

Testing Models of Decision under Risk: The Case of Horserace Bettors in France

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Abstract

One of the most robust findings in the literature using data on horseraces bets is that odds associated to horses reflect their chances of winning very well, with the exception that favorites are underbet while outsiders are overbet. Expected utility theory and behavioral theories of decision under risk compete to explain this finding. This paper seeks to discriminate between the two classes of models by testing which is the most suited to explaining the behavior of bettors observed in the data. Using a unique dataset of bets on horseraces in France, I find that behavioral theories of decision under risk better fit my data than expected utility. This result shows that behavioral theories provide a better representation of choice behavior than expected utility.

Key words: Decision Making, Expected Utility, Cumulative Prospect Theory, Rank-Dependent Utility, Probability Weighting Function, Risk-Aversion, Representative Bettor, Favorite-Longshot Bias.

JEL classification: D81, L83.

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1 Introduction

Horseshoe betting markets provide a real-life laboratory to study decisions taken in situations of risk. First, wagering on a horse involves making a choice between clearly identified alternatives, each alternative being associated with a monetary outcome. Second, the occurring alternative is observed publicly after a short period of time. Third, choices are made repeatedly by a large number of participants. Fourth, extensive information is available on probabilities of outcomes. Horseshoe betting markets hence offer the opportunity to test the theoretical framework of decision under risk in a simple, yet real-life situation. In particular, they share many characteristics with very simple financial markets.

A large number of papers have taken advantage of these characteristics. They have studied whether prices associated to horses (odds) reflect their intrinsic values (chances of winning in a given race). One of the most robust findings of the literature is that odds associated to horses indeed reflect their intrinsic values very well, with the exception that favorites (horses with a high chance of winning) tend to be underbet while outsiders (horses with a relatively small chance of winning) are overbet (Sauer (1998)). As a result, the expected returns on outsiders are lower than on favorites. An abundant literature tries to explain the existence of this empirical regularity, called the favorite-longshot bias (see Ottaviani and Sorensen (2008) for a review of the main explanations). In particular, two theories of decision under risk compete to this purpose.

On the one hand, the standard theory of individual choice in economics (expected utility theory by von Neumann and Morgenstern (1947)) can rationalize the bias by posing that bettors have, at least locally, a convex utility function for money outcomes. On the other hand, behavioral theories are able to explain the bias by incorporating that decision makers transform probabilities when assessing the value of risky prospects.

The goal of this paper is to discriminate between these two classes of models. The question under study seeks to identify which model best explains the favorite-longshot bias. In a broader perspective, it tests whether expected utility provides a sufficiently accurate representation of actual choice behavior or whether it should be replaced by alternative theories (see Starmer (2000)). Using the wrong model might prevent from understanding commonly observed behaviors. Behaviors which can only be explain using behavioral models include for example the

equity premium puzzle in finance, the choice of some menu of premium/deductible in insurance or the labor supply of cab drivers in labor economics.

Our analysis relies on a unique dataset of bets on horseraces of the French betting operator from 2013 to 2015. The main novelty of the paper lies in the dataset. It is used to study the existence of the favorite-longshot bias in France and determine whether the results of Jullien and Salanié (2000) hold in a different context and at a different time period.

I first show that the favorite-longshot bias exists in France. I further find that behavioral theories of decision under risk, that is both rank-dependent and cumulative prospect theories are better suited to explaining the behavior of bettors observed in the data than expected utility. This result provides evidence that bettors weight probabilities non-linearly when making choices. Using cumulative prospect theory, I find significant weighting of probabilities in the domain of losses and linear weighting of probabilities in the domain of gains, which is consistent with the result of Jullien and Salanié (2000) but contradicts results from experiments which find similar weighting of probabilities in the gains and losses domains (Kahneman and Tversky (1992), Abdellaoui (2000)).

This paper fits into a considerable theoretical and experimental literature motivated by the observation that, in laboratory experiments, people make choices systematically inconsistent with expected utility theory (Allais (1953), Kahneman and Tversky (1979)). Cumulative prospect theory has emerged as the favorite model from the experimental literature. Evidence that decision makers weight probabilities non linearly as in cumulative prospect theory were provided by many experiments (see Camerer and Ho (1994), Tversky and Kahneman (1992), Wu and Gonzalez (1996, 1999) and Abdellaoui (1998)). The theory was also found to be able to rationalize behaviors observed in laboratories that could not be explained by expected utility. One such example is probabilistic insurance (see Wakker, Thaler and Tversky (1997)). This type of insurance policy involves a small probability (say 1 %) that the consumer will not be reimbursed. According to expected utility theory (and whatever the concavity of the utility function), people should pay approximately 99% times as much for probabilistic insurance as they pay for full insurance. But experimental responses show that people are willing to pay much less to compensate for the low chance that the claim will not be paid. This behavior cannot be explained by expected utility but is consistent with the overweighting of small probabilities of

prospect theory.

Some people have questioned whether the findings of the experimental literature generalize to real-world data (see List (2004), Levitt and List (2008)). They believe that biases are less likely in the presence of large stakes, experience and competition.

Existing studies using real-world settings typically rely on insurance, finance and bets or games market data. While Cicchetti and Dubin (1994) present evidence that decisions to purchase insurance against the risk of telephone line malfunction at home are consistent with expected utility theory, O'Donoghue et al. (2010) show that non linear probability weighting plays a role in the behavior of households in the choice of auto and home insurance. Kliger and Levy (2009) also find that cumulative prospect theory better fits their data than expected utility and rank-dependent utility relying on data on call options on the S&P500 index. Using data from game shows, Post et al (2008) show that preferences are reference dependent so that they exhibit characteristics of the cumulative prospect theory model. List (2003, 2004) provides evidence that although inexperienced consumers behave as in prospect theory, market experience brings experienced traders' behavior close to neoclassical predictions. On the contrary Pope and Schweitzer (2011) show that highly experienced professional golfers who face high stakes payoffs and intense competition exhibit loss aversion as predicted by prospect theory.

My paper is closely related to the literature using horserace data. Jullien and Salanié (2000) show, by focusing on win bets in the UK, that cumulative prospect theory describes the behavior of a representative agent better than expected utility and rank-dependent utility theories. However contrary to the usual inversed S-shaped probability weighting function, they find little evidence for the existence of a certainty effect and of a change in concavity of the probability weighting functions that they estimate. They also establish that rank-dependent utility does not improve on expected utility. The present paper seeks to confirm their results by reestimating their model on a different dataset.

Snowberg and Wolfers (2010) use an impressively large dataset of pari-mutuel bets in the United-States to test predictions derived for expected utility and cumulative prospect theories in the particular case of win bets on complex bets. Their approach is based on that the two theories yield different implications for the prices of complex bets so by comparing predictions with real prices, the best model can be identified. They find that the model with non linear

probability weighting provides the best description of the data, which suggests that prospect theory permits a better description of the data than expected utility.

The present paper is organized as follows. Section 2 explains how horseraces bets are organized in France. Section 3 describes the data. Section 4 introduces the model and the estimation procedure. Section 5 presents the results. The last section concludes.

2 Horse race betting in France

The betting market on horseraces in France is exclusively a pari-mutuel system. The concept of pari-mutuel consists in pooling together all bets corresponding to a race and a bet type, removing a share to cover the taxes and expenses of the betting operator and redistributing the remaining among winning bettors in proportion to their bets. Final payoffs hence depend exclusively on the total pool, the share kept by the betting operator (the “take”)² and the stakes attracted by each horse.

The more stakes a horse attracts relative to the total pool, the lower the payoff of a bet on this horse. Payoffs on horses are called odds. In the simplest type of bets (which are the focus of the paper), which consist in finding the winner of a given race, odds of 1.2 on a given horse and race means that a 1 unit winning bet on that horse returns the bet (1) plus 1.2. Odds hence correspond to net returns of a unit bet. A horse cannot have odds inferior to 0.1.

For a race happening on a particular day, the market opens online as soon as the ultimate race of the previous day ends. For a bettor which prefers to go to a specialized store, it starts on the day of the race at the opening of stores. A bettor at the track can only bet about thirty minutes before the beginning of the race. The market closes right before the start of the race. Because of the way odds are computed in the pari-mutuel system, bettors only have access to temporary odds which are computed with the current state of bets and are updated about every minute online.

²In addition to the “take”, French operators also enjoy “breakage”, which is the gain from rounding payoffs downwards to the nearest ten cents.

3 Data

Data were collected from pmu.fr between April 2013 and May 2015. PMU (pari-mutuel urbain) is the main operator of bets in France. Online, it gathers 84.8% of the total pool and in-store, it is a legal monopoly. The dataset records information on bets, races, horses and tracks for races which were supports of bets offered by the PMU. It contains 33,196 races.

For each race, the dataset encompasses the final win payoff of each horse, its rank in the race and many of its characteristics such as the number of races run by the horse in its career or the amount won. In addition to the time of day, date and track, races are also characterized by their discipline and types. Data also includes the total pool, dividends of winning horses, the number of winners and information on tracks. The data only contains payoffs of each horse for winning bets, which are the focus in this paper.

Since I am interested in modeling the process of decision making regarding the choice of a specific horse in a race, I drop the 7,919 races in which two or more horses in a given race belong to a team, which happens when horses have the same owner or the same trainer. In this case all the horses of the team have the same payoff and if one of them wins the race, a bet on any of the horses in the team also wins. Hence the payoff of a horse that is part of a team does not reflect its probability of winning, but rather the probability that any horse in the team wins.

I also remove races in which several horses arrive in the first position, called deadheats, because I model a race in which only one horse wins the race. I drop races for which payoffs are incomplete or erroneous. It includes races which are not recorded as being over, in which at least one running horse has a missing payoff and for which the final payoff of the winning horse does not correspond to the dividend. I am left with 23,462 races.

As the following table shows, the average number of races per day amounts to 32. During some days, 80 races take place, while on other days only 13 do. The average number of running horses in a given race is 12. The minimum is 2 and the maximum 24. Half of the races count between 9 and 14 running horses.

The distribution of odds covers a wide range. The maximum reaches 998 while the minimum is 0.1. The median amounts to 15.4 and the mean to 27. 90% of odds range between 0.1 and 68.4.

< Table 1 about here >

The following figure shows that the sample contains large favorites, with odds between 0.1 and 0.5 (0.2% of the sample), and very long outsiders with odds above 50 (19% of the sample).

< Figure 1 about here >

Using these definitions, 64% of large favorites and 0.6% of very long outsiders won their race. Alternatively, defining large favorites as horses that attract twice more bets than the second-more-bet horse in their race, large favorites win 44% of the time.

4 Model and estimation procedure

4.1 The theoretical model

The model is similar to Jullien and Salanié (2000). It describes the decision of a representative bettor who bets a in a given race and is endowed with an initial wealth M^3 . The choice of a particular horse in the race depends only on its probability of winning and final odds. In a given race c with N horses, the bettor is hence presented with a menu of probabilities and odds $((O_1, p_1), (O_2, p_2), \dots, (O_N, p_N))_c$, probabilities being non negative and summing to one.

I assume that the menu is known to the bettor when he makes his choice. This assumption is potentially problematic since final odds are not known until the beginning of the race and the bettor does not have perfect knowledge of probabilities of winning. However, previous studies of horserace bettors show that they are very sophisticated and exploit the sheer amount of information available to infer chances of winning of horses. Additionally, odds are quite stable at the end of the betting period.

Writing H_i the action of betting on horse i , the overall value of a bet on horse i , $W(H_i)$, is a number such that the bettor prefers horse i to horse j or is indifferent between horses i and j if and only if $W(H_i) \geq W(H_j)$. The representative bettor is rational in that he bets on the horse with the highest overall value.

³The data do not contain information on the amount bet or on the wealth of bettors.

Furthermore, I assume that the race is only won by one horse. In this perspective, the few races won by several horses were removed from the sample.

Given the stated assumptions, bettors continue to bet in a given race until odds make them indifferent between betting on any horse in the race and not betting. So in equilibrium:

$$\forall i \in 1, \dots, N, W(H_i) = w, w \text{ constant} \quad (1)$$

In general terms, the overall value of a bet on horse i can be written :

$$W(p_i, a, O_i) = \varphi^+(p_i) * u(M + aO_i) + \varphi^-(1 - p_i) * u(M - a) \quad (2)$$

The specific expression of the overall value of a bet on horse i depends on the model of decision making studied. In expected utility, $\varphi^+(p_i) = \varphi^-(p_i) = p_i$. In rank-dependent theory, $\varphi^-(1 - p_i) = 1 - \varphi^+(p_i)$ and in cumulative prospect theory $M = 0$, so that I assume that the reference point is not betting⁴. More details on each model are given in Appendix A.

The model is solved using the procedure of Jullien and Salanié (2000) which consists in computing w and then the probability of winning of the horse which actually won the race (p_1), which is in turn used to compute the likelihood function. The procedure used to obtain p_1 is explained for each model in Appendix B.

4.2 Estimation

Let θ, α, β be the parameters of the utility and of the probability weighting functions. p_1^r is the probability of winning of the horse which actually won race r , with $r = 1, \dots, M$. For each r , the likelihood to observe horse 1 win the race is $l(\theta, \alpha, \beta; O_1^r) = p_1(O_1^r; \theta, \alpha, \beta)$.

Because outcomes of races are independent, the probability of observing the sample under study, assuming that the expressions of p_1 derived from the model are correct, is the product of the M individual densities, which corresponds to the following likelihood function:

$$\prod_{r=1}^M p_1(O_1^r; \theta, \alpha, \beta) = L(\theta, \alpha, \beta | O_1^r)$$

Hence the log-likelihood function:

⁴This assumption is in line with the literature which commonly assumes that the reference point is the *status quo*. For a discussion on how people think about gains and losses, see Köszegi and Rabin (2006, 2007, 2009).

$$LL = \ln L(\theta, \alpha, \beta | O_1^r) = \sum_{r=1}^M \ln p_1(O_1^r; \theta, \alpha, \beta)$$

The maximum likelihood estimator has the usual asymptotic properties. It is consistent, asymptotically normal, asymptotically efficient and invariant.

$$\hat{\theta} \sim N(\theta_0, [I(\theta_0)]^{-1}), I(\theta_0) = -E_0(\partial^2 \ln L / \partial \theta_0 \partial \theta_0') = -E_0(\partial \ln L / \partial \theta_0 * \partial \ln L / \partial \theta_0')$$

Standard-errors are computed both using a bootstrap procedure and the previous formula.

4.3 Functional form of the utility function

Following Jullien and Salanié (2000), I assume that the utility function has the following CARA form throughout the paper: $u(x, \theta) = \frac{1 - e^{-\theta x}}{\theta}$.

The CARA form allows the estimation of the level of absolute risk aversion θ under the assumption that it is constant. Bettors are risk-loving if $\theta < 0$ and risk-averse if $\theta > 0$. The expression of u retained is convenient since M , which is not observed, cancels out in the expression of the probability used in the likelihood function.

4.4 Functional forms of the probability weighting functions

The common functional forms presented in Table 2 are tested. More information on these probability weighting functions are available in Appendix C.

< Table 2 about here >

5 Results

5.1 The favorite-longshot bias

The favorite-longshot bias is the finding that betting on favorites (horses with small odds) yields a higher expected returns than betting on longshots (horses with relatively high odds). It has been shown in a large number of papers, starting with Griffith (1949). It has been found across different types of races and at different times in North America (McGlothlin(1956), Weitzman (1965), Ali (1977), Snyder (1978), Asch et al. (1982), Snowberg and Wolfers (2010)) where

the pari-mutuel system prevails, in the UK in both the pari-mutuel and bookmaker systems (Vaughan Williams and Paton (1997), Jullien and Salanié (2000)), in Australia in both the pari-mutuel (Coleman (2002)) and bookmaker systems (Bird et al. (1987)) and in New Zealand in the pari-mutuel system (Coleman (2002), Gandar et al. (2001)).

The first result of the paper is that the favorite-longshot bias also exists in France. The expected returns for a 1-unit bet on horse i is $R_i = \pi_i * O_i + (1 - \pi_i) * (-1)$, where π_i is the probability of winning of horse i and O_i corresponds to its final odds. Probabilities of winning of horses, which is the proportion of times the horse would win the same race repeated an infinitely large number of times, are unknown so I compute rates of returns using the approach commonly adopted in the literature (see Coleman (2004)), which consists in grouping all horses of the dataset by either intervals of odds or favorite order (the favorite is in the first group, the second favorite in the second group, etc.) and computing the percentage of winners and the average odds in each group.

Expected returns are graphed in the following figure, horses were grouped by odds percentiles and data are presented on a log-odds scale.

< Figure 2 about here >

Figure 2 shows that returns are not equated across betting odds: betting on favorites yields a higher rate of returns than betting on outsiders. The expected returns of betting horses with odds of 127 to 1 is -0.6 , whereas it is -0.07 for horses with odds 1.43. Hence payoffs of favorites are not low enough to compensate for their high probabilities of winning, or equivalently favorites are underbet compared to their probability of winning. On the contrary, payoffs of outsiders are not high enough to compensate for their low probabilities of winning, or equivalently, they are overbet.

Hence, in a simple model with linear utility and probability weighting functions, the data shows that, in equilibrium, rates of returns are not equalized across horses in a race. Many propositions have been made to explain this bias, one of which being that the simple model does not properly account for the tastes and beliefs of bettors.

5.2 Tests of models of decision making under risk

5.2.1 Expected utility model

The results of the parameters obtained in the expected utility framework, as well as the maximum value of the log-likelihood function are presented in table 3.

< Table 3 about here >

$a\theta$ is significantly negative and has a small absolute value, meaning that at least statistically and in the income range in which I test them, bettors exhibit a small and significant taste for risk. The parameter is smaller to the one obtained by Jullien and Salanié (2000) (-0.055) but similar to that of Snowberg and Wolfers (2010) who fit their data with a CARA utility function of parameter -0.017 .

A bet of €20 on a horse with odds 10 and probability of winning 25% is equivalent to the lottery winning €200 with probability 25% and losing 20 with probability 75%. The estimated risk-attitude parameter makes a bettor indifferent between this lottery and the sure amount of €38. A risk neutral bettor would be indifferent between the same lottery and the sure amount €35 so the behavior of bettors exhibits some risk-love.

5.2.2 Rank-dependent utility model

The results for all functional forms of the probability weighting functions tested are presented in Table 4 and graphed in Figure 3.

< Table 4 and Figure 3 about here >

The risk-attitude parameter of the utility function is negative, statistically significant and small in every specification of the rank-dependent utility model.

The statistically significant power coefficient differs from and is slightly inferior to 1 (column 1 of Table 4) so $\varphi(p) > p$, which reflects optimism. In the Cicchetti and Dubin function (column 2), a_1 is not significantly different from 1 although it is precisely estimated so p_0 is not identified and we are back to the expected utility model. In the Lattimore, Baker and Witte function

(column 3), γ does not significantly differ from 1 while δ is large and statistically significantly different from 1 at the 5% level. The function is concave and above the 45 degree line, which suggests optimism. In the Prelec function (column 4), α is close to but statistically different from 1, β is large and also statistically different from 1. The function is convex and below the 45 degree line, showing pessimism. In the Kahneman and Tversky function (column 5), γ is close to but statistically different from 1.

All models are nested within the expected utility model. Likelihood ratio tests between the latter and the rank-dependent utility models show that the rank-dependent utility model statistically significantly better fit the data than the expected utility model in four specifications (power, Lattimore et al., Prelec and Kahneman and Kahneman and Tversky). Hence the first conclusion of the analysis is that the rank-dependent utility model better fits the data than the expected utility model. This conclusion differs from Jullien and Salanié (2000) who concluded that rank-dependent utility models did not improve on the expected utility model because they found that only the Prelec specification fitted their data better than expected utility.

The results do not permit to conclude on the overall attitude toward risk of bettors, which combines both the risk attitude parameter of the utility function and the weights associated to probabilities. As Figure 3 suggests, the power and the Kahneman and Tversky functions are extremely close to the diagonal. Additionally, Table 4 shows that attitude toward risk is similar in the expected utility model and the rank-dependent models with power and Kahneman and Tversky weighting functions. The difference between the expected utility model and these models hence does not change anything in terms of behavior of bettors: bettors exhibit a small taste for risk. In Lattimore et al., the risk attitude parameter suggests that bettors are risk-lovers, which is reinforced by the overweighting of probabilities. In, the Prelec specification, the risk attitude parameter suggests risk-love but bettors underweight probabilities so that in the end their behavior exhibits risk-aversion. To illustrate, the estimated parameters in the Prelec case makes a bettor indifferent between the lottery winning €200 with probability 25% and losing 20 with probability 75% and the sure amount of €22. In the Lattimore et al. specification, it makes the bettor indifferent between the same lottery and the sure amount of €54. Because risk-neutrality corresponds to the sure amount of €35, the bettor exhibits risk aversion in the first case and risk-love in the second. I am not able to discriminate between

the models within the rank-dependent theory because AIC and BIC criteria and Vuong tests between models are inconclusive. I can only establish that the Prelec model, which is nested within the power model, performs better than the power function.

5.2.3 Cumulative prospect theory

The results for all functional forms of the probability weighting functions tested are presented in Table 5. Probability weighting functions for gains are graphed in Figure 4 and probability weighting functions for losses in Figure 5.

< Table 4 and Figures 4 and 5 about here >

The risk-attitude parameter of the utility function is negative, statistically significant and small in every specification of the cumulative prospect theory model.

The statistically significant power coefficient differs from and is slightly above 1 in the power probability weighting function of gains and below 1 in the power probability weighting function of losses (column 1 of Table 5). In the case of gains, it is very close to the diagonal, showing almost no weighting of probabilities. In the case of losses, it is well above the 45-degree line, showing clearly that bettors overweight probabilities of losses, which reflects pessimism.

In the Cicchetti and Dubin function (column 2), a_1 is not significantly different from 1 in the case of gains so p_0 is not identified and the model is equivalent to expected utility. The estimated parameters of the Cicchetti and Dubin probability weighting function of losses have very high standard-errors so that I cannot draw any conclusion from their values. Parameters are also too imprecisely estimated in the Lattimore, Baker and Witte (column 3) and the Prelec (column 4) specifications to draw any conclusion. The statistical significance of the parameters does not authorize to reject expected utility even if likelihood ratio tests favor these specifications over expected utility.

In the Kahneman and Tversky function (column 5), γ does not differ from 1 in the probability weighting function of gains but it does concerning the probability weighting function of losses, with $\gamma' = 0.77$. The function slightly overweights small probabilities and underweights high probabilities, being hence inverse S-shaped. The curvature of the function is less pronounced

than in experimental studies such as Camerer and Ho (1994) who estimate a probability weighting function of gains with parameter 0.56, Tversky and Kahneman (1992) who found 0.61 for gains and 0.69 for losses, Wu and Gonzalez (1996) who found 0.71 for gains and Abdellaoui (2000) who found 0.60 for gains and 0.70 for losses. I hence find more sensitivity to changes in probabilities far from 0 and 1 than those studies and less pronounced certainty and possibility effects.

Likelihood ratio tests between the expected utility model and the cumulative prospect theory models show that the latter significantly better fits the data than the former in all specifications so that expected utility is clearly rejected. This result is consistent with the conclusions of Jullien and Salanié (2000).

Jullien and Salanié (2000) further explain that their data does not support changing concavity in the probability weighting functions and that the probability weighting function for losses is concave while the weighting function for gains is linear. They find this result surprising given the many experiments that find evidence of changing concavity. Camerer (2000) interprets this result as a new explanation for the favorite-longshot bias : “*Bettors like longshots because they have a convex utility and weight their high chances of losing and small chances of winning roughly linearly. But they hate favorites because they like to gamble ($u(x)$ is convex), but are disproportionately afraid of the small chance of losing when they bet on a heavy favorite*”.

I do not reach the same conclusion. I find like Jullien and Salanié (2000) a clear difference between probability weighting of gains, which is quasi linear, and probability weighting of losses, which departs from the 45-degree line. The power model gives me the same result as Jullien and Salanié (2000) but I also estimate the Kahneman and Tversky function, which was not estimated by Jullien and Salanié (2000) and tells a different story. I indeed estimate an inverse S-shaped function for losses in this model. I am not able to discriminate between the two models. AIC and BIC of the power and the Kahneman and Tversky models are very close. Furthermore, the Vuong statistic for non nested models lies in the inconclusive region. I hence cannot conclude on the way bettors weight probabilities. I also cannot conclude on the risk attitude of bettors since the two models have different implications in terms of behaviors of bettors. To illustrate, the estimated parameters in the Kahneman and Tversky model makes a bettor indifferent between the lottery winning €200 with probability 25% and losing 20 with probability 75% and the sure

amount of €40. In the power model, it makes the bettor indifferent between the same lottery and the sure amount of €31. In the first-case the bettor hence exhibits a slightly risk-loving attitude. In the second case he is clearly risk-averse.

6 Conclusion

This paper relies on the model of Jullien and Salanié (2000) to compare the fit of expected utility theory, rank-dependent utility theory and cumulative prospect theory to French data on horserace bets. It shows that the favorite-longshot bias exists in France. It additionally establishes that both rank-dependent utility and cumulative prospect theory are better suited to explaining the data than expected utility, which suggests that bettors weight probabilities non-linearly when they make choices. In rank-dependent utility, my results contradict those of Jullien and Salanié (2000) who found no improvement in fit with this model. In cumulative prospect theory, my results confirm those of Jullien and Salanié (2000).

My analysis however suffers from one limitation. I do not have data on individual bettors so I have to study the behavior of a representative bettor. This is potentially problematic since bettors might differ with respect to their attitude toward risk and their beliefs.

Two interesting research paths could be pursued to complement the study. First, I do not test several shapes of the value function but rather focus on the probability weighting function. Jullien and Salanié (2000) initiated this possible venue of research by testing a HARA utility function. However because they did not have data on the wealth of bettors, they could not pursue further and finally choose to use a CARA utility function for convenience. I face the same limitations due to the data. Second, my data does not allow to test loss aversion, which is one of the main characteristics of cumulative prospect theory. Information on the amount bet could allow to follow this path in future research.

Table 1: Summary statistics

Variable	Mean	Std. Dev.	Min.	Max.	N
Races per day	32	9.29	13	80	735
Running horses per race	12	3.31	2	24	23,464
Odds	27	31.79	0.1	998	279,792

Figure 1: Distribution of odds (90 % of odds only)

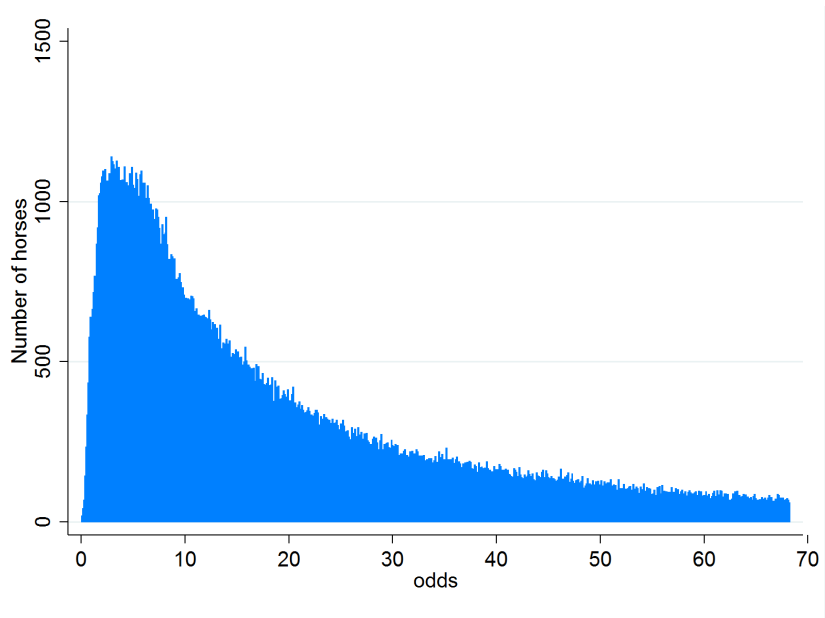


Table 2: Probability weighting functions

Name	Expression	Restrictions on parameters
Power	$\varphi(p) = p^\alpha$	$\alpha > 0$
Kahneman and Tversky (1992)	$\varphi(p) = \frac{p^\gamma}{[p^\gamma + (1-p)^\gamma]^{1/\gamma}}$	$\gamma > 0$
Cicchetti and Dubin (1994)	$\frac{\varphi(p)}{1-\varphi(p)} = \left(\frac{p}{1-p}\right)^{a_1} * \left(\frac{p_0}{1-p_0}\right)^{1-a_1}$	$a_1 \geq 0, p_0 \in [0, 1]$
Prelec (1998)	$\varphi(p) = e^{-\beta(-\ln p)^\alpha}$	$0 < \alpha < 1, \beta > 0$
Lattimore, Baker and Witte (1992)	$\varphi(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$	$\gamma > 0, \delta > 0$

Figure 2: Rates of returns by odds

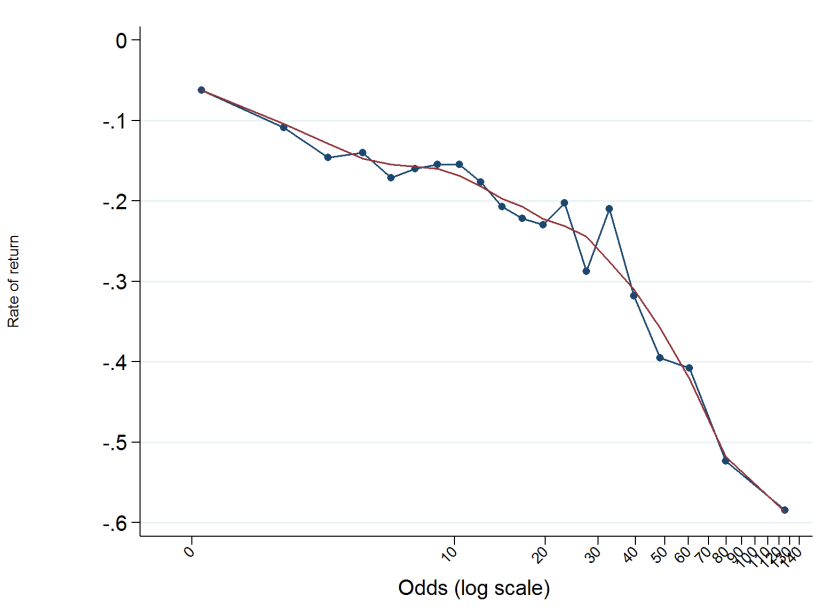


Table 3: Expected utility model

	EU model
$a\theta$	-0.014*** (0.0011)
Max LL	-45,977.315

Table 4: Rank-dependent utility models

	Power	CD	LBW	Prelec	KT
$a\theta$	-0.011*** (0.0019)	-0.011*** (0.0026)	-0.016*** (0.0029)	-0.013*** (0.0038)	-0.011*** (0.0018)
α	0.97*** (0.014)			0.91*** (0.045)	
a_1		0.99*** (0.026)			
p_0		0.99*** (0.22)			
γ			1.05*** (0.035)		0.98*** (0.011)
δ			1.48*** (0.237)		
β				1.28*** (0.077)	
Max LL	-45,975.289	-45,974.628	-45,972.658	-45,972.481	-45,975.363

Figure 3: Rank-dependent theory - Estimated probability weighting functions

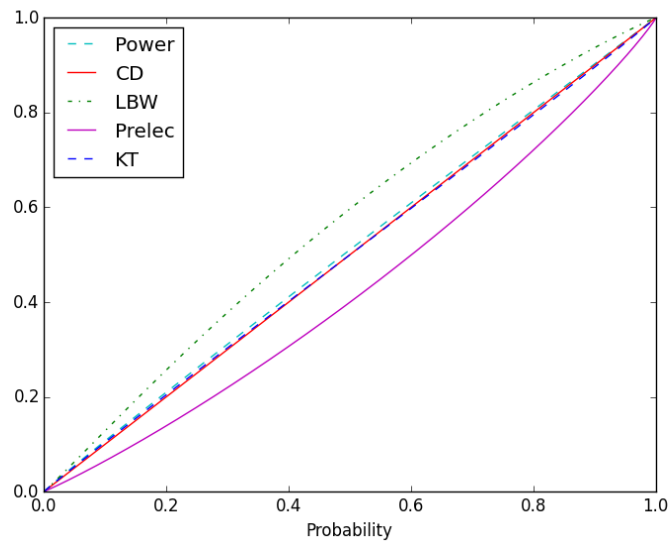


Table 5: Cumulative Prospect Theory models

	Power	CD	LBW	Prelec	KT
$a\theta$	-0,017*** (0.0027)	-0.016*** (0.0048)	-0.015*** (0.0038)	-0.0089** (0.0044)	-0.014*** (0.0021)
α	1,078*** (0.036)			0.75*** (0.26)	
β	0,474*** (0.123)			1.79 (1.35)	
α'				0.99 (2.94)	
β'				0.084 (0.40)	
α_1		1.05*** (0.15)			
p_0		0.99*** (0.25)			
α'_1		0.74 (0.85)			
p'_0		0.99*** (0.31)			
γ			1.05*** (0.085)		0.99*** (0.012)
δ			0.39 (1.00)		
γ'			0.96 (2.35)		0.77*** (0.072)
δ'			16.94 (123.00)		
Max LL	-45,972.178	-45,971.818	45,970.667	-45,970.320	-45,972.613

Figure 4: Cumulative Prospect Theory - Estimated probability weighting functions for gains

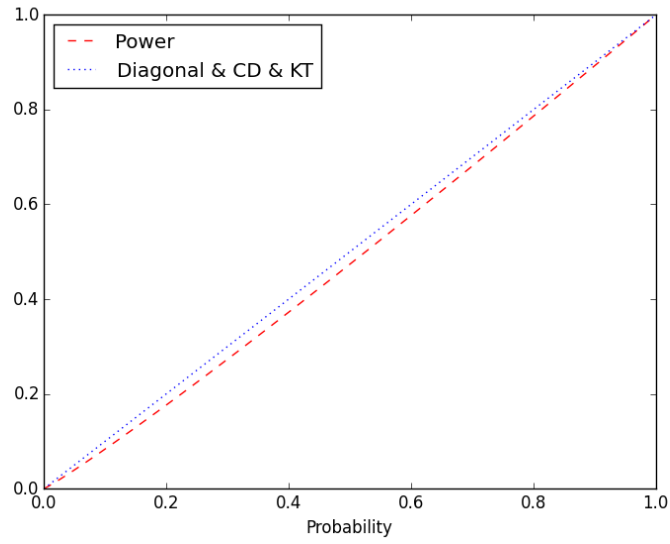
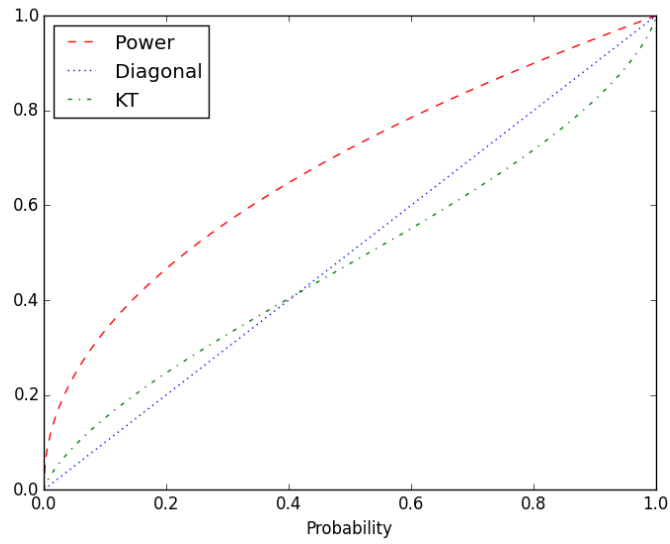


Figure 5: Cumulative Prospect Theory - Estimated probability weighting functions for losses



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Appendices

A Overall value of a bet in each model

The representative bettor bets a in each race and is endowed with an initial wealth M . The choice of a particular horse depends only on the probability of winning of the horse and final odds written $(O_1, p_1), (O_2, p_2), \dots, (O_N, p_N)$ for the N horses of a race.

A.1 Expected utility model

The final state of endowment of a bettor wagering on horse i can either be $M - a$ if the horse loses the race or $M + aO_i$ if the horse finishes first. The former occurs with probability $1 - p_i$ and the latter with probability p_i . The overall value of a bet on horse i is hence $W(H_i) = p_i * u(M + aO_i) + (1 - p_i) * u(M - a)$, where u is a continuous and strictly increasing utility function with $u(0) = 0$.

A.2 Rank-dependent utility model

In the rank-dependent utility model (Quiggin (1982)), the overall value of a prospect with two possible outcomes equals the utility derived from the worst outcome, which the decision maker is sure to get, plus the possible increase in utility from obtaining the best outcome, weighted by the weighted probability of obtaining the best outcome.

The overall value of a bet on horse i is hence $W(p_i, a, M, O_i) = \varphi(p_i) * u(M + aO_i) + (1 - \varphi(p_i)) * u(M - a)$, where φ is the probability weighting function, continuous and strictly increasing from $[0, 1]$ to $[0, 1]$ and satisfying $\varphi(0) = 0$ and $\varphi(1) = 1$.

A.3 Cumulative prospect theory model

The cumulative prospect theory model developed by Kahneman and Tversky (1992) departs from the rank-dependent utility model in that outcomes are perceived as gains and losses with respect to a reference point. Additionally, the value function differs for gains and for losses. It is generally concave for gains and convex for losses; and steeper for losses than for gains to reflect loss aversion.

The probability weighting function overweights small probabilities and underweights moderate and high probabilities. It is inverse S-shaped, meaning concave then convex. The more

curved it is, the more sensitivity to small probabilities changes near the extreme of the probability scale. This property is called diminishing sensitivity. The point where the function intersects the diagonal lies at a probability level of approximately 1/3.

A winning bettor obtains aO_i . This happens with probability p_i . Losses amount to a and occur with probability $1 - p_i$. The probabilities weighting function of gains is written φ^+ and that of losses φ^- . Both functions are strictly continuous and increasing from the unit interval into itself and satisfy $\varphi^+(0) = \varphi^-(0) = 0$ and $\varphi^+(1) = \varphi^-(1) = 1$. The overall value of a bet on horse i is $W(p_i, a, O_i) = \varphi^+(p_i) * u(aO_i) + \varphi^-(1 - p_i) * u(-a)$.

Note that utility is the same for gains and losses. The reason is that a is not observed so I do not have enough data to identify u in the domain of losses. Hence loss aversion is not modeled here. Note also that the reference point is the *status quo* which corresponds to not betting. Two key aspects of cumulative prospect theory are hence modeled here: reference-dependence and different probability weighting of gains and losses.

B Obtaining p_1 to estimate the parameters of the models

B.1 Expected utility model

In equilibrium, $\forall i \in 1, \dots, N$, $p_i * u(M + aO_i) + (1 - p_i) * u(M - a) = w$.

So that:

$$p_i = \frac{w - u(M - a)}{u(M + aO_i) - (M - a)} \quad (3)$$

Because $\sum_{i=1}^N p_i = 1$,

$$w = u(M - a) + \frac{1}{\sum_{j=1}^n \frac{1}{u(M + aO_j) - u(M - a)}} \quad (4)$$

Combining equations 3 and 4 solves the model for p_i :

$$p_i = \frac{1}{u(M + aO_i) - u(M - a)} * \frac{1}{\sum_{j=1}^n \frac{1}{u(M + aO_j) - u(M - a)}} \quad (5)$$

Given equation 5 and the shape of u ,

$$p_i = \frac{1}{e^{a\theta} - e^{-a\theta O_i}} * \frac{1}{\sum_{j=1}^n \frac{1}{e^{a\theta} - e^{-a\theta O_j}}}$$

Note that a , which is not observed in the data, cannot be disentangled from θ .

B.2 Rank-dependent utility model

In equilibrium $\forall i \in 1, \dots, N$, $\varphi(p_i) * u(M + aO_i) + (1 - \varphi(p_i)) * u(M - a) = w$.

So that:

$$\varphi(p_i) = \frac{w - u(M - a)}{u(M + aO_i) - (M - a)} \quad (6)$$

Writing Ψ the reciprocal of φ (which exists since φ is strictly increasing):

$$p_i = \Psi\left(\frac{w - u(M - a)}{u(M + aO_i) - u(M - a)}\right) \quad (7)$$

Because $\sum_{i=1}^N p_i = 1$:

$$\sum_{j=1}^n \Psi\left(\frac{w - u(M - a)}{u(M + aO_j) - u(M - a)}\right) = 1 \quad (8)$$

Solving this equation, which cannot be done analytically, gives w . Replacing w in equation 7 solves the model for p_i .

B.3 Cumulative prospect theory

In equilibrium,

$$\forall i \in 1, \dots, N, \varphi^+(p_i) * u(aO_i) + \varphi^-(1 - p_i) * u(-a) = w. \quad (9)$$

Using the fact that $\sum_{i=1}^N p_i = 1$ and combining it with equation 9 solves the model for p_i . It can not be done in a closed form so p_1 is obtained numerically.

C Details on probability weighting functions by model

C.1 Rank-dependent utility model

C.1.1 Power probability weighting function

The probability weighting function has the shape:

$$\varphi(p) = p^\alpha$$

where $\alpha \geq 0$. If the weighted probability of winning the bet is inferior to the real probability ($\alpha > 1$), bettors underestimate the overall value of a bet, they are pessimistic. The expected utility model is nested within this model for $\alpha = 1$.

C.1.2 Cicchetti and Dubin probability weighting function

The function introduced by Cicchetti and Dubin (1994) is :

$$\frac{\varphi(p)}{1 - \varphi(p)} = \left(\frac{p}{1-p}\right)^{a_1} * \left(\frac{p_0}{1-p_0}\right)^{1-a_1}$$

$\varphi(p)$ crosses the diagonal in p_0 . a_1 is positive. If $a_1 < 1$, the function is convex, then concave. Inversely, if $a_1 > 1$, it is first concave, then convex. The closer a_1 is to 1, the closer to the diagonal the function is, which means relatively little sensitivity to small probabilities changes near the extreme of the probability scale and high sensitivity far off the extremes of the probability scale. When $a_1 = 1$, we are back to the expected utility model and p_0 is not identified.

This function is strictly increasing, its inverse is :

$$\Psi(p) = \frac{\left(\frac{p}{(1-p)*A}\right)^{1/a_1}}{1 + \left(\frac{p}{(1-p)*A}\right)^{1/a_1}} \text{ with } A = \left(\frac{p_0}{(1-p_0)}\right)^{1-a_1}$$

C.1.3 Lattimore, Baker, Witte probability weighting function

The function proposed by Lattimore, Baker and Witte (1992) is:

$$\varphi(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$$

where δ and γ are strictly positive. δ primarily controls the elevation of the function. It captures the extent of pessimism or optimism. γ primarily controls curvature (i.e., sensitivity to changes in probabilities). When $\delta = 1$ and $\gamma = 1$, we are back to the expected utility model. This function is strictly increasing, let Ψ be its inverse.

$$\Psi(p) = \frac{\left(\frac{p}{\delta(1-p)}\right)^{1/\gamma}}{1 + \left(\frac{p}{\delta(1-p)}\right)^{1/\gamma}}$$

C.1.4 Prelec probability weighting function

The probability weighting function proposed by Prelec (1998) is:

$$\varphi(p) = e^{-\beta(-\ln p)^\alpha}$$

where $0 < \alpha < 1$ and $\beta > 0$. If $\beta = 1$ and $\alpha = 1$, we are back to expected utility. It nests the power specification for $\alpha = 1$. α represents the sensitivity to probabilities: the smaller alpha is, the more curved the function. $\beta < 1$ shows optimism, $\beta > 1$ pessimism. β hence controls the elevation of the function.

This function is strictly increasing, I write its inverse Ψ .

$$\Psi(p) = \exp\left(-\left(-\frac{\ln p}{\beta}\right)^{1/\alpha}\right)$$

C.1.5 Kahneman and Tversky probability weighting function

The probability weighting function of Kahneman and Tversky (1992) is :

$$\varphi(p) = \frac{p^\gamma}{[p^\gamma + (1-p)^\gamma]^{1/\gamma}}$$

If $\gamma = 1$, we are back to the expected utility hypothesis.

C.2 Cumulative prospect theory

C.2.1 Power probability weighting functions

Probability weighting functions have the following shape:

$$\varphi^+(p) = p^\alpha, \varphi^-(p) = p^\beta$$

If $\alpha = 1$ and $\beta = 1$, we are back to the expected utility hypothesis, except that utility applies to gains and losses rather than to final wealth. Since M cancels out in the expected utility model, it is nested within this model.

C.2.2 Cicchetti and Dubin probability weighting functions

Assuming that the probability weighting functions have the following shapes:

$$\frac{\varphi^+(p)}{1 - \varphi^+(p)} = \left(\frac{p}{1-p}\right)^{a_1} * \left(\frac{p_0}{1-p_0}\right)^{1-a_1}$$

$$\frac{\varphi^-(p)}{1 - \varphi^-(p)} = \left(\frac{p}{1-p}\right)^{a'_1} * \left(\frac{p'_0}{1-p'_0}\right)^{1-a'_1}$$

These functions cross the diagonal in p_0 and p'_0 . a_1 and a'_1 are positive. If $a_1 = a'_1 = 1$, we are back to the expected utility hypothesis. $a_1 = a'_1$ and $p_0 + p'_0 = 1$ is equivalent to rank-dependent utility with the Cicchetti and Dubin probability weighting function. The reflection case occurs when $a_1 = a'_1$ and $p_0 = p'_0$.

C.2.3 Lattimore, Baker, Witte probability weighting functions

Assuming that the probability weighting functions have the following shapes:

$$\varphi^+(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$$

$$\varphi^-(p) = \frac{\delta' p^{\gamma'}}{\delta' p^{\gamma'} + (1-p)^{\gamma'}}$$

where δ , γ , δ' and γ' are strictly positive. When $\delta = \gamma = \delta' = \gamma' = 1$, we are back to the expected utility hypothesis. The reflection case occurs when $\delta = \delta'$ and $\gamma = \gamma'$.

C.2.4 Prelec probability weighting functions

Assuming that the probability weighting functions have the following shapes:

$$\varphi(p) = e^{-\beta(-\ln p)^\alpha}$$

$$\varphi(p) = e^{-\beta'(-\ln p)^{\alpha'}}$$

where $0 < \alpha < 1$, $\beta > 0$, $0 < \alpha' < 1$ and $\beta' > 0$. If $\beta = \alpha = \beta' = \alpha' = 1$, we are back to expected utility. It nests the power specification for $\alpha = \alpha' = 1$. The reflection case occurs when $\alpha = \alpha'$ and $\beta = \beta'$.

C.2.5 Kahneman and Tversky probability weighting functions

Assuming that the probability weighting functions have the following shapes:

$$\varphi^+(p) = \frac{p^\gamma}{[p^\gamma + (1-p)^\gamma]^{1/\gamma}}$$

$$\varphi^-(p) = \frac{p^{\gamma'}}{[p^{\gamma'} + (1-p)^{\gamma'}]^{1/\gamma'}}$$

γ and γ' are positive. If $\gamma = \gamma' = 1$, we are back to the expected utility hypothesis. The reflection case occurs when $\gamma = \gamma'$.