Land Resources, Industrialization and the Feedback between Industry and Agriculture

(Preliminary version)

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February 27, 2017

Abstract

By using a simple model of structural change and of interactions between the primary and the secondary sector, this paper gives a simple explanation to the ambiguity that natural resources in the agricultural sector play in industrialization. In a closed economy, a low resource endowment will hinder industrialization if the country's population is close to a state of starvation. In a price-taking, open economy, a low resource endowment will accelerate industrialization, but this earlier take-off should not have welfare consequences. Additionnally, by postulating a simple feedback between industry and agriculture through an increased variety of agricultural inputs, we show that Engel's law creates a situation of multiple equilibria which might account for the poverty trap of countries facing a low agricultural productivity.

1 Introduction

An ongoing question of development economics is the link between land resources and economic take-off. Although one might be tempted to argue that a high natural resource endowment in agriculture is a facilitator of economic growth, sylized facts of structural change for both developed and developing countries have repeatedly called this insight into question. It is a well-known fact among economic historians that countries endowed with higher than average land resources usually industrialize later, because of a high opportunity cost of labor (Chenery, 1988). At the same time, numerous authors (Drechsel *et alii*, 2001; Sachs, 2001; ELD Initiative, 2015), have emphasized that a low agricultural productivity linked with low soil fertility could be a serious obstacle to economic growth. This paper is an attempt to reconcile both lines of argument into a single theoretical framework with features of structural change.

Previous theoretical work on structural change include the works of Matsuyama (1992), Laitner (2000), Kongsamut, Rebelo & Xie (2001), Gollin, Parente & Rodgerson (2002), Ngai and Pissarides (2004), Irz & Roe (2005). Matsuyama (1992) and Gollin, Parente & Rodgerson (2002) insist on a high total factor productivity (TFP) in agriculture to shift labor from agriculture to industry and industrialization to result. Matsuyama however states that a high agricultural productivity might be damaging in the context of an open economy and learning-by-doing effects in the manufacturing sector. Laitner (2000) shows that features of structural change produce an endogenous increase in the saving rate and a relative reallocation of wealth in

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reproducible capital. Kongsamut, Rebelo & Xie (2001) prove that a balanced growth path is compatible with the main features of structural change and among them, the rise of the employment and output share of services. Ngai & Pissarides give simple relationships between the long-run growth rate of TFPs in different sectors and their relative prices and share in employment, and show that all employment converge to the sector with the lowest growth in TFP and the sector producing investment goods. Finally, Irz and Roe (2005) use a numerical procedure to state that land resource per capita ought to affect positively the share of labor in manufacturing and to improve the rate of capital accumulation.

None of these papers give clear theoretical statements on the role of land resource endowment in industrialization, and none of them endogenize improvements in the total factor productivity of agriculture. This is the aim of the present paper, along with studying the role of openness to trade. Section 2 presents a simple model with no feedback between industry and agriculture and discuss the role that land resource will play under different assumptions on the elasticity of substitution between land and labor in agriculture. Section 3 builds a more sophisticated version of the earlier model by assuming that the development of the manufacturing sector increase the variety of inputs available to farmers. Section 4 asks how the frameworks developped in section 1 and 2 would translate into an open economy and discuss how international trade change our perception of the role of natural resources. Section 5 concludes the paper.

2 The basic framework

We start by a simple framework where there are no feedback effects between agriculture and industry. There are only two sectors of productions. The primary sector (Y^A), agriculture, uses labor (L^A) and land (S). By land, we do not mean merely the surface of arable land but also land fertility, a cricital component of agricultural yields (Sachs 2001, ELD Initiative 2015). S is thus interpreted as a product of land surface with average land fertility. For now, we do not focus on land utilization through harvesting and simply assume that land enters the production function directly. We simplify the analysis by assuming no capital accumulation. This sector is therefore to be seen as the "traditional" primary, labor-intensive sector where most of the population would work initially. The secondary sector (Y^M), the modern sector, is identified with manufacturing and services and uses only labor (L^M) as input. Additionally, the labor force (identified with population) is constant.

$$L^A + L^M \leq L \tag{1}$$

$$Y^M = A_0 F(L^M) \tag{2}$$

$$Y^A = B\,G(S, L^A) \tag{3}$$

Therefore at this stage both manufacturing and agricultural outputs are entirely consumed. We complete the description of the technology by fairly general hypotheses on the production functions, that will be made more specific along the way:

$$F_{L^M}, F_K, G_{L^A}, G_S > 0$$
$$\text{F, G homogeneous of degree one} \tag{4}$$
$$\text{F, G concave}$$

The assumption of homogeneity of degree one immediately imply :

$$Y^M = A_0 A_1 L^M = A L^M \tag{5}$$

Additionnally, we rewrite the production functions in per capital terms, assuming that labor is fully utilized :

$$l^A + l^M = 1 \tag{6}$$

$$y^M = A l^M = A(1 - l^A) \tag{7}$$

$$y^A = B G(s, l^A) \tag{8}$$

Where small letters stand for per capita terms. We assume the existence of a representative household having perfect foresight and taking prices as given. Its preferences are given by the following utilitarian framework:

$$W = \int_0^\infty e^{-(\rho - n)t} \frac{u(t)^{1-\theta}}{1-\theta} dt \tag{9}$$

$$\text{where} \quad u(t) = (c^A(t) - \lambda)^\nu (c^M(t))^{1-\nu}, \quad \nu \in (0, 1) \tag{10}$$

Note that we did not allow for capital accumulation or any type of intertemporal choice yet, so the role of the discount rate is to be very limited-making welfare comparison of different consumption paths- and will not affect short-term and long term variables. The felicity function is a simple form embodying Engels' law that was first used by Matsuyama (1992) in the context of structural change. This minimum food level is by aggregation an average of the minimum food level of the whole population.

The representative household owns the land (the per capita amount of land to be precise) and is endowed with one unit of labor. We normalize the price of the manufacturing good to one and denote the price of the agricultural good by p^A, the price of the rent on land by s. Thus the budget constraint of the representative household is the following :

$$p^A c^A + c^M = w + rs \tag{11}$$

For now, land is evolving exogenously and the representative household has no grip on its per-capita level. (9) and (11) imply the following interior solution of utility maximization :

$$\frac{u_{c^A}}{u_{c^M}} = p^A \tag{12}$$

Production decisions in agriculture and manufacturing are taken by profit-maximizing entities, acting as price-takers:

$$A = w$$

$$p^A B G_{l^A}(s, l^A) = w \tag{13}$$

$$p^A B G_s(s, l^A) = r$$

Using (12) and (13) we obtain:

$$\frac{u_{c^A}}{u_{c^M}} = \frac{A}{B G_{l^A}} \tag{14}$$

An equality between marginal rate of substitution and marginal rates of transformation. To complete the description of this market economy, market clearing involves :

$$c^M = y^M = A(1 - l^A) \tag{15}$$

$$c^A = y^A = B G(s, l^A) \tag{16}$$

We now want to determine the equilibrium share of labor in agriculture and manufacturing. Using (10) in (14) we have

$$\frac{v c^M}{(1-v)(c^A - \lambda)} = \frac{A}{B G_{l^A}} \tag{17}$$

Replacing c^M and c^A using (15) and (16) and rearranging:

$$\frac{v}{(1-v)}(1 - l^A) = \frac{B G(s, l^A) - \lambda}{B G_{l^A}(s, l^A)} \tag{18}$$

Let us write the right-hand side term as a single function :

$$\frac{v}{(1-v)}(1 - l^A) = g(s, l^A, B, \lambda) \tag{19}$$

We assume l^A to be interior. g has the following derivatives:

$$\frac{\partial g}{\partial l^A} = 1 + \frac{(B G - \lambda)(-G_{l^{A2}})}{B(G_{l^A})^2} > 0 \tag{20}$$

$$\frac{\partial g}{\partial s} = \frac{B G_s G_{l^A} + (B G - \lambda)(-G_{l^A s})}{B(G_{l^A})^2} \tag{21}$$

$$\frac{\partial g}{\partial B} = \frac{\lambda}{B^2 G_{l^A}} > 0 \tag{22}$$

$$\frac{\partial g}{\partial \lambda} = -\frac{1}{B G_{l^A}} < 0 \tag{23}$$

Where the signs are deduced from the set of assumptions (4). Next notice that the implicit differential of l^A with respect to any argument different than v writes :

$$\frac{\partial l^A}{\partial x} = -\frac{g_x}{g_{l^A} + \frac{v}{1-v}} \tag{24}$$

And since the denominator of (24) is positive, l^A has derivatives with opposite sign than the derivatives of g. Finally, the derivative of l^A with respect to v is :

$$\frac{\partial l^A}{\partial v} = \frac{(1-l^A)}{(1-v)[(1-v)g_{l^A} + v]} > 0 \tag{25}$$

To summarize :

$$\frac{\partial l^A}{\partial s} > 0 \text{ or } < 0$$

$$\frac{\partial l^A}{\partial B} < 0$$

$$\frac{\partial l^A}{\partial \lambda} > 0 \tag{26}$$

$$\frac{\partial l^A}{\partial v} > 0$$

The signs of l^A with respect to B, λ and v have a straightforward interpretation. What remains to determine is the sign of l^A with respect to s , in words conditions under which an abundance of natural resources will shift labor from agriculture to manufacturing or the contrary. The relationship of l^A to s is ambiguous, unless we make the uncomfortable assumption of negative cross effects in the agricultural sector, that is:

$$G_{l^A n} \leq 0 \quad \Rightarrow \quad \frac{\partial l^A}{\partial s} < 0 \tag{27}$$

But there is no reason to believe that such a relationship holds. Turning to a simple form like the Cobb-Douglas form is tempting but would most likely be a mistake since we cannot expect strong subtitution possibilities in a natural-resource oriented sector. As land surface or its fertility decline it seems difficult to keep the same level of production by simply adding even a large amount of worker. That is why we can expect natural resources and worker to be close complements rather than imperfect substitutes, most likely having an elasticity of substitution less than one. For the sake of intuition, it will be useful to work on a CES production function:

$$G(s, l^A) = (\beta s^{\frac{\sigma-1}{\sigma}} + (1-\beta)(l^A)^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \tag{28}$$

As a starting point, one might want to know if l^A is uniquely defined in terms of s. Appendix A shows that, with this fairly general specification, it turns out to be the case. Hence,

we now that there is a *unique* way to optimally allocate l^A for each s.

Second, is l^A a monotonic function of s ? Implicit differentiation shows in which conditions we must answer by the negative. Solving for the implicit differential of l^A with respect to s:

$$\frac{\partial l^A}{\partial s} = \frac{\frac{BG-\lambda}{BG} - \sigma}{\frac{BG-\lambda}{BG}\frac{s}{l^A} + \epsilon_0\left(\frac{s}{l^A}\right)^{\frac{1}{\sigma}}} \tag{29}$$

with

$$\epsilon_0 = \frac{(1-\beta)}{\beta}\frac{\sigma}{(1-\nu)} \tag{30}$$

Since $\frac{BG-\lambda}{BG} < 1$, l^A will be monotonic if σ is greater than one. All the analysis is summarized in the following proposition, which is proven in the appendix.

Proposition 1 *Let G be defined as in (28) and l^A be defined implicitly by the relation (29). Define $\bar{s}$ such that $l^A(\bar{s}) = 1$. The following holds:*

l^A is uniquely defined, continuously derivable for any value of $s \in [\bar{s}; +\infty[$. Besides,

- *If $\underline{\sigma \geq 1}$, then $l^A(s)$ is convex strictly decreasing.*

- *If $\underline{\sigma < 1}$, then*

  if $\underline{\lambda = 0}$, $l^A(s)$ is concave strictly increasing.

  if $\underline{\lambda > 0}$, there is a unique $\hat{s} \in]\bar{s}; +\infty[$ such that $\frac{\partial l^A}{\partial s}(\hat{s}) = 0$; l^A is strictly decreasing on $[\bar{s}; \hat{s}]$, then strictly increasing on $[\hat{s}; +\infty[$. Concave or convex regions depend on the parameters β, ν chosen.

The case $\sigma < 1$-the more likely, as argued before-together with $\lambda > 0$ provide for interesting dynamics. An increase in land surface or fertility will first shift labor from agriculture to industry and then back again into agriculture. To understand, one must first study the case $\lambda = 0$. In this situation, labor and land are close complements, and an increase in land resources call for more labor to be invested in agriculture to develop the land. Hence the share of labor devoted to agriculture quickly rise with land resources then stabilize around an asymptote as in Figure 1. In this situation, however, as land resources diminish, substitution between agricultural and manufacturing goods happens entirely smoothly. It means that close to the y-axis the representative household will consume an amount of food as small as one could imagine, continuously substituting small amounts of food with more manufacturing goods. This surely is not a realistic situation, because individuals would die out of hunger below some threshold. When taking into account this threshold, the substitution is non-smooth. When agricultural output come close to the minimum food requirement λ, shifting additional labor from agriculture to industry would cause agricultural output to equal λ and the marginal utility of agricultural goods to become infinite. Labor in the agricultural sector wil then increase so that the representative household can keep above λ. In words, when s is close to zero, even if the marginal productivity of a laborer *in terms of agricultural output* is virtually nil, this laborer cannot possibly be put to work in manufacturing because that would put the population in a state of starvation.

To conclude, no matter how little are the substitution possibilities between l^A and s, there is always a neighborhood of zero, corresponding to a state of near starvation, where l^A is a strictly decreasing convex function.

Even so it is not possible to solve explicitly for the value $\hat{s}$, it is simple to tell using agricultural output when this turning over of the function occurs. Looking at equation (30) it can be expressed as:

$$\frac{\partial l^A}{\partial s} = 0 \Leftrightarrow B\,G = \frac{\lambda}{1 - \sigma} \tag{31}$$

If σ is close to one, an extremely high G is required for l^A to become increasing in s. If we restrict the analysis to some bounded range of resource per capita, we then have a decreasing relationship all along. Hence, the "length" of the initial decreasing relationship depends positively on λ and σ.

This simple model already gives a good idea of why a closed economy with poor land endowment per capita might have a hard time industrializing quickly, given that most labor is "trapped" into the resource sector. Before we add sophistication to it, another important question is yet to be answered. Having in mind the increasing relationship between l^A and s after the agricultural output threshold (32) is reached, one might wonder if having a huge resource endowment might also slow down the industrialization process.

Looking at calibration of the model with reasonable range of parameters, it doesn't seem that the share of labor devoted to agriculture increase very sharply once the turning point has been reached. The increase would be limited to one or two percentage points, provided the utility weight of manufacturing is reasonable. Even with limited qualitative importance, the idea that arises here-and that will become critical when we look at an open economy- is that an important endowment of resource creates a high opportunity cost of labor and makes it difficult for the manufacturing sector to hire, an idea that has been part of the stylized fact of development economics for a while.

3   The feedback between industry and agriculture

Let us now consider what would happen if the agricultural sector was to benefit from inputs or capital goods such as fertilizer, hybridized seeds and irrigation infrastructures. What we need to take into account is how the development of the manufacturing sector will increase the *availability* of substitutes to the agricultural sectors, provided that such inputs were not available at early stages of industry development. The rational for this is that, nascent industries do not have enough experience to produce sophisticated goods, and usually start by very simple manufactured products such as clothes or cigars. As the industry matures, an heavy industry emerge that provides for all kind of chemicals and metals and make the way for more sophisticated products. In the present case of interest, these sophisticated products would include fertilizer, water pipelines, improved seeds, agricultural machines, greenhouses, and so on.

Our starting point in modelizing this phenoma is the assumption of an increased availability of *resource-augmenting* inputs that substitute imperfectly with land resources. We thus define the *augmented resource* by :

$$\hat{S} = \left(\int_0^{J(t)} I(t,u)^{1-\gamma}\,du \right) S^{\gamma} \tag{32}$$

where $I(t,u)$ is the amount of input of type u at time t, and J is the total variety of inputs. This formulation is part of the well-known Dixit-Stiglitz framework of increasing input variety. We assume that inputs are produced out of manufacturing goods with a unique, constant returns to scale technology:

$$I^S(t,u) = \eta M(t,u), \quad \forall u \in [0,J], \; \eta > 0 \tag{33}$$

where $Y(t,u)$ is the supply of input of variety u at time t and $M(t,u)$ is the amout of manufacturing goods used in producing this input. Let us now consider the demand for input u at time t by farmers. First, the agricultural production function rewrites:

$$Y^A = G(\hat{S}, L^A) = G\left(\left(\int_0^{J(t)} I^D(t,u)^{1-\gamma} du\right) S^\gamma, L^A\right) \tag{34}$$

Where we have assumed $B = 1$ for simplicity. Notice that the function is homogeneous of degree one with respect to labor, land, and inputs. Using per capita variables:

$$c^A = y^A = G\left(\left(\int_0^{J(t)} i^D(t,u)^{1-\gamma} du\right) s^\gamma, l^A\right) \tag{35}$$

where $i(t,u)$ refers to per capita input of variety u at time t. Demand for inputs derive from profit maximization of the representative agricultural firm:

$$\text{Maximize} \quad p_A\, G\left(\left(\int_0^{J(t)} i^D(t,u)^{1-\gamma} du\right) s^\gamma, l^A\right) - w l^A - \frac{1}{\eta}\int_0^{J(t)} i^D(t,u)du$$

$$\text{with respect to} \quad i \in [0;J]$$

Wich gives the following first-order conditions:

$$p_A\, G_{\hat{s}}\, s^\gamma (1-\gamma)\, i^D(t,u)^{-\gamma} = \frac{1}{\eta} \tag{36}$$

$$p^A\, G_{l^A}(s, l^A) = w \tag{37}$$

Using $A = w$ and (37) to replace p_A in (36) gives:

$$A \frac{G_{\hat{s}}}{G_{l^A}} s^\gamma (1-\gamma)\, i^D(t,u)^{-\gamma} = \frac{1}{\eta} \tag{38}$$

For now, we choose a Cobb-Douglas form for G. This is indeed an exageration of the

true substitution possibilities between land resources and labor, but will serve as a starting point for understanding more complex dynamics.

$$G = (\hat{s})^\beta (l^A)^{1-\beta} \tag{39}$$

Using this in (38) gives

$$A \frac{\beta}{(1-\beta)} (1-\gamma) l^A(t) \frac{i^D(t,u)^{-\gamma}}{\int_0^{J(t)} i^D(t,u)^{1-\gamma} dv} = \frac{1}{\eta} \tag{40}$$

The demand for each input is the same.

$$i^D(t,u) = i^D(t) \tag{41}$$

Rewriting (40):

$$A \frac{\beta}{(1-\beta)} (1-\gamma) l^A(t) \frac{1}{J(t) i^D(t,u)} = \frac{1}{\eta} \tag{42}$$

The demand for input is then :

$$i^D(t) = A(1-\gamma)\eta \frac{\beta}{(1-\beta)} \frac{l^A(t)}{J(t)} \tag{43}$$

And by equality of supply (33) and demand (43) :

$$m(t) = A(1-\gamma) \frac{\beta}{(1-\beta)} \frac{l^A(t)}{J(t)} \tag{44}$$

where m is the per-capita volume of manufacturing goods used as inputs for the production of one type of agricultural input.
Omitting the time argument for convenience, market clearing for manufacturing now writes :

$$y^M = c^M + Jm \tag{45}$$

Which gives :

$$c^M = A[1 - (1+\epsilon_1)l^A] \tag{46}$$

$$\text{where} \quad \epsilon_1 = \frac{\beta}{(1-\beta)} (1-\gamma)$$

From the previous definitions :

$$\hat{s} = (Js)^\gamma \left(A(1-\gamma)\eta \frac{\beta}{(1-\beta)} l^A \right)^{1-\gamma} \tag{47}$$

And plugging back (47) into (39):

$$G = \hat{B}(Js)^{\beta\gamma}(l^A)^{1-\beta\gamma} = \hat{B}\hat{G}(Js, l^A) \tag{48}$$

$$\text{where} \quad \hat{B} = (A\eta\epsilon_1)^{\beta(1-\gamma)} \tag{49}$$

Analogously

$$G_{l^A} = \frac{1-\beta}{1-\beta\gamma}\,\hat{B}\,\hat{G}_{l^A}(Js, l^A) \tag{50}$$

We can now derive a more sophisticated version of equation (18):

$$\frac{1-\beta}{1-\beta\gamma}\frac{\nu}{(1-\nu)}[1-(1+\epsilon_1)l^A] = \frac{\hat{B}\hat{G}(Js, l^A) - \lambda}{\hat{B}\hat{G}_{l^A}(Js, l^A)} \tag{51}$$

An equation which has similar features as equation (18), with minor modifications; in particular, proposition 1 fully applies, and since $\hat{G}$ is Cobb-Douglas, the analysis reduces to the case $\sigma = 1$. The only fundamental change, -not withstanding new parameters and a new interpretation- is that what was a relationship between l^A and s is now a relationship between l^A and Js.

We now need to specify how the variety J of inputs evolve through time. To understand what kind of intersectoral relationship emerge with the current framework, let us first assume a simple specification of the evolution of J :

$$J = \zeta y^M, \qquad \zeta > 0 \tag{52}$$

Note that we assume J to depend on a per-capita term, since we would expect the manufacturing sector to gradually gain sophistication as the per-capita manufacturing output increase, and not simply as a result of an expanding labor force.
Let us denote Js by $\bar{s}$ and assume that s is constant. Explicitating (52), we now have a simple relationship between l^A and $\bar{s}$:

$$l^A = 1 - \frac{1}{A\zeta s}\bar{s} \tag{53}$$

An equilibrium in the present framework is defined as a couple $(l^A_*, \bar{s}_*)$, that verify equations (51) and (52). These equations are entirely static, so the economy will start on a couple $(l^A_*, \bar{s}_*)$ and will stay there forever. Let us rewrite the set of equilibria of this economy with the new

notation $Js \equiv \bar{s}$:

$$
\begin{cases}
l^A = 1 - \dfrac{1}{A\zeta s}\bar{s} \qquad (a) \\[4mm]
\epsilon_2[1 - (1 + \epsilon_1)l^A] = \dfrac{\hat{B}\hat{G}(\bar{s}, l^A) - \lambda}{\hat{B}\hat{G}_{l^A}(\bar{s}, l^A)} \qquad (b)
\end{cases}
\tag{54}
$$

$$
\text{where} \quad \epsilon_2 = \frac{1 - \beta}{1 - \beta\gamma}\frac{\nu}{(1 - \nu)}
$$

This system of equations will give rise to either zero, one or two equilibria. The following proposition, proven in the appendix, summarizes what can be known about this system, given that it is not possible to solve for these equilibria in a closed form.

Proposition 2 *Consider the market equilibrium characterized by* (54). *For ζ is high enough, two equilibria $\{(l_1^A, \bar{s}_1); (l_2^A, \bar{s}_2)\}$ exists. Then the following holds:*

$$
\bar{s}_1 < \bar{s}_2 \Rightarrow
\begin{cases}
J_1 < J_2 \\
l_1^A > l_2^A \\
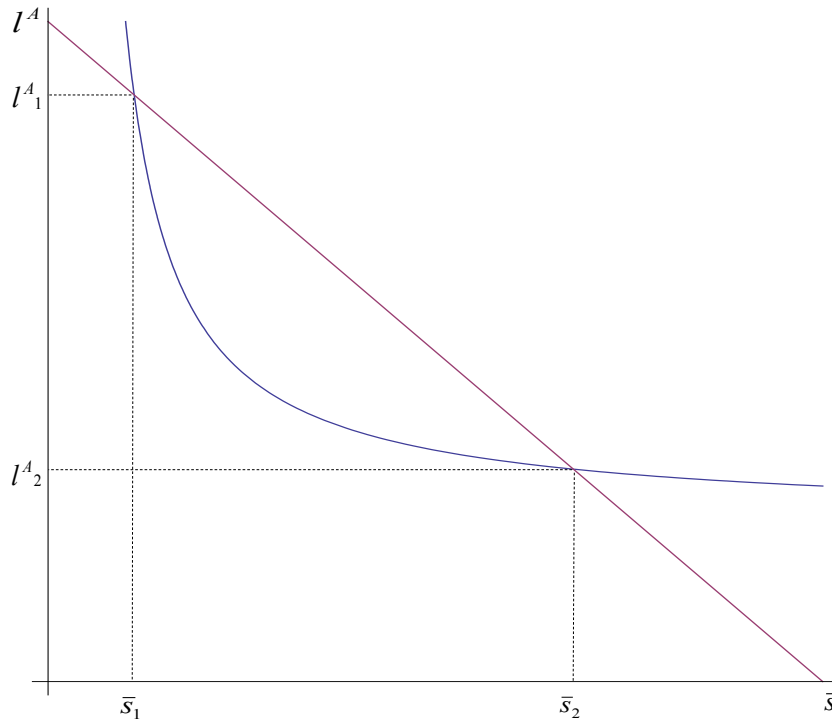Y_1^M < Y_2^M \\
Y_1^A < Y_2^A
\end{cases}
$$

Figure 1 : Multiple equilibria and the poverty trap

Visually, this multiple equilibria phenomenon is easy to see. In figure 1 above, the red line represent equation (a) while the blue line is equation (b) in (54). Because of the interdependence between industry and agriculture, any economy with a high enough ζ can either get stuck in equilibrium 1 with a low augmented resource and a high share of labor in agriculture or "jump" to equilibrium 2 with a high augmented resource and a low share of labor in agriculture. Equilibrium 1 has both low manufacturing output and low agricultural output with respect to equilibrium 2. Thus, the interdepence that we have hypothesized between industry and agriculture creates the possibility of a poverty trap. This mechanism is easy to grasp. The development of agriculture contributes to the growth of the manufacturing sector through Engel's law, by shifting labor from agriculture to industry. The development of the industrial sector on the other hand increases the availability of agricultural inputs and contributes to the growth of agriculture. It is then possible to be trapped in a situation of locked-in where both sectors are atrophied.

To insist, Engel's law (a positive λ) is necessary for a situation of multiple equilibria to happen. In the Cobb-Douglas case, a zero λ actually means that the share of labor devoted to agriculture is constant. But provided l^A is constant, there can be only one equilibrium, as figure 2 illustrates.
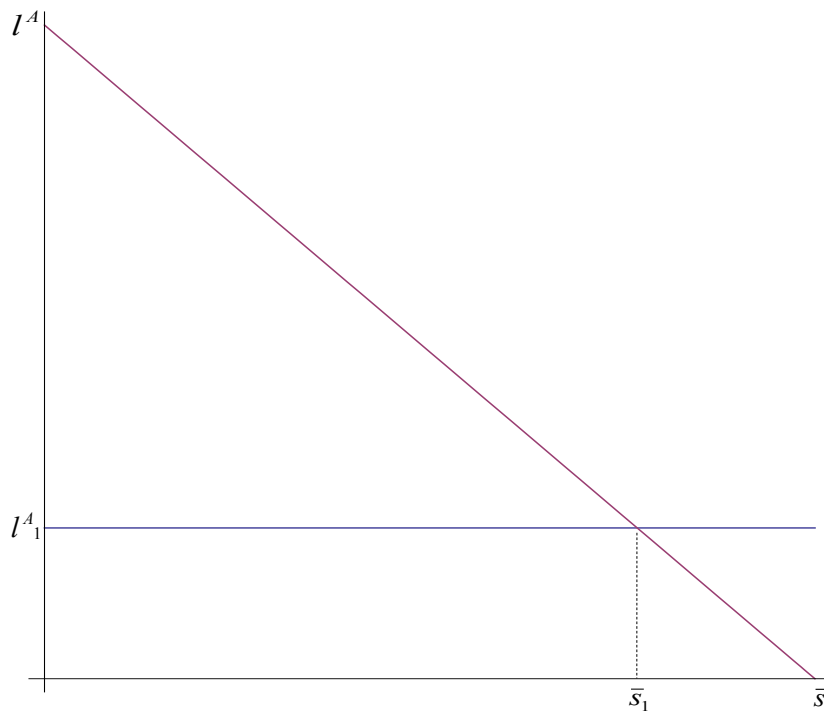

Figure 2 : No minimum food requirement ($\lambda = 0$), a unique equilibrium prevails

Another interesting feature of the new set of equilibria is that total factor productivity in manufacturing is now affecting the allocation of labor through $\hat{B}$ (see equation (49)), while the

allocation of labor was independent of A in the baseline case with no agricultural inputs. To understand what this implies, assume $B \neq 1$ again and recall the equation for total agricultural ouput:

$$Y^A = BG = B(A\eta\epsilon_1)^{\beta(1-\gamma)}(JS)^{\beta\gamma}(L^A)^{1-\beta\gamma} \tag{55}$$

And using equation (52):

$$Y^A = BG = B(A\eta\epsilon_1)^{\beta(1-\gamma)}\zeta^{\beta\gamma}(y^M S)^{\beta\gamma}(L^A)^{1-\beta\gamma} \tag{56}$$

In econometric estimates of agricultural output, economists have been using per-capita manufacturing output as a proxy of the availability of inputs. Imagine that an econometrician estimates the agricultural production function Y^A over S, L^A and y^M. Then the *apparent* total factor productivity will be:

$$\tilde{B} = B(A\eta\epsilon_1)^{\beta(1-\gamma)}\zeta^{\beta\gamma} \tag{57}$$

We have endogenized the apparent TFP of the agricultural sector as being the result of an expanded volume of agricultural inputs used. Suppose B is constant, i.e. productivity improvements in agriculture only comes from the extent of agricultural inputs. Rewriting the preceding equation in terms of growth rates:

$$\frac{\dot{\tilde{B}}}{\tilde{B}} = \beta(1-\gamma)\frac{\dot{A}}{A}, \qquad 0 < \beta(1-\gamma) < 1 \tag{58}$$

The growth of apparent TFP in agriculture is a fraction of the growth of TFP in manufacturing, an insight consistent with empirical findings (Chenery, 1988).

We now turn to a dynamic framework that takes into account population growth. J is now growing as a result of the current per capita manufacturing output.

$$\frac{\dot{J}}{J} = \zeta y^M, \qquad \zeta > 0 \tag{59}$$

The per-capita augmented resource is evolving as a result of the growth of the variety of agricultural inputs, the absolute increase of the land resource, and decrease with population growth:

$$\frac{\dot{\tilde{s}}}{\tilde{s}} = \frac{\dot{J}}{J} + \frac{\dot{S}}{S} - \frac{\dot{N}}{N} \tag{60}$$

We assume that $\frac{\dot{S}}{S} - \frac{\dot{N}}{N}$ is constant and exogenous and use the notation:

$$\frac{\dot{S}}{S} - \frac{\dot{N}}{N} = d \qquad (61)$$

d might be positive or negative. Reasons for a change in the absolute size of land resource (S) are diverse and include geographical extension, desertification, climate change, armed conflicts etc. Geographical extension through land development (deforestation, wetlands draining) is usually described as an important source of increase in land resource during the early stages of industrialization. Climatic factors such as desertification, usually made worse by the current global warming, are serious in most Sub-Saharan countries. Thus this term is likely to be important in practice.

For now, increase in the land resource through geographical extension is considered costless. Note that this is not a benign assumption since the development of new land may require investing an important amount of labor, if draining a swamp is required to cultivate the new land for example. We do not know of any economic study giving a cost to such effort at a country scale.

This yields

$$\frac{\dot{\bar{s}}}{\bar{s}} = \zeta A(1 - l^A) + d \qquad (62)$$

The dynamics are simple to understand:

$* \, d \geq 0$

If $l^A(0) = 1$ then we are back to the case of a subsistence economy; agricultural output is equal to lambda forever and manufacturing output is zero.

If $l^A(0) \neq 1$, $\bar{s}$ grows continuously and l^A decreases along the blue curve of figure 1. As l^A diminishes $\frac{\dot{\bar{s}}}{\bar{s}}$ increases, further accelerating the shift of labor out of agriculture. Over the long-run l^A stabilize around its asymptotic value, agricultural output keeps growing through the development of J and manufacturing output stabilizes.

$* \, d < 0$

If $\bar{s}(0)$ is high enough, then we are back to the preceding case.

If $\bar{s}(0)$ is such that $\frac{\dot{\bar{s}}}{\bar{s}} = 0$ then neither l^A nor $\bar{s}$ move and the economy is static.

If $\bar{s}(0)$ is low enough so that $\frac{\dot{\bar{s}}}{\bar{s}}$ is negative, then the economy will collapse in a finite time, meaning eventually agricultural output will go below the minimum food requirement lambda.

Such a stark, "on the edge" picture of the economy is caused by the constant growth rate of land resource per capita. Ideally, one would like to provide more nuance by assuming more sophisticated dynamics. For example, say $\frac{\dot{S}}{S}$ is zero for simplicity, and we denote by N_{max} the long-run level of population. A logistic growth rate for population yields:

$$\frac{\dot{s}}{s} = \zeta A(1 - l^A) - b(N_{max} - N), \qquad b > 0 \tag{63}$$

An assumption close to the stylized facts of population growth. However, it is difficult to make precise what kind of relationship between b, $N_{m}ax$ and N_0 would make the economy collapse or converge to an industrialized state.

4 International trade

Would all these insights carry over to an economy open to trade ? To understand, let us begin with the simplest framework, one in which the representative agents acts as a price-taker with respect to world prices. It means that $p^A = p^{A^*}$ where p^{A^*} is the world price. We also assume that both labor and land are immobile, but there may be technology diffusion, that is, we can assume $A = A^*$ and $B = B^*$ without modifying any sensitive result.
Recalling (13):

$$A = w$$
$$p^A B G_{l^A}(s, l^A) = w \tag{64}$$

Hence

$$\frac{A}{B G_{l^A}(s, l^A)} = p^{A^*} \tag{65}$$

This gives

$$\frac{\partial l^A}{\partial s} = -\frac{G_{l^A s}}{G_{l^A 2}} \tag{66}$$

Provided the cross-marginal product of G is positive, then an increase in land resource will increase the share of labor devoted to agriculture. Note that assuming a non-homogeneous production function for manufacturing would not change anything to this conclusion, provided this production function is concave. For the case of a CES production function, no matter the value of the elasticity of substitution σ, $G_{l^A s}$ is always positive and there is a strictly increasing relationship between l^A and s. To emphasize the symmetry with proposition 1, we state the following proposition:

Proposition 3 *Let G be defined as in (28) and l^A be defined implicitly by the relation (72). Then the following holds:*

$$l^A = \left[\frac{\beta \epsilon_3^{\sigma-1}}{1 - (1 - \beta)\epsilon_3^{\sigma-1}} \right]^{\frac{\sigma}{\sigma-1}} s, \qquad \sigma > 0 \tag{67}$$

$$where \qquad \epsilon_3 = \frac{B}{A}(1 - \beta)p^{A^*} \tag{68}$$

and

$$\frac{\partial l^A}{\partial \epsilon_3} > 0, \qquad \sigma > 0 \tag{69}$$

l^A is now a linear function of s. Note that l^A is now a function of A, the total factor productivity of the manufacturing sector, while this term was not involved in the determination of l^A in the closed economy case. This much simpler form come from the fact that l^A plays no role in the determination of the relative price p^A.

Also we might wonder why there is no room for a decreasing relationship between l^A and s, like in the closed economy case. This stems from the fact that l^A do not affect the supply of food anymore, and a low l^A cannot push food prices up. Any increase in s increases the marginal productivity of an agricultural worker. But now given the positive cross marginal productivity of l^A and s, this will call in more labor in agriculture to adjust the marginal productivity of agricultural labor downward and preserve the equilibrium. In the closed economy case, this effect was potentially offset by the rising price of food as agricultural output diminishes. In fact, when $\sigma \geq 1$, we have proven that the rising price of food always dominate, even for large quantities of s. To see that more clearly we put in comparison the implicit differentials of the closed economy (left) and the open economy (right).

$$\frac{\partial l^{A\,\text{closed}}}{\partial s} = \frac{(BG - \lambda)G_{l^A s} - BG_s G_{l^A}}{(BG - \lambda)(-G_{l^{A2}}) + B(G_{l^A})^2 \frac{1}{1-\nu}} \qquad \frac{\partial l^{A\,\text{open}}}{\partial s} = \frac{G_{l^A s}}{(-G_{l^{A2}})} \tag{70}$$

The effect of the rising food prices is represented by the term $BG_s G_{l^A}$ on the denominator of the implicit differential on the left, while terms on the left of the denominator and the numerator bear a clear similarity to the terms of the implicit differential on the right. As the term $(BG - \lambda)$ indicates, the argument of *productive efficiency* i.e. the force maximizing output at constant prices, become less pregnant as agricultural output come close to λ. This is another way of saying that as agricultural output comes close to the minimum food threshold, there is more room for a decreasing relationship between l^A and s due to a *price effect*.

Another interesting feature of this situation l^A now depends positively on $\frac{B}{A}$ through equations (68) and (69), so an increase in the total factor productivity (TFP) of agriculture will now *increase* the share of labor devoted to agriculture. But over the course of most development paths, the growth of the TFP in manufacturing rises above the growth of TFP in agriculture, contributing to a downward trend in l^A. Looking at equation (68) again, another reason for a gradual decrease in l^A is a long-run downward trend in the world price of food. These two factors would explain why even countries with high land resource endowment gradually shift labor from agriculture to industry but usually industrialize *later* than countries with lower land resource (Chenery 1988), provided the latter are not subject to the kind of poverty trap studied earlier.

Note that the higher share of labor in the agricultural sector attached to a higher endowment of land is an entirely efficient behavior, and can only be welfare-improving, since it is maximizing income at given prices. Thus, although trade opening could slow industrialization (here understood as the development of the secondary sector) it should not be attached to any negative perception. This is an important remark since late industrialization is sometimes linked with the *resource curse*, that is, the fact that countries having a high endowment of natural resources (broadly defined, not only in agriculture) tend to consume the resource rent instead of investing it in productive capitals. Using yet another idea, the resource curse has also been linked with *learning-by-doing* effects in the manufacturing sector (see Matsuyama, 1992). Even so the resource curse seems a serious problem, it is important to realize that a late industrialization could originate in a purely efficient behavior (independently of preferences: proposition 3 do not depend on any specific utility function). But given what we have said about the role of $\frac{B}{A}$ on l^A, provided there is technology diffusion from the rest of the world, one would

expect industrialization to be merely delayed, not to be hindered at a worrying state.

Is there reasons to believe that the multiple equilibria phenomenon we studied earlier is still there in a small open economy ? A reasonable assumption is to assume that agricultural inputs are freely traded as well, and that countries use the same types of agricultural inputs. This might seem like an innocuous assumption, except that agricultural inputs, as emphasized by Borlaug (1970), are usually climate-specific. For example, fertilizers that have been developed in temperate regions usually cannot be used in tropical or sub-tropical regions, a problem which is still at the core of agricultural research. We therefore assume implicitly that adapting inputs from the world economy to the domestic economy is costless. In this case the variety of agricultural inputs depend on world income:

$$J = \zeta \, y^{M*} = J^*, \qquad \zeta > 0 \tag{71}$$

where y^{M*} stands for world per capita manufacturing output. Again working on the Cobb-Douglas case (which does not change the results qualitatively because of proposition 3) and assuming $B = 1$ to simplify notation, the equivalent of equation (51) is:

$$l^A = (\bar{\epsilon}_3)^{\frac{1}{\beta \gamma}} \, \bar{s} \tag{72}$$

$$\text{where} \qquad \bar{\epsilon}_3 = \frac{\hat{B}}{A} (1 - \beta) p^{A*} \qquad \text{and} \qquad \hat{B} = (A \eta \epsilon_1)^{\beta(1-\gamma)}$$

Using again the notation $Js \equiv \bar{s}$. Plugging equation (71) into (72):

$$l^A = \zeta \, y^{M*} (\bar{\epsilon}_3)^{\frac{1}{\beta \gamma}} \, s \tag{73}$$

l^A is unique and is a linear function of s. How will l^A evolve with a changing A ? Rewriting $\bar{\epsilon}_3$:

$$\bar{\epsilon}_3 = A^{\beta(1-\gamma)-1} (\eta \epsilon_1)^{\beta(1-\gamma)} (1-\beta) p^{A*} \tag{74}$$

An increase in A will again shift labor out of agriculture, but with a *smaller* order of magnitude. This stems from the fact that the manufacturing sector contributes to the productivity of the resource sector through the availability of inputs, and thus an increase in the TFP of the industry is partially offset by an increase in the augmented land resource.

5   Conclusion

We have shown that a closed economy can suffer from both types of problems: if its population is close to starving, no matter how little substitution possibilities there are between labor and land, potentially all the workforce will be trapped in agriculture and this will prevent industrialization. Furthermore, assuming that the development of the manufacturing sector increases the types of inputs that farmers can use, the closed economy is subject to a problem of multiple equilibria where both agriculture and manufacturing can't grow because of their interdependence. This interdependence also endogenize the apparent TFP of the agricultural sector as simply being the result of an expanded volume of agricultural inputs used. An open economy is not in principle subject to the first kind of problem, since a low resource endowment do not generate any price effect and would actually push this country to allocate more labor in industry. However, note that high world food prices would keep labor in the agricultural sector and potentially put the population in a state of starvation. Provided agricultural inputs are freely traded internationally, this open economy will also escape the multiple equilibria problem. However, as noted before, agricultural inputs do not easily adapt to other climate, so trade would be beneficial with more advanced countries sharing the same soil conditions, or countries having an export industry compatible with foreign soil conditions. Overall, our study clearly makes a point for international trade as a way of escaping poverty, provided world food prices are not too high.

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