$

where the dependent variable s_t is the primary balance-to-GDP ratio, b_{t-1} is the public debt-to-GDP ratio at end of period $t - 1$ and X_t is a vector of control variables. It includes a constant, output gap \hat{y}_t , cyclical government spending \hat{g}_t , as suggested by [Bohn \(1998\)](#). Then, we include a dummy variable FinCrisis_t equal to one for years 2008–2013 in order to account for severe recession years. To account for potential non-linearities regarding the level of debt, we also estimate fiscal rules as polynomial functions of debt-to-GDP ratio following [Bohn \(1998\)](#) and [Mendoza and Ostry \(2008\)](#). Finally, we account for a potential deterministic time trend, as suggested by previous unit-root and stationarity tests. In presence of serial correlation in the residuals, we correct for serially correlated residuals of order one or two, depending on the model estimated.

Table 1 presents the results. Based on OLS estimation of constant-parameters fiscal policy rules, we find no evidence in favor of fiscal sustainability¹⁴. Models (1)–(2) give no positive feedback effect, but rather negative though non-significant estimates for γ . We do not find any evidence in favor of a polynomial specification of the fiscal policy rule, since coefficients on debt b_{t-1} and quadratic debt b_{t-1}^2 are never significant. Still, point estimates for polynomial specifications would imply a "flattening" of the fiscal policy rule for high debt-output ratio.

Unit-root and stationarity tests (available upon request) conclude to the potential presence of deterministic time trends respectively negative in s_t and positive in b_t . Thus we control for a deterministic

¹⁴This result contrasts with [Fincke and Greiner \(2012\)](#) who find a significant positive reaction of the primary surplus to debt. Two differences with our approach are worth mentioning. First, Fincke and Greiner do not strictly reproduce Bohn' fiscal rule: they limit cyclical public spending to spending related to the social insurance system though some of these expenditures may be structural; second, their sample is shorter (1970-2008) than ours.

Table 1: Constant-parameters Fiscal policy rules, OLS estimates

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Initial Debt b_{t-1}	-0.0121 (-0.71)	-0.0058 (-0.35)	0.0283 (0.93)	0.0300* (-1.72)	0.0962 (1.50)	0.0547 (0.86)	0.0735 (1.38)
Quadratic debt b_{t-1}^2	-0.0555 (-1.18)	-0.0429 (-0.87)	-0.0367 (-0.86)
Constant	-0.0025 (-0.24)	-0.0052 (-0.57)	0.0014 (0.16)	-0.0065 (-0.96)	-0.0190 (-1.01)	-0.0219 (-1.15)	-0.0179 (-1.19)
Output gap \hat{y}_t	0.4190*** (3.38)	0.3807*** (3.23)	0.4800*** (3.91)	0.4527*** (3.56)	0.4565*** (3.64)	0.4163*** (3.21)	0.4360*** (3.38)
Temporary spending \hat{g}_t	-0.4053*** (-3.18)	-0.3667*** (-3.09)	-0.3448*** (-2.73)	-0.3763*** (-2.91)	-0.3754*** (-2.85)	-0.4147*** (-3.08)	-0.3982*** (-2.98)
FinCrisis $_t$.	-0.0179*** (-2.95)	.	-0.0160** (-2.25)	.	-0.0112 (-1.37)	-0.0131 (-1.70)
Trend	.	.	-0.0009 (-1.55)	-0.0006** (-2.16)	-0.0008 (-1.61)	.	-0.0006** (-2.06)
DW	1.98	1.99	1.70	1.87	1.68	1.81	1.83
Adj. R^2	0.70	0.75	0.72	0.73	0.72	0.70	0.72
Observations	49	49	50	50	50	50	50

Notes: t-stats are in parentheses. Results are significant at 1% level (***), 5% level (***) and 10% level (*). Models (1)–(2) are controlling for second-order serial correlation in the residuals. Model (3)–(7) control for first-order serial correlation in the residuals.

trend in equation (41), in models (3)–(5) and (7). When estimating the fiscal rule with a time trend, the feedback coefficient on initial debt turns out to be positive, but never significant at 5% level. Only model (4) shows a positive but weakly significant (at 10% level) feedback response of primary surplus to initial debt. Moreover, deterministic trends enter negatively in all equations, which would imply $\lim_{t \rightarrow +\infty} s_t = -\infty$, thus obviously violating the PVBC. Still, a significant and negative time trend in surplus-to-GDP ratio remains puzzling.

4.3 Regime-Switching Model-Based Sustainability test

We estimate the following Markov-switching fiscal rule by direct maximisation of the log likelihood (Hamilton, 1989):

$$s_t = \gamma(z_t)b_{t-1} + \alpha(z_t) + \alpha_y(z_t)\hat{y}_t + \alpha_g(z_t)\hat{g}_t + u_t \quad (42)$$

where s_t is primary balance-to-GDP ratio, b_{t-1} is public debt-to-GDP ratio at end of period $t - 1$, \hat{y}_t and \hat{g}_t are measures of output gap and temporary real government spending. Except the autoregressive residuals and the error variance¹⁵, all remaining parameters can periodically shift between two values, according to a hidden two-state Markov-process z_t .

Numerical optimization of the log likelihood function is usually difficult, raising identification issues, so we choose the following estimation strategy¹⁶. First, we estimate the most general model, allowing *all parameters, including error variance* to switch between regimes 1 and 2, thus being agnostic on the true structural form of the regime-dependent fiscal rule. At this stage, if the maximization algorithm converges, we can already appreciate how precise the resulting estimates are, both across regimes and in the long-run through the computation of the ergodic value of each parameter. This can be achieved through basic t-statistics and F-statistics analysis. We also look carefully at estimated regimes' properties: transition probabilities associated to the Markov process and filtered and smoothed regime probabilities. We

¹⁵To account for first-order serial correlation in the data, we assume: $(1 - \rho)u_t = \sigma \varepsilon_t$ with an i.i.d error term $\varepsilon_t \sim \mathcal{N}(0, 1)$

¹⁶We also randomize the estimation algorithm by drawing 500 starting values and running initial ML estimations with 100 iterations on each draw, in order to reduce the dependence of the ML algorithm on starting values and thus the risk of reaching a local maximum of the log likelihood function. The main estimation algorithm starts using the starting values for which the maximization algorithm reached the highest value of the log likelihood function among the 500 initial random draws.

check, in particular, if they are consistent with historical knowledge on fiscal policy shifts, and if they are sufficiently persistent, regarding the timing of fiscal policy. If any subset of parameters were non-significantly different from zero in *both regimes* it would be a strong motivation to estimate a restricted model in which this subset of parameters would be regime-invariant. Thus, if any restricted model can be successfully estimated, that is, if the maximization algorithm successfully converges, then the same procedure as described before can be applied. Given the short length of the sample, we acknowledge that ML estimates must be considered with caution. Still, given the potential presence of unit-root in the debt-to-GDP ratio, with stationary primary balance-to-GDP ratio, OLS estimates of a *constant* fiscal policy rule would be equally dubious. But this paper builds on the idea a non-linear fiscal policy behavior implies periodical explosive dynamics of public debt-to-GDP, without necessarily implying either instability of public debt-to-GDP ratio, or Ponzi schemes, in the *long run*. As a result of our estimation strategy, equation (42) seems to be the best specification of the Markov-switching fiscal policy rule.¹⁷

Table 2 presents estimation results of equation (42). We report estimated parameters for each regime and we also compute implied long-run estimates of regime-switching parameters using ergodic probabilities. Standard deviations of long-run estimates are obtained using standard deviations and covariance of regime-specific parameters: for any regime-switching parameter $\alpha(z_t)$ which takes two values (α_1, α_2) , with associated standard deviations $(\sigma_{\alpha_1}, \sigma_{\alpha_2})$ and covariance $\text{Cov}(\alpha_1, \alpha_2)$, we compute the long-run (ergodic) estimate α using ergodic probabilities (π_1, π_2) by:

$$\alpha \equiv \alpha_1 \pi_1 + \alpha_2 \pi_2$$

and with standard deviation:

$$\sigma_\alpha \equiv \sqrt{(\pi_1 \sigma_{\alpha_1})^2 + (\pi_2 \sigma_{\alpha_2})^2 + 2\pi_1 \pi_2 \text{Cov}(\alpha_1, \alpha_2)}$$

The results raise some comments. First, France's fiscal policy is well described by a two-state Markov-switching policy rule. One regime is sustainable with a strongly positive and significant correlation between primary balance s_t and initial debt b_{t-1} , implying a stable debt-to-GDP ratio, while the other one shows a non-significant positive correlation. As expected, the constant is significantly negative in the sustainable regime, which is consistent with a debt-stabilizing fiscal policy, while non-significant in the unsustainable regime, as explained in section 3. Second, both regimes appear to be strongly persistent with respective expected durations of 8.1 and 11.9 years, respectively for sustainable and unsustainable regimes. This would explain why OLS estimates were inconclusive about the long-run correlation between primary surplus and initial debt in table 1.

Figure 2 represents estimated smoothed and filtered probabilities for regime 1 which we label as sustainable. Results show a succession of periods of unsustainable or sustainable fiscal policies with marked decades. Public finances in the 1970s were sustainable over the most part. In sharp contrast, France's fiscal policy has been mostly unsustainable during the 1980s. Still, filtered probabilities show a small and transitory increase in the probability of being in a sustainable regime during the so-called "Tournant de la rigueur" of 1983-1986 when the Socialist government turned to disinflation and deficit-reduction policies. Overall, results are consistent with a comprehensive and historical analysis of France's fiscal policy. In the 1990s, results report that France's fiscal policy became gradually sustainable (or passive

¹⁷In a previous version of this paper, we estimated an alternative specification with regime-heteroskedasticity. While the MLE successfully converged, our results appeared a posteriori to be highly dependent on initial values for estimation algorithm and they might be a local maximum of the log likelihood function. That is the main reason why we increased the number of random draws at the start of the estimation process. We also estimated a model with a regime-invariant deterministic trend. We conclude to a non-significant (at 5% level) deterministic trend, so we choose to exclude the deterministic trend from our baseline specification.

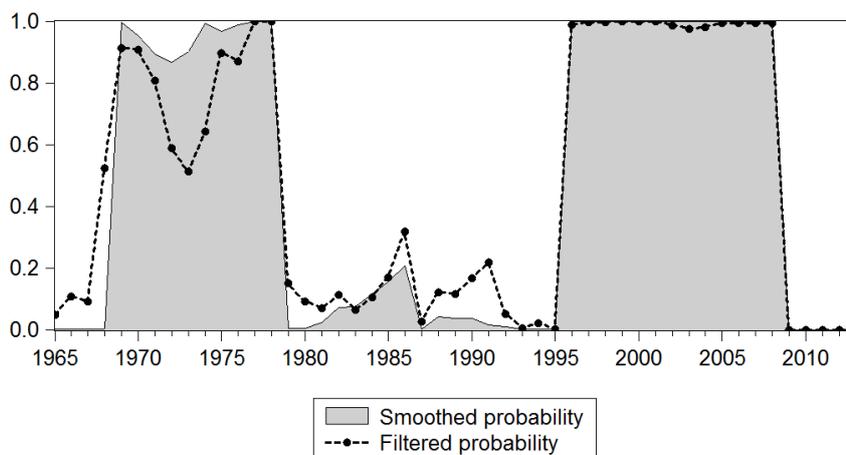
Table 2: Estimated Markov-switching fiscal rule for France (1965–2013)

Regime-switching parameters	Regime 1	Regime 2	Long-run estimates
Debt b_{t-1}	0.0889*** (3.08)	0.0017 (0.07)	0.0370 (1.51)
Constant	-0.0608* (-1.90)	-0.0256 (-0.84)	-0.0399 (-1.30)
Output gap \hat{y}_t	0.4214*** (3.30)	0.2894** (2.39)	0.3429*** (4.08)
Temporary spending \hat{g}_t	-0.0637 (-0.50)	-0.5491*** (-5.33)	-0.3524*** (-4.20)
Regime-invariant parameters			
AR(1)	0.9443*** (13.10)	0.9443*** (13.10)	-
Standard-error σ	0.0046*** (8.34)	0.0046*** (8.34)	-
Regimes properties			
	Transition probabilities p_{ij}	Ergodic probabilities π_i	Expected durations (years) d_i
i=1	0.8770	0.4051	8.1
i=2	0.9162	0.5949	11.9
Durbin-Watson statistic	1.7724	Akaike info criterion	-6.8723
Log likelihood	180.3709	Schwarz criterion	-6.4090
Number of observations	49	Hannan-Quinn criter.	-6.6965

Notes: t-stats are in parentheses. Results are significant at 1% level (***), 5% level (***) and 10% level (*). We control for regime-invariant first-order serial correlation in the residuals. Basically, estimates for $\hat{\sigma}$ were obtained as $\log \hat{\sigma}$: consequently, standard errors and t-statistics are obtained applying the Delta method. For regime-switching parameters we compute "long-run estimates" as defined earlier. We report estimates for regime-invariant parameters twice in columns "Regime 1" and "Regime 2", for clarity purposes since they are constant in each regime-specific equation.

to use Leeper's terminology) and actually so by 1996, until 2008 and the advent of the Great Recession. This finding supports the view that the Maastricht Treaty and the Stability and Growth Pact (SGP) actually made France's fiscal policy more sustainable, despite it being under an Excessive Deficit Procedure from 2003 to 2007. In contrast with [Weichenrieder and Zimmer \(2014\)](#) who show that Euro membership of France has reduced the responsiveness of the primary surplus to debt, our results show that the 1999-2011 period (Euro membership years in Weichenrieder and Zimmer) was heterogeneous as regards fiscal responsiveness: it was positive until 2008 and then negative.

Figure 2: Estimated sustainable regime, France (1965-2013)



The long-term estimate of $\gamma\pi$ is positive, equal to 0.037 but non-significant (with a p-value equal to 0.1394). Still, this results calls two comments. First, the long-run estimate of $\gamma\pi$ appears non-significant

Table 3: Regime-Switching MBS: unilateral versus bilateral tests

Student tests for...	t-stat	Bilateral test p-value	Unilateral test p-value
No-Ponzi Game condition (28)	3.0841	0.0039	0.0020
Stable long-run debt-to-GDP ratio (34)			
...using apparent interest rate: $\frac{r-y}{1+y} = -3.19\%$	5.8214	p<0.001	p<0.001
...using market interest rate: $\frac{r-y}{1+y} = 0.33\%$	2.8008	0.00801	0.0041

Notes: these Student tests assume γ_{NS} is virtually equal to 0. Both apparent and market real interest rates are ex-post real interest rates, obtained using the implicit GDP deflator from OECD Economic Outlook database.

mainly from the fact estimate of γ_{NS} is strongly non-significant (i.e. with a large estimated standard-error), and thus might be considered as virtually equal to 0. Second, significance tests are not appropriate to test for NPG and debt-stabilizing conditions on $\gamma\pi$ since they are *bilateral* tests. On the contrary, Propositions 1 and 2 call for *unilateral* tests for which critical values are lower with respect to bilateral tests.¹⁸

Beyond the appropriateness of the bilateral vs. unilateral test, a question arises: what is the appropriate measure for the growth-adjusted real interest rate? Should we use the *market value*¹⁹ of long-term real interest rate or the *apparent*²⁰ real interest rate on public debt? Using the market value of long-term interest rate yields a mean growth-adjusted real interest rate equal to 0.33% over the sample while using apparent interest rate yields a growth-adjusted real interest rate equal to -3.19%. This difference mainly comes from the strong divergence between these two rates²¹ before the late 1980s, see figure 3: the apparent rate was much smaller than the market rate. Figure 3 shows that France benefited from *large* and *negative* growth-adjusted real interest rates until 1980, if we consider the market long-term rate, and until 1987 if we consider the apparent rate. This snowball effect in reverse explains *other things equal* the rapid and persistent decrease in France's public debt-output ratio during the 1960s and the 1970s. Consequently, we propose to test for a stable debt-output ratio using two different measures of the growth-adjusted real interest rate on public debt: the average apparent real rate and the average market long-term real rate.

Assuming that γ_{NS} is virtually equal to 0, we find significant evidence that France's fiscal policy not only satisfies the No-Ponzi Game condition (Proposition 1) but also the Debt-stabilizing condition (Proposition 2), using both measures of the interest rate. In other words, given past history of French fiscal policy and fiscal regimes, we find significant evidence that France's fiscal policy might have been sustainable overall the period 1965-2013, despite a prolonged period of unsustainability from 1979 to 1995.

Finally, using point estimates reported in table 2 and historical average for the real interest rate and real GDP growth rate, table 4 reports the expected debt-to-GDP ratios, neglecting the covariance terms, under two alternative scenarios. In scenario 1, we suppose sustainable regimes last longer and we increase their expected duration (or persistence) while keeping the expected duration of unsustainable regimes constant and equal to their estimated value. In scenario 2, we suppose unsustainable regimes are shorter and we decrease their expected duration while keeping the expected duration of sustainable regimes constant and equal to their estimated value.

¹⁸For instance, a bilateral test of the NPG condition on the parameter $\gamma\pi$ is build upon the null hypothesis $\gamma\pi = 0$ against the alternative $\gamma\pi \neq 0$, while the unilateral test is build upon the null hypothesis $\gamma\pi = 0$ against the alternative $\gamma\pi > 0$ which is a more adequate testing hypothesis in the sustainability context.

¹⁹We label "market value" of long-term interest rate the yield on 10-years government bonds.

²⁰Here, we compute the apparent interest rate on public debt the ratio of total interests paid on total gross financial debt, from OECD Economic Outlook data.

²¹Both growth-adjusted measures of real interest rate are computed using real GDP and GDP deflator from OECD Economic Outlook database, hence divergences only come from the nominal rate.

When using the market interest rate, our estimates indicate France's gross public debt-to-GDP ratio would reach an average value of 121% across fiscal regimes, which may be interpreted as too high to prevent sovereign default. First, this approach does not pretend sovereign default would be ruled out with certainty by a debt-stabilizing fiscal policy rule. We agree with [Daniel and Shiamptanis \(2013, p.2308\)](#) who argue that "*a country following a responsible fiscal rule could still encounter solvency problems due to negative shocks or due to future plans which are insolvent. However, a country following a fiscal rule which is not responsible will encounter solvency problems with certainty.*" Using regime-switching models, this paper proposes a new non-linear test to discriminate between obviously unsustainable fiscal policies and most probable sustainable ones, given taking into account fiscal policy can periodically deviate from sustainability requirements. But we do not propose any measure of "fiscal space" or "fiscal vulnerability". Second, this figure cannot be interpreted as a long-run steady-state ratio, in the usual sense. It represents a long-run average between a regime where public debt follows stable dynamics and a regime with explosive public debt. In particular, assuming $d_S \rightarrow +\infty$ or equivalently $d_{NS} = 0$, we obtain the underlying debt-to-GDP target ratio $b_S^* = 71\%$ towards which public debt converges during sustainable regimes²².

As expected, using the apparent interest rate yields a lower expected debt-to-GDP ratio than when using the market interest rate. Given our estimate, the long-run debt-to-GDP ratio would reach 61% in this case. This divergence mainly comes from large negative apparent interest rates during the 1970s. In [table 5](#), we show how the debt-to-GDP ratio vary with the level of the growth-adjusted real interest rate.

5 Conclusions

This paper introduces a Regime-Switching Model-Based Sustainability test for fiscal policy, building on Bohn's Model-Based Sustainability (MBS) framework and on the literature on Markov-switching fiscal policy rules. We assume a Markov-switching fiscal policy rule that stochastically switches between sustainable and unsustainable regimes, where by unsustainable regime we mean a periodic and persistent *negative or null* feedback effect of initial public debt on primary surplus, i.e. violation of Bohn's sustainability condition. Consequently, the public debt-to-GDP ratio becomes periodically and persistently explosive during unsustainable regimes, and fiscal regimes thus matter for fiscal sustainability analysis.

The Regime-Switching MBS test is then applied to French data over a 51-year horizon and compared to standard MBS tests. Our results are threefold. First, we estimate various specifications of Bohn's constant-parameters fiscal policy rule. These estimates do not allow to reject unsustainability: the feedback coefficient on public debt-to-GDP is rarely positive and never significant, according standard MBS tests. Second, we estimate a Markov-switching fiscal policy rule. We identify two different fiscal regimes over the period: one regime is sustainable, with a strong positive and significant feedback effect of lagged public debt-to-GDP on primary surplus-to-GDP, while the second one is unsustainable with no significant feedback effect. In addition, identified fiscal regimes are found to be strongly persistent. In particular, our findings support the view that the Maastricht Treaty and the Stability and Growth Pact (SGP) actually made France's fiscal policy more sustainable, and notably, despite being under an Excessive Deficit Procedure from 2003 to 2007. Third, we perform RS-MBS tests for No-Ponzi Game and Stationary debt-output ratio. We conclude to reject the null hypothesis of a Ponzi Scheme as well as

²²This level cannot be compared to Maastricht criterion of 60% of gross public debt. Indeed, we used the OECD's gross government financial liabilities in our estimates rather than Maastricht gross public debt, for data availability reasons. These two measures of gross public debt differ in terms of debt instruments and valuation methods. As a result, Maastricht debt is generally much lower than gross government financial liabilities.

the null of an explosive public debt-to-GDP ratio, using two measures of the real interest rate paid on public debt.

Future research may now move towards the analysis of the interactions between monetary policy and fiscal policy, in presence of regime-switching policy rules, and their consequences on fiscal sustainability. In contrast with early attempts (see Davig and Leeper (2011) for example), a euro-area country like France cannot be described by a domestic monetary policy. Theoretical research is thus required to match domestic fiscal policy with a federal monetary policy. Beyond that, the question of fiscal sustainability in a Regime-switching MBS framework could be embedded in an open-economy framework. It would introduce another determinant of fiscal sustainability, namely cooperative or non-cooperative fiscal behaviours.

Table 4: Expected regime durations and Debt-GDP ratios using market long-term interest rate

<i>Scenario 1: Increasing expected duration of sustainable regime</i>						
d_S	π_S	π_{NS}	$\gamma\pi$	NPG condition	Stable debt-GDP ratio	$\mathbb{E}[b_t]$
2	0.14	0.86	1.27%	Satisfied	Yes	313%
4	0.25	0.75	2.23%	Satisfied	Yes	178%
7	0.37	0.63	3.28%	Satisfied	Yes	129%
8.1	0.40	0.60	3.59%	Satisfied	Yes	121%
15	0.56	0.44	4.94%	Satisfied	Yes	97%
30	0.72	0.28	6.35%	Satisfied	Yes	84%
60	0.83	0.17	7.41%	Satisfied	Yes	77%
∞	1.00	0.00	8.88%	Satisfied	Yes	71%
<i>Scenario 2: Decreasing expected duration of unsustainable regime</i>						
d_{NS}	π_S	π_{NS}	$\gamma\pi$	NPG condition	Stable debt-GDP ratio	$\mathbb{E}[b_t]$
50	0.14	0.86	1.24%	Satisfied	Yes	322%
30	0.21	0.79	1.89%	Satisfied	Yes	207%
15	0.35	0.65	3.12%	Satisfied	Yes	134%
11.9	0.41	0.59	3.60%	Satisfied	Yes	121%
6	0.58	0.42	5.11%	Satisfied	Yes	95%
3	0.73	0.27	6.49%	Satisfied	Yes	83%
1	0.89	0.11	7.91%	Satisfied	Yes	75%
0	1.00	0.00	8.88%	Satisfied	Yes	71%

Notes: Debt-output ratios are computed from equation (35) neglecting covariance terms. For scenarios 1 and 2, we use average market long-term interest rate $r = 3\%$, average real growth rate $y = 2.68\%$ and $r - y = 0.32\%$ (sample: 1963-2013). In scenario 1, we compute expected debt-output ratios under various values of d_S and for $d_{NS} = 11.9$. In scenario 2, we compute expected debt-output ratios under various values of d_{NS} and for $d_S = 8.1$. All others parameters are constant and equal to point estimates obtained in table 2, except γ_{NS} which is set to 0.

Table 5: Expected regime durations and Debt-GDP ratios using apparent interest rate

<i>Scenario 1: Increasing expected duration of sustainable regime</i>						
d_S	π_S	π_{NS}	$\gamma\pi$	NPG condition	Stable debt-GDP ratio	$\mathbb{E}[b_t]$
2	0.14	0.86	1.27%	Satisfied	Yes	72%
4	0.25	0.75	2.23%	Satisfied	Yes	66%
7	0.37	0.63	3.28%	Satisfied	Yes	62%
8.1	0.40	0.60	3.59%	Satisfied	Yes	61%
15	0.56	0.44	4.94%	Satisfied	Yes	57%
30	0.72	0.28	6.35%	Satisfied	Yes	55%
60	0.83	0.17	7.41%	Satisfied	Yes	53%
∞	1.00	0.00	8.88%	Satisfied	Yes	51%
<i>Scenario 2: Decreasing expected duration of unsustainable regime</i>						
d_{NS}	π_S	π_{NS}	$\gamma\pi$	NPG condition	Stable debt-GDP ratio	$\mathbb{E}[b_t]$
50	0.14	0.86	1.24%	Satisfied	Yes	73%
30	0.21	0.79	1.89%	Satisfied	Yes	68%
15	0.35	0.65	3.12%	Satisfied	Yes	62%
11.9	0.41	0.59	3.60%	Satisfied	Yes	61%
6	0.58	0.42	5.11%	Satisfied	Yes	57%
3	0.73	0.27	6.49%	Satisfied	Yes	54%
1	0.89	0.11	7.91%	Satisfied	Yes	52%
0	1.00	0.00	8.88%	Satisfied	Yes	51%

Notes: Debt-output ratios are computed from equation (35) neglecting covariance terms. For scenarios 1 and 2, we use average apparent interest rate $r = -0.43\%$, average real growth rate $y = 2.68\%$ and $r - y = -3.11\%$ (sample: 1963-2013). In scenario 1, we compute expected debt-output ratios under various values of d_S and for $d_{NS} = 11.9$. In scenario 2, we compute expected debt-output ratios under various values of d_{NS} and for $d_S = 8.1$. All others parameters are constant and equal to point estimates obtained in table 2, except γ_{NS} which is set to 0.

Table 6: Growth-adjusted real rates and Debt-GDP ratios

$\frac{r-y}{1+y}$	Stable debt-GDP ratio	$\mathbb{E}[b_t]$
2.5%	Yes	334%
2.0%	Yes	235%
1.5%	Yes	183%
1.0%	Yes	150%
0.5%	Yes	127%
0.0%	Yes	111%
-0.5%	Yes	98%
-1.0%	Yes	89%
-1.4%	Yes	81%
-1.9%	Yes	74%
-2.4%	Yes	69%
-2.8%	Yes	64%

Notes: Debt-output ratios are computed from equation (35) neglecting covariance terms. We use point estimates of γ_S , α_S , α_{NS} , except for γ_{NS} which is set to 0, and we use expected durations of regime d_S and d_{NS} from table 2. Then, we set $r = 3\%$ and compute expected debt-output ratios for various real GDP growth rate.

A Appendix

A.1 Proof of Proposition 1 (No-Ponzi Game)

We show that a strictly positive long-run feedback effect, i.e. (27)

$$\gamma\pi > 0$$

is a *sufficient* condition for the NPG (26) to hold, in a dynamically efficient economy and a bounded innovation process $\mu(z_t)$, following Bohn (1998, see online appendix). Using (17) and iterating of (14) yields:

$$b_{t+T} = \prod_{i=0}^T \frac{1+r_{t+i}}{1+y_{t+i}} \left(1 - (1+y_{t+i})\gamma(z_{t+i})\right) b_{t-1} - \sum_{k=0}^T (1+r_{t+k}) \left(\prod_{j=k+1}^T \frac{1+r_{t+j}}{1+y_{t+j}} \left(1 - (1+y_{t+j})\gamma(z_{t+j})\right) \right) \mu_{t+k}(z_{t+k}) \quad (43)$$

Then, multiplying by (25), one gets an expression for the discounted debt-output ratio at time $t+T$:

$$\mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T} = \mathbb{E}_t \prod_{i=0}^T \left(1 - (1+y_{t+i})\gamma(z_{t+i})\right) b_{t-1} - \mathbb{E}_t \sum_{k=0}^T \left(\prod_{j=k+1}^T \left(1 - (1+y_{t+j})\gamma(z_{t+j})\right) \right) a_{t,k} \quad (44)$$

with $a_{t,k} = (1+y_{t+k})\tilde{Q}_{t,k}\mu_{t+k}(z_{t+k})$. Taking the absolute value²³ of (44) and using triangle inequality yields:

$$\left| \mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T} \right| \leq \mathbb{E}_t \left| \prod_{i=0}^T \left(1 - (1+y_{t+i})\gamma(z_{t+i})\right) b_{t-1} \right| + \underbrace{\mathbb{E}_t \left| \sum_{k=0}^T \left(\prod_{j=k+1}^T \left(1 - (1+y_{t+j})\gamma(z_{t+j})\right) \right) a_{t,k} \right|}_{W_t} \quad (45)$$

and applying the triangle inequality on W_t allow us to give an upper bound to the absolute value of (44):

$$\left| \mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T} \right| \leq \mathbb{E}_t \prod_{i=0}^T \left| 1 - (1+y_{t+i})\gamma(z_{t+i}) \right| |b_{t-1}| + \mathbb{E}_t \sum_{k=0}^T \left| \prod_{j=k+1}^T \left(1 - (1+y_{t+j})\gamma(z_{t+j})\right) \right| |a_{t,k}| \quad (46)$$

An important step is to give a tractable expression for

$$\mathbb{E}_t \prod_{i=0}^T \left| 1 - (1+y_{t+i})\gamma(z_{t+i}) \right| \quad (47)$$

in order to study the limit property of equation (44). Thus remark that:

$$\begin{aligned} \mathbb{E}_t \prod_{i=0}^T \left| 1 - (1+y_{t+i})\gamma(z_{t+i}) \right| &= \mathbb{E}_t \left[\exp \left(\ln \prod_{i=0}^T \left| 1 - (1+y_{t+i})\gamma(z_{t+i}) \right| \right) \right] \\ &= \mathbb{E}_t \left[\exp \left(T \times \frac{1}{T} \sum_{i=0}^T \ln \left| 1 - (1+y_{t+i})\gamma(z_{t+i}) \right| \right) \right] \end{aligned} \quad (48)$$

²³Remark that $f(x) = |x|$ is convex, then Jensen inequality yields for any random variable X:

$$|\mathbb{E}[X]| \leq \mathbb{E}[|X|]$$

where $\frac{1}{T} \sum_{i=0}^T \ln|1 - (1 + y_{t+i})\gamma(z_{t+i})|$ is the Lyapunov exponent associated to the present-value debt-output ratio. Since both $(1 + y_t)$ and z_t are stationary-ergodic, then we know that:

$$\lim_{T \rightarrow +\infty} \frac{1}{T} \sum_{i=0}^T \ln|1 - (1 + y_{t+i})\gamma(z_{t+i})| = \mathbb{E} \left[\ln|1 - (1 + y_t)\gamma(z_t)| \right] \quad (49)$$

which is measurable at time t . If one assumes $(1 + y_t)\gamma(z_t) < 1$ ²⁴ then it yields $\ln|1 - (1 + y_t)\gamma(z_t)| = \ln(1 - (1 + y_t)\gamma(z_t))$. Applying Jensen's inequality on the logarithm function and the expectation operator yields an upper-bound for

$$\mathbb{E} \ln(1 - (1 + y_t)\gamma(z_t)) \leq \ln(1 - \mathbb{E}(1 + y_t)\gamma(z_t)) \quad (50)$$

From what precedes²⁵, we deduce it exists an arbitrarily high $N \in \mathbb{N}$ such that:

$$\forall T \geq N, \quad \mathbb{E}_t \prod_{i=0}^T |1 - (1 + y_{t+i})\gamma(z_{t+i})| \leq \exp \left[\ln(1 - \mathbb{E}(1 + y_t)\gamma(z_t))^T \right] \quad (51)$$

which allows us to conclude

$$\mathbb{E}_t \prod_{i=0}^T |1 - (1 + y_{t+i})\gamma(z_{t+i})| \leq (1 - \mathbb{E}(1 + y_t)\gamma(z_t))^T \quad (52)$$

Finally, we define the following upper bound for equation (47):

$$\mathbb{E}_t \prod_{i=0}^T |1 - (1 + y_{t+i})\gamma(z_{t+i})| \leq (1 - (1 + y)\gamma\pi - (\gamma_S - \gamma_{NS}) \text{Cov}(y_t, z_t))^T \quad (53)$$

where $\text{Cov}(y_t, z_t)$ is the unconditional covariance of y_t and z_t .

At this stage, we need two assumptions to proceed further.

Assumption 1 Following [Bohn \(1998\)](#), we assume dynamic efficiency which implies present-value of income is finite:

$$\lim_{T \rightarrow +\infty} Y_t \sum_{i=1}^T \mathbb{E}_t \tilde{Q}_{t,i} = \bar{Y}$$

implying $\lim_{T \rightarrow +\infty} \mathbb{E}_t \tilde{Q}_{t,T} = 0$, by convergence of the serie $\sum_{i=1}^T \mathbb{E}_t \tilde{Q}_{t,i}$.

Assumption 2 Following [Bohn \(1998\)](#), we assume the innovation process $\mu_t(z_t)$ is bounded $|\mu_t(z_t)| \leq M$.

Assumptions 1-2 jointly imply $\lim_{T \rightarrow +\infty} \mathbb{E}_t a_{t,k} = 0$ ²⁶ that is:

$$\forall \delta > 0, \quad \exists K \in \mathbb{N} \quad / \quad \forall k > K, \quad |\mathbb{E}_t a_{t,k}| \leq \delta \quad (54)$$

²⁴This assumption is actually purely technical, since it mainly relies on the assumption $|\gamma(z_t)|$ is close to zero, about the size of a small interest rate and $(1 + y_t)$ is close to 1.

²⁵In particular, remark Jensen inequality implies that:

$$\frac{1}{T} \sum_{i=0}^T \ln|1 - (1 + y_{t+i})\gamma(z_{t+i})| \leq \ln \left(\frac{1}{T} \sum_{i=0}^T |1 - (1 + y_{t+i})\gamma(z_{t+i})| \right)$$

allow us to define an upper-bound for $\mathbb{E}_t \prod_{i=0}^T |1 - (1 + y_{t+i})\gamma(z_{t+i})|$.

²⁶Given that $\lim_{T \rightarrow +\infty} \mathbb{E}_t \tilde{Q}_{t,T} = 0$ also implies $\lim_{T \rightarrow +\infty} \mathbb{E}_t (1 + y_T) \tilde{Q}_{t,T} = 0$

Then, using assumptions 1-2 along with equation (53) yields:

$$\begin{aligned} |\mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T}| \leq & \left(1 - (1+y)\gamma\pi - (\gamma_S - \gamma_{NS})\text{Cov}(y_t, z_t)\right)^T |b_{t-1}| + \Omega \left(1 - (1+y)\gamma\pi - (\gamma_S - \gamma_{NS})\text{Cov}(y_t, z_t)\right)^{T-K} \\ & + \sum_{k=K}^T \left(1 - (1+y)\gamma\pi - (\gamma_S - \gamma_{NS})\text{Cov}(y_t, z_t)\right)^{T-k} \delta \end{aligned} \quad (55)$$

where $\Omega = \sum_{k=0}^{K-1} E_t \prod_{j=k+1}^{K-1} |1 - (1+y_{t+i})\gamma(z_{t+i})| |\mathbb{E}_t a_{t,k}|$ is finite. Finally, rearranging the last expression allows us to write:

$$\begin{aligned} |\mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T}| \leq & \left(1 - (1+y)\gamma\pi - (\gamma_S - \gamma_{NS})\text{Cov}(y_t, z_t)\right)^T |b_{t-1}| + \Omega \left(1 - (1+y)\gamma\pi - (\gamma_S - \gamma_{NS})\text{Cov}(y_t, z_t)\right)^{T-K} \\ & + \frac{\delta}{(1+y)\gamma\pi + (\gamma_S - \gamma_{NS})\text{Cov}(y_t, z_t)} \end{aligned} \quad (56)$$

Assumption 3 In a purely Ricardian economy, we assume the fiscal regime z_t is independent of the real growth rate of the economy y_t , i.e. $\text{Cov}(y_t, z_t) = 0$.

Therefore, under assumption 3, a sufficient condition for the NPG condition only requires:

$$\gamma\pi > 0 \quad (57)$$

which implies $(1+y)\gamma\pi > 0$. Therefore, we find that

$$\forall \hat{\epsilon} > 0, \quad \exists K \in \mathbb{N} \quad / \quad \forall T \geq K \quad |\mathbb{E}_t \tilde{Q}_{t,T+1} b_T| < \hat{\epsilon}$$

provided one sets $\hat{\epsilon} = \frac{\delta}{|(1+y)\gamma\pi|}$, from which we conclude that:

$$\lim_{T \rightarrow +\infty} \mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T} = 0 \quad (58)$$

Discussion. In a more general framework with $\text{Cov}(y_t, z_t) \neq 0$, a sufficient condition to rule out Ponzi schemes, given a Markov-switching fiscal rule such as (17), would be:

$$\gamma\pi > -\frac{(\gamma_S - \gamma_{NS})\text{Cov}(y_t, z_t)}{1+y} \quad (59)$$

and would critically depends on the covariance term $\text{Cov}(y_t, z_t)$. If positive (i.e. if sustainable regimes are positively correlated to higher growth), it implies that a strictly positive $\gamma\pi$ would not be required to rule out Ponzi schemes; if negative, on the contrary, it would not be sufficient. Still, our empirical results provide an *ex post* validation for assuming $\text{Cov}(y_t, z_t) = 0$, since the estimated unconditional covariance between smoothed probabilities of a sustainable regime (i.e. the empirical counterpart of z_t) and the growth rate of real GDP is non-significantly different from zero, with a positive point estimate.

A.2 Proof of Proposition 2 (Debt-stabilizing condition)

Using the sufficient condition for a *strictly* stationary Markov-switching autoregressive process of order one, we show a strictly larger feedback effect than the average growth-adjusted real interest rate, i.e. (33), is a sufficient condition for the debt-output ratio process (32) to be strictly stationary and fluctuate around its ergodic mean (35).

Considering stochastic processes $\{x_t\}$ described by:

$$x_t = \phi_0 + \phi(z_t)x_{t-1} + \varepsilon_t \quad (60)$$

where z_t is a discrete-time Markov process, defined on the state-space $z(\Omega)$. We know from [Kesten \(1973\)](#) that a sufficient condition for *strict stationarity* is:

$$\mathbb{E}[\ln|\phi(z_t)|] \equiv \sum_{i \in z(\Omega)} \ln|\phi(i)|\pi(i) < 0 \quad (61)$$

which means that a globally stationary process $\{x_t\}$ can be locally (or periodically) non-stationary. This condition ensures that $\{x_t\}$ is strictly (or strongly) stationary implying its joint-probability distribution does not change over time. Strict stationarity only implies $\{x_t\}$ has a finite mean but does not imply necessarily a finite variance. Since weak stationarity requires finite variance, this condition is not sufficient for weak stationarity. For a finite variance, this process must verify a stronger condition. Define $\Phi \equiv \text{diag}(\phi(i), \forall i \in z(\Omega))$ and $\rho(M)$ the spectral radius of any square-matrix M . Then, for this strictly stationary process to admit a unique stationary solution at second-order, it must satisfy the following condition:

$$\rho(\Phi^2 P) < 1 \quad (62)$$

where P is the transition matrix of the underlying Markov-chain.

Applying condition (61) to equation (32) yields the following condition:

$$\mathbb{E}[\ln|\phi(z_t)|] = \mathbb{E}\left[\ln\left|\frac{1+r_t}{1+y_t}\right| + \ln|1 - (1+y_t)\gamma(z_t)|\right] < 0 \quad (63)$$

Hence, using usual approximation $\ln(1+x) \sim x$ when $x \rightarrow 0$ and taking unconditional expectations of r_t , y_t and $\gamma(z_t)$, we find a sufficient condition for strict stationarity of process $\{b_t\}$ is:

$$\gamma\pi > \frac{r-y}{1+y} \quad (64)$$

assuming that $\text{Cov}(y_t, z_t) = 0$.

Therefore, process $\{b_t\}$ has an ergodic mean equal to

$$\begin{aligned} \mathbb{E}[b_t] &= \frac{-\mathbb{E}[(1+r_t)\alpha(z_t)] + \text{Cov}(\phi(z_t), b_{t-1})}{\mathbb{E}[1 - \Phi(z_t)]} \\ &= \frac{-(1+r)\mathbb{E}\alpha(z_t) - (\alpha_S - \alpha_{NS})\text{Cov}(r_t, z_t) + \text{Cov}(\phi(z_t), b_{t-1})}{(1+r)\gamma\pi + (\gamma_S - \gamma_{NS})\text{Cov}(r_t, z_t) - \frac{r-y}{1+y}} \end{aligned} \quad (65)$$

which we approximate by

$$\mathbb{E}[b_t] \simeq \frac{-(1+r)\mathbb{E}\alpha(z_t)}{(1+r)\gamma\pi - \frac{r-y}{1+y}} \quad (66)$$

neglecting covariance terms, following [Bohn \(1998, 2008\)](#) and [Mendoza and Ostry \(2008\)](#).

A.3 Data on real interest rates and real GDP growth rate

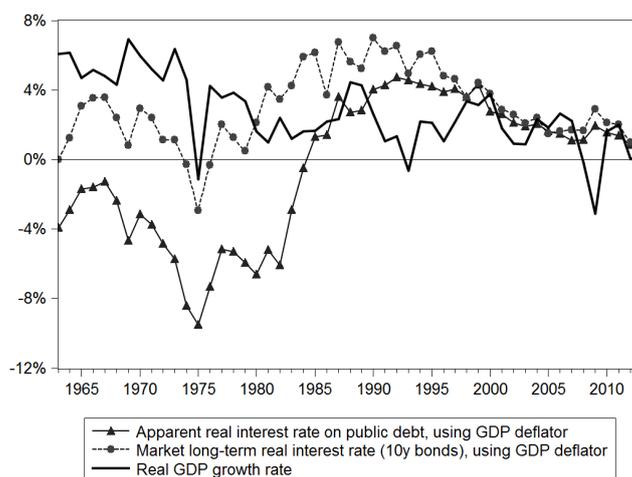
Table 7 presents descriptive statistics on real interest rate measures and real GDP growth. Figure 3 plots each time series and growth-adjusted real interest rates.

Table 7: Descriptive statistics on real interest rates and real GDP growth, 1963-2013

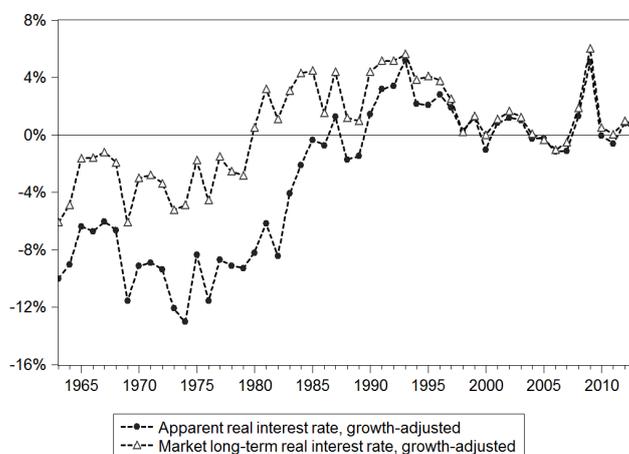
	Apparent real rate	Market long-term real rate	Real GDP growth rate
Mean	-0.43%	3.00%	2.68%
Median	1.13%	2.86%	2.31%
Maximum	4.75%	6.99%	6.91%
Minimum	-9.49%	-2.94%	-3.11%
Std. Dev.	4.0%	2.2%	2.1%
Skewness	-0.515	-0.060	-0.125
Kurtosis	2.025	2.759	2.994
Jarque-Bera normality test	4.276	0.154	0.132
p-value	0.118	0.926	0.936
Sum	-0.221	1.530	1.366
Sum Sq. Dev.	0.080	0.023	0.022
Observations	51	51	51

Figure 3: Apparent and market real interest rates and real GDP growth rate

(a) Real interest rates and real GDP growth



(b) Growth-adjusted real interest rates



References

- Afonso, António and João Tovar Jalles**, “The elusive character of fiscal sustainability,” *Applied Economics*, January 2016, 0 (0), 1–14.
- **and Priscilla Toffano**, “Fiscal regimes in the EU,” Working Paper Series 1529, European Central Bank 2013.
- Afonso, Antonio**, “Fiscal Sustainability: The Unpleasant European Case,” *FinanzArchiv: Public Finance Analysis*, 2005, 61 (1).
- Auerbach, Alan J.**, “Is there a role for discretionary fiscal policy?,” *Proceedings - Economic Policy Symposium - Jackson Hole*, 2002, pp. 109–150.
- Barro, Robert J.**, “On the Determination of the Public Debt,” *Journal of Political Economy*, 1979, 87 (5), 940–71.
- Bi, Huixin**, “Sovereign default risk premia, fiscal limits, and fiscal policy,” *European Economic Review*, 2012, 56 (3), 389–410.
- **and Eric Leeper**, “Analyzing Fiscal Sustainability,” Working Paper, Bank of Canada 2013.
- **and Nora Traum**, “Estimating Sovereign Default Risk,” *American Economic Review*, 2012, 102 (3), 161–66.
- Bianchi, Francesco**, “Regime Switches, Agents’ Beliefs, and Post-World War II U.S. Macroeconomic Dynamics,” *The Review of Economic Studies*, September 2012, p. rds032.
- Blot, Christophe, Bruno Ducoudré, and Xavier Timbeau**, “Sovereign debt spread and default in a model with self-fulfilling prophecies and asymmetric information,” *Journal of Macroeconomics*, March 2016, 47, Part B, 281–299.
- Bohn, Henning**, “The Sustainability of Budget Deficits in a Stochastic Economy,” *Journal of Money, Credit and Banking*, February 1995, 27 (1), 257–271.
- , “The Behavior Of U.S. Public Debt And Deficits,” *The Quarterly Journal of Economics*, 1998, 113 (3), 949–963.
- , “Are stationarity and cointegration restrictions really necessary for the intertemporal budget constraint?,” *Journal of Monetary Economics*, 2007, 54 (7), 1837–1847.
- , “The Sustainability of Fiscal Policy in the United States,” in “Sustainability of Public Debt,” MIT Press, 2008, pp. 15–49.
- Borio, Claudio E. V., Piti Disyatat, and Mikael Juselius**, “A Parsimonious Approach to Incorporating Economic Information in Measures of Potential Output,” SSRN Scholarly Paper ID 2397702, Social Science Research Network, Rochester, NY February 2014.
- Burger, Philippe and Marina Marinkov**, “Fiscal rules and regime-dependent fiscal reaction functions,” *OECD Journal on Budgeting*, 2012, 12 (1), 1–29.
- Canzoneri, Matthew B., Robert E. Cumby, and Behzad T. Diba**, “Is the Price Level Determined by the Needs of Fiscal Solvency?,” *American Economic Review*, 2001, 91 (5), 1221–1238.
- Canzoneri, Matthew, Robert Cumby, and Behzad Diba**, “The Interaction Between Monetary and Fiscal Policy,” *Handbook of Monetary Economics*, Elsevier 2010.

- Chen, Shyh-Wei**, “Testing for fiscal sustainability: New evidence from the G-7 and some European countries,” *Economic Modelling*, 2014, 37 (C), 1–15.
- Chung, Hess, Troy Davig, and Eric M. Leeper**, “Monetary and Fiscal Policy Switching,” *Journal of Money, Credit and Banking*, 2007, 39 (4), 809–842.
- Clarida, Richard, Jordi Galí, and Mark Gertler**, “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory,” *The Quarterly Journal of Economics*, February 2000, 115 (1), 147–180.
- Cochrane, John H.**, “Long-Term Debt and Optimal Policy in the Fiscal Theory of the Price Level,” *Econometrica*, 2001, 69 (1), 69–116.
- , “Money as stock,” *Journal of Monetary Economics*, 2005, 52 (3), 501–528.
- Cotis, Jean-Philippe, Jørgen Elmeskov, and Annabelle Mourougane**, “Estimates of potential output: benefits and pitfalls from a policy perspective,” in “Euro Area Business Cycle: Stylized Facts and Measurement Issues,” cepr ed., London: L. Reichlin, 2005.
- Creel, Jerome and Herve Le Bihan**, “Using structural balance data to test the fiscal theory of the price level: Some international evidence,” *Journal of Macroeconomics*, 2006, 28 (2), 338–360.
- Daniel, Betty C.**, “A graceful return of the drachma,” *European Economic Review*, October 2014, 71, 228–243.
- **and Christos Shiamptanis**, “Pushing the limit? Fiscal policy in the European Monetary Union,” *Journal of Economic Dynamics and Control*, November 2013, 37 (11), 2307–2321.
- Davig, Troy**, “Periodically expanding discounted debt: a threat to fiscal policy sustainability?,” *Journal of Applied Econometrics*, 2005, 20 (7), 829–840.
- **and Eric M. Leeper**, “Fluctuating Macro Policies and the Fiscal Theory,” NBER Chapters, National Bureau of Economic Research, Inc 2007.
- **and Eric M Leeper**, “Generalizing the Taylor Principle,” *American Economic Review*, June 2007, 97 (3), 607–635.
- **and Eric M. Leeper**, “Monetary-fiscal policy interactions and fiscal stimulus,” *European Economic Review*, 2011, 55 (2), 211–227.
- , —, **and Todd B. Walker**, “Inflation and the fiscal limit,” *European Economic Review*, 2011, 55 (1), 31–47.
- Favero, Carlo A. and Tommaso Monacelli**, “Fiscal Policy Rules and Regime (In)Stability: Evidence from the U.S.,” SSRN Scholarly Paper ID 665506, Social Science Research Network, Rochester, NY January 2005.
- Fincke, Bettina and Alfred Greiner**, “How to assess debt sustainability? Some theory and empirical evidence for selected euro area countries,” *Applied Economics*, 2012, 44 (28), 3717–3724.
- Ghosh, Atish R., Jonathan D. Ostry, and Mahvash S. Qureshi**, “Fiscal space and sovereign risk pricing in a currency union,” *Journal of International Money and Finance*, April 2013, 34, 131–163.
- , **Jun I. Kim, Enrique G. Mendoza, Jonathan D. Ostry, and Mahvash S. Qureshi**, “Fiscal Fatigue, Fiscal Space and Debt Sustainability in Advanced Economies,” *The Economic Journal*, 2013, 123 (566), F4–F30.

- Greiner, Alfred, Uwe Köller, and Willi Semmler**, “Debt sustainability in the European Monetary Union: Theory and empirical evidence for selected countries,” *Oxford Economic Papers*, 2007, 59 (2), 194–218.
- Hamilton, James D.**, “A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle,” *Econometrica*, 1989, 57 (2), 357–84.
- Hamilton, James D and Marjorie A Flavin**, “On the Limitations of Government Borrowing: A Framework for Empirical Testing,” *American Economic Review*, 1986, 76 (4), 808–19.
- Hamilton, James Douglas**, *Time Series Analysis*, Princeton University Press, January 1994.
- Kesten, Harry**, “Random difference equations and Renewal theory for products of random matrices,” *Acta Mathematica*, 1973, 131 (1), 207–248.
- Lamé, G., M. Lequien, and P.-A. Pionnier**, “Interpretation and limits of sustainability tests in public finance,” *Applied Economics*, 2014, 46 (6), 616–628.
- Leeper, Eric M.**, “Equilibria under ‘active’ and ‘passive’ monetary and fiscal policies,” *Journal of Monetary Economics*, 1991, 27 (1), 129–147.
- Lubik, Thomas A. and Frank Schorfheide**, “Testing for Indeterminacy: An Application to U.S. Monetary Policy,” *American Economic Review*, 2004, 94 (1), 190–217.
- Mendoza, Enrique G. and Jonathan D. Ostry**, “International evidence on fiscal solvency: Is fiscal policy “responsible”?,” *Journal of Monetary Economics*, 2008, 55 (6), 1081–1093.
- Quintos, Carmela E.**, “Sustainability of the Deficit Process with Structural Shifts,” *Journal of Business & Economic Statistics*, 1995, 13 (4), 409–17.
- Schoder, Christian**, “The fundamentals of sovereign debt sustainability: evidence from 15 OECD countries,” *Empirica*, 2014, 41 (2), 247–271.
- Sims, Christopher A.**, “A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy,” *Economic Theory*, 1994, 4 (3), 381–99.
- Trehan, Bharat and Carl E Walsh**, “Common trends, the government’s budget constraint, and revenue smoothing,” *Journal of Economic Dynamics and Control*, 1988, 12 (2-3), 425–444.
- **and Carl E. Walsh**, “Testing Intertemporal Budget Constraints: Theory and Applications to U.S. Federal Budget and Current Account Deficits,” *Journal of Money, Credit and Banking*, 1991, 23 (2), 206–23.
- Weichenrieder, Alfons and Jochen Zimmer**, “Euro membership and fiscal reaction functions,” *International Tax and Public Finance*, 2014, 21 (4), 598–613.
- Wickens, M. R and Merih Uctum**, “The sustainability of current account deficits : A test of the US intertemporal budget constraint,” *Journal of Economic Dynamics and Control*, 1993, 17 (3), 423–441.
- Wilcox, David W.**, “The Sustainability of Government Deficits: Implications of the Present-Value Borrowing Constraint,” *Journal of Money, Credit and Banking*, 1989, 21 (3), 291–306.
- Woodford, Michael**, “Price-level determinacy without control of a monetary aggregate,” *Carnegie-Rochester Conference Series on Public Policy*, 1995, 43 (1), 1–46.