Abstract

The demand of weather-sensitive products such as beverages varies with the temperature. But when temperatures change, product managers lack a framework to adapt marketing-mix elements, such as pricing and advertising. This article contributes to this framework with theoretical and empirical elements. First, we develop an analytical model linking demand to temperature. The firm decides on price and advertising for branded products and on price for non-branded products. We show that pricing adapts differently to temperature for branded and non-branded products. Second, we empirically investigate the market of branded and non-branded drinking yogurts with an original three-year weekly panel data from French retailers. We find evidence that price and advertising increase with higher temperatures, suggesting complementarity between marketing-mix elements and temperature. Following a temperature rise, sales and price increase even more for branded drinking yogurts, revealing that higher temperatures accentuate the market power of the brand.

Keywords: Pricing, advertising, temperature, weather-sensitivity, marketing mix

Code JEL: D21, D22, M21, M37, L81

1 Introduction

A shopper’s mood, interest, and behavior depend on outdoor temperature (Mittal et al., 2004). Directly tied to weather conditions, temperature is part of a group of seasonal factors that has been widely integrated within demand in sales models (see Herrmann and Roeder 1998 for food products, Parker and Tavassoli 2000 for hedonic goods such as alcohol and coffee, and Bruyneel et al. 2005 for hazardous games). Seasonal factors correspond to uncontrollable variables, but managers also have to monitor controllable variables of the marketing-mix, such as price and advertising (Lam et al., 2001). Monitoring price and advertising should accommodate seasonality to enhance profitability. Despite its impact on profit, the relationship between weather and the marketing-mix is rarely studied (an example is Marion and Walker 1978, who examine the link be-
tween temperature and price-elasticity). More broadly, a comprehensive framework of
the marketing-mix adaptation to weather enhancing demand is still missing. This article
fills the gap, investigating how price and advertising policies of weather-sensitive products
should adjust to temperature change.

This article examines the pricing and advertising policies of temperature-sensitive
products using a theoretical and an empirical perspective. We propose an analytical
framework of dynamic pricing and advertising when demand depends on temperature.
Non-branded products (managed solely by price) and branded products (managed by
price and advertising) are distinguished. We propose an empirical analysis of the French
drinking yogurt market with non-branded and branded products using weekly obser-
vations over a three years period. The literature on sales models integrating weather
elements informs this research.

In prior literature, sales have been widely studied with respect to seasonal factors and
weather conditions. Temperature is integrated into sales models (Kök and Fisher, 2007)
because of its influence on consumer preferences and choices (Mittal et al., 2004; Fergus,
1999). Seasonal factors largely affect food sales (Parker and Tavassoli, 2000), but the
impact of seasonal factors on price remains controversial (Herrmann and Roeder, 1998;
Torrisi et al., 2006). From a managerial point of view, weather-related decisions are more
about a firm’s organization in terms of its operational hedging techniques (Chen and
Yano, 2010), among which are increased production flexibility (Fisher and Raman, 1996),
resource sharing (Van Mieghem, 2003) and the use of derivative financial instruments
(Chod et al., 2010). Change in price-mix also appears as an option for managers to
adapt to temperature, since shopper’s mood and willingness to pay vary with seasonal
factors (Murray et al., 2010). In the same vein, Bertrand et al. (2015) wonder how
efficient are such seasonal changes. These changes can be promotions such as conditional
rebates related to weather (Gao et al., 2012). Regarding advertising, the opportunity to
adjust advertising to temperature is criticized (Deckinger, 1948). As a matter of fact, the
efficiency of such adjustments is principally reviewed for in-store media (Steele, 1951).
The impact of temperature on sales is widely acknowledged in prior literature, but the
adaptation of price and advertising to temperature remains unaddressed.

This article contributes to extant literature by examining how price and advertising adapt when temperature shifts. First, we develop a theoretical model stressing the optimal dynamic policies of price and advertising for temperature-sensitive products. Results reveal that for a general demand function, price and advertising must not increase with temperature. Yet, for an additively separable demand function, price and advertising do increase with temperature. Second, we conduct an empirical analysis for the specific case of drinking yogurts with branded (Yop) and non-branded (retailer name) products. A temperature increase of 10% generates a sales growth of 6.8% in drinking yogurts, and sales increase 81.5% more for branded than for non-branded products. Further, price and advertising increase along with the temperature, the price augmentation being larger for the branded product. The results support the following managerial implications in the drinking yogurt market: Temperature strengthens the market power of the brand for which the product sales and price increase more than for the retailer. Also, price and advertising are complementary with temperature, with whom they increase. This article provides a more comprehensive understanding of the marketing-mix of weather-sensitive products. A product manager who ignores weather implications would charge and advertise inadequately, thereby losing money by disregarding profitable relationships between weather and the marketing-mix.

2 Literature Review

2.1 Integration of Seasonal Factors in Sales Modeling

2.1.1 Impact of Seasonal Factors on Sales

Sales models have been developed to include seasonal factors. In the context of assortment optimization, Kök and Fisher (2007) integrate seasonal factors into demand: the outside weather is included with other parameters, such as the number of clients, daily sales, holidays, and marketing levers such as price and promotion. Seasonal factors are categorized as external factors with an impact on sales. Ramanathan and Muyldermans
identify three groups of external factors: special days (such as holidays), consumer preferences (including for a certain type of products such as innovative products) and seasonal factors (including temperature and period of the year). External factors can also be considered to be uncontrollable variables (see Lam et al. 2001 concerning the retailing sector). In effect, uncontrollable variables (seasonality, day of week, time, economic conditions, and the like) differ in nature from controllable variables (price, promotion, advertising, sales force, and the like). Whereas seasonal factors are included in sales models, further analysis enables to better understand the mechanism of their impact on sales.

Impact mechanisms of seasonal factors on sales are tied to consumer behavior. Indeed, seasonal factors influence human behavior. Consumer needs and choices vary with climate (see Fergus 1999 for examples in the construction sector where needs vary depending on climate). Consumer satisfaction depends on preference criteria that vary throughout geographic data including temperature (Mittal et al., 2004). Inside the shop, customers’ emotional state may be modified by the temperature, with an impact on buying behavior (Baker et al., 1992). In the finance sector, seasonal factors affect investor behavior: analysis of stock prices reveals that investor psychology is correlated to the weather outside. Thus, outside weather influences asset prices (see Saunders 1993 about the New York Stock Exchange and Hirshleifer and Shumway 2003 about worldwide stock exchanges). It is known that seasonal factors influence consumer decisions, though it is hard to measure the precise impact of seasonal factors on sales. At best, sales models isolate seasonal factors: forecasting models require simple extrapolative models (excluding seasonal impact, which are then combined with market information from experts; see Fildes et al. 2008). Therefore, non-seasonal sales baselines estimate the impact of promotions on sales (for instance Cooper et al. 1999 use a baseline at the sub-category level based on 130 weeks). For better inventory planning, seasonal effects should adjust to a non-seasonal sales baseline (Achabal et al., 2000). If sales are correlated to seasonal factors as influencing behaviors, their impact is hard to quantify. Prior research on food markets brings more robust conclusions.
2.1.2 Impact of Seasonal Factors on Food Sales

Seasonal factors influence demand for food products. They influence eating behaviors as do other factors, such as education, age, regional localization, gender, and state of health (Herrmann and Roeder, 1998). For example, in the Italian wine market, temperature is taken into account for linear regression (Torrisi et al., 2006). Moreover, weather conditions influence hedonic goods consumption, such as alcohol and coffee (Parker and Tavassoli, 2000) or lottery tickets (Bruyneel et al., 2005). In addition, price elasticity of food products varies by the season (Marion and Walker, 1978): price-elasticity of beef demand varies with paydays, the number of weeks during the month, and season and temperature. Nevertheless, daily price volatility remains hard to explain. Roll (1984) found an unaccountable daily price volatility in the analysis of future markets on oranges. Food product markets require a thorough analysis in order to better assess seasonal factors and their impact on demand.

Impact of seasonal factors on food sales has to be put into perspective with the specificities of the food product market. Preferences and health drivers interact with seasonal factors. For instance, qualification of a product or service as “good for health” is an increasing determining factor in the shopper’s decision-making process (Caswell, 1998). Thus, changes of preferences (modeled by attitude and knowledge) relativize the importance of price and revenue factors. As a result, demand for food products in industrialized economies may be considered to be non-elastic (Herrmann and Roeder, 1998). Low price elasticity of demand for food products in industrialized countries (Blundell, 1988; Moschini and Moro, 1996) to some extent supports Engel’s law. However, this might not be verified at a micro level because of price wars between stores (Herrmann and Roeder, 1998). As a managerial consequence, the relationship between price and sales of food products requires a deep study of the price-elasticity of demand at the level of point of sale. Thus, the relationship between seasonal factors and sales requires a comprehensive understanding of market specificities. Other relationships contribute to a more accurate assessment impact of seasonal factors’ on demand.
2.2 Relationships between Seasonal Factors and Price and Advertising

2.2.1 Seasonal Factors and Price

Seasonal factors modify demand responses to price. Changes in price relative to seasonal factors are of interest because sunshine affects shoppers’ mood, and their willingness to pay (Murray et al., 2010). Conditional rebates varying according to weather generate a shift in shoppers’ purchasing decisions. Conditional rebates may compensate dissatisfied shoppers because of weather conditions (for example, a rebate could be granted to the buyer of a convertible car if the summer is abnormally rainy). Therefore, weather-conditional rebates induce early sales. As a tool for better discrimination, these rebates may increase profit as well (Gao et al., 2012). Promotional rebate efficiency also might be correlated with temperature if the rebates are activated at the same time as vacation (Ramanathan and Muyldermans, 2010; Cooper et al., 1999). In addition, promotion efficiency can be moderated by season and by product format. Relationships between promotional actions and temperature in the soft drink market are positive but only for a specific soft drink format (Ramanathan and Muyldermans, 2010). From a manager’s perspective, promotions should be planned precisely during the year according to the format (for example, a manager should boost 500ml soft drink sales with promotions during the summer). Yet, as far as price variations with seasons is concerned, manager latitude is limited.

The effectiveness of weather-related managerial decisions is ambiguous. Price levers do not necessarily have a significant ability to compensate negative influences of temperature on sales (Bertrand et al., 2015). In clothing market, managerial decisions following weather abnormalities do not have a systematically significant impact (impact is significant only when temperatures are below normal in the fall season). Hence, weather-related managerial actions on price would not affect sales at an aggregated level. Relationships between seasonal factors and price remain strategic, though for management, the same also applies to advertising.
2.2.2 Seasonal Factors and Advertising

The impact of advertising is related to seasonal factors. For this purpose, measuring efficiency of in-store advertising should take into account seasonal factors: potential future sales take into account temperature, but then are compared to actual sales (Steele, 1951). However, advertising actions independent of seasonal factors may also affect sales. For example, it may be more efficient for a radio program to extend its sponsoring of a brand all year-long, even during the summer when audiences decline, rather than suspending sponsoring or changing radio programs (Deckinger, 1948). Seasonal adaptation of advertising should be done in conjunction with seasonal adaptation of regular marketing-mix.

In general, marketing-mix policy should adapt to seasonal demand sensitivity. But there is a controversy about the use of specific levers, such as advertising. In their study of hedonic consumption, Sun et al. (2009) conclude that marketing-mix should depend on a regional-specific correlation between sales and seasonality. Yet, adaptations of marketing-mix to seasonal factors rarely mention price or advertising but rather specific techniques: in order to adapt to economic uncertainties caused by weather variations, distributors develop operational hedging techniques (Chen and Yano, 2010), including adjustment of product range (Devinney and Stewart, 1988), greater flexibility in terms of production capacity (Fisher and Raman, 1996), delayed product differentiation (Lee and Tang, 1997), diversification and resource sharing (Van Mieghem, 2003), advanced logistics technologies or late season price premium. Financial instruments such as derivatives (see Chen and Yano 2010 for rebates contracts adapted to climate conditions) encourage non risk-averse managers (Chod et al., 2010). Not adapting price or advertising to seasonal factors may result from rigidities in business procedures. Indeed, seasonal variations disturb retailers’ sales planning and their relationships with suppliers (Thomas et al., 2010). Smith et al. (1998) note that retailers suffer from rigidities in planning markdowns and ads. Thus, seasonal adaptations of marketing levers, such as advertising, require a deep analysis of the relationships among the demand factors.
3 Modeling

Section 2 stresses that weather conditions affect demand of meteo-sensitive products. In particular, higher temperature enhances beverage sales. But, the literature review does not expose how the marketing-mix of price and advertising should adapt to higher temperature. After temperature changes, it is not obvious if price and advertising should increase, decrease, or hold constant. In terms of price when temperature increases, the firm could benefit from higher demand to charge more, augmenting markup. Alternatively, it could charge less to enhance even more the demand, generating higher sales. The trade-off between markup and sales drives an ambiguous price-temperature relationship.

In terms of advertising, because temperature (free as a gift of nature) and advertising (costly due to advertisement expenses) both enhance demand, the firm may compensate lower temperature by higher advertising to maintain sales and reduce advertising when temperature raises, substituting temperature for advertising. Conversely, when demand increases following higher temperature, the firm may advertise even more to benefit from a synergy phenomenon, complementing temperature with advertising. The competition between these substitutability and complementarity of advertising and temperature prevents clear deductions about the advertising-temperature relationship. The link between price and advertising may also complicate this analysis further. An analytical modeling is therefore useful to clarify price-temperature and advertising-temperature linkages.

In a stylized theoretical setup, we distinguish two baseline models. The first model describes the case of a non-branded product, for which only price is decided. The second model characterizes the case of a branded product, for which price and advertising decisions are made. We suppose that a monopoly (the manufacturer and the retailer are vertically integrated within one firm) makes these decisions and sells both products in distinct markets (we disregard product line issues). This approach simplifies the reality in which 1) two players—a manufacturer and a retailer—commercialize their product(s) following their own objective, 2) branded and non-branded products may be substitutes, and 3) the advertisement of the branded-product may exert influence on the demand of the non-branded product. These simple baseline models aim to clarify the marketing-mix
adaptation for meteo-sensitive products. We acknowledge though that the cost of such simplifying assumptions represents a loss of realism. However, more realistic models are not necessary to highlight the opposing effects at the demand level, which are sufficient to explain both positive and negative price-temperature and advertising-temperature relationships.

Table 1 defines the notations used in the model analysis.

Table 1: Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$p(t)$</td>
<td>unit price at time $t$ (decision variable)</td>
</tr>
<tr>
<td>$a(t)$</td>
<td>advertising expense at time $t$ (decision variable)</td>
</tr>
<tr>
<td>$w(t)$</td>
<td>temperature (weather) at time $t$ (exogeneous variable)</td>
</tr>
<tr>
<td>$D(p, w)$</td>
<td>demand of non-branded product</td>
</tr>
<tr>
<td>$\pi(p, w)$</td>
<td>profit of non-branded product</td>
</tr>
<tr>
<td>$D(p, a, w)$</td>
<td>demand of branded product</td>
</tr>
<tr>
<td>$\pi(p, a, w)$</td>
<td>profit of branded product</td>
</tr>
</tbody>
</table>

Note. For simplicity, we do not differentiate the demand and profit notations for non-branded and branded products, which refer to different subsections.

3.1 Non-Branded Product

A firm sells a product over time $t \in [0, \infty)$. The firm decides the price $p(t) \in \mathbb{R}^+$. Temperature is a proxy for weather conditions. Temperature $w(t) \in \mathbb{R}$ is given (exogeneous variable) and it changes over time. The demand function $D : \mathbb{R}^+ \times \mathbb{R} \rightarrow \mathbb{R}^+$ is twice continuously differentiable.

The demand $D$ depends on the price $p(t)$ and temperature $w(t)$.

$$D = D(p(t), w(t)).$$

For simplicity, we omit hereafter the functional arguments, where there is no confusion. Also, a subscript variable below a function notes the derivative of that function with respect to that variable. Demand falls with price. The demand is meteo-sensitive in the sense that it augments with temperature. Moreover, it is more difficult to increase the

\footnote{To avoid confusion with time noted $t$, we note temperature $w$, recalling that it approximates weather conditions.}
demand by reducing the price when temperature is high compared to when temperature is low:

\[ D_p < 0, \quad D_w > 0, \quad D_{pw} \leq 0. \]  

These demand assumptions are satisfied with regular demand functions. They cope with the linear demand function \( D = k_0 + k_1p + k_2w \), the Cobb-Douglas (iso-elastic) demand function \( D = k_0p^{k_1}w^{k_2} \), and the exponential demand function \( D = k_0e^{k_1p}e^{k_2w} \) with parameters \( k_1 < 0, k_0, k_2 > 0 \).

The profit function \( \pi : \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}^+ \) is twice continuously differentiable. It reads

\[ \pi(p, w) = pD(p, w). \]

We seek an interior solution for the price, assuming its existence. The profit is thus assumed concave with respect to price \( p \). The necessary and sufficient first- and second-order conditions for \( p \) impose

\[
\begin{align*}
\pi_p &= 0 \implies D + pD_p = 0 \implies e_p = 1, \\
\pi_{pp} &\leq 0 \implies D_p + D + pD_{pp} \leq 0 \implies -2D_p - pD_{pp} \geq 0, 
\end{align*}
\]

with the price elasticity of demand \( e_p = -\frac{D_p}{D} \).

Condition (2a) states that the firm charges a price such that the price elasticity of demand is unitary. Condition (2b) imposes that the demand is not “too” convex with respect to the price, otherwise the concavity assumption of profit is violated. Following Chenavaz (2016a) and Chenavaz (2016b), deeper insights are provided with the decomposition of the condition (2a) with respect to time. A dot above a variable notes the time derivative.

**Proposition 1.** For a non-branded product, the impact of higher temperature on price is
characterized by

\[ \dot{p}( -2D_p - pD_{pp} ) + \dot{w}( D_w + pD_{pw} ) , \]

for all \( t > 0 \).

**Proof.** Differentiate (2a) with respect to time \( t \) and rearrange. \( \square \)

On the left-hand side, the second factor is positive because of (2b). Therefore, the relationship between the signs of \( \dot{p} \) and \( \dot{w} \) is tied to the two terms of the second factor of the right-hand side. The direct effect of temperature on demand \( D_w \) pushes toward a positive relationship between price and temperature. The firm, benefiting from higher demand, can achieve greater rents with a higher price. Conversely, the value of an indirect effect on demand of temperature with price \( pD_{pw} \) drives a negative price-temperature relationship. Indeed, demand increases with temperature, but it could increase even more with a lower price. Thus, the impact of temperature on price is ambiguous, depending on the strength of the direct and indirect effects of temperature on demand. If the indirect effect does not exert influence, the result becomes stronger.

**Corollary 1.** For a non-branded product, if there is no indirect effect at the demand level \( (D_{pw} = 0) \), the impact of higher temperature on price is characterized by

\[ \dot{p}( -2D_p - pD_{pp} ) + \dot{w}( D_w ) , \]

for all \( t > 0 \).

**Proof.** Substitute \( D_{pw} = 0 \) in Proposition 1. \( \square \)

According to Corollary 1, price increases with temperature. If the firm considers the sole direct effect of temperature on demand, then it profits from demand increase to charge more when temperature increases. Note that for additive separable demand function \( D = h(p) + l(w) \), the indirect effect vanishes \( (D_{pw} = 0) \) and Corollary 1 applies.
An instance of additive separable demand function is the linear demand function \( D = k_0 - k_1 p + k_2 w \) already mentioned.

### 3.2 Branded Product

For a branded-product, the firm sets price as for the non-branded product, and it also decides on advertising expense \( a \in \mathbb{R}^+ \). The characteristics of advertising is to increase demand at some cost (Jørgensen et al., 2009; Karray and Martín-Herrán, 2009; Chutani and Sethi, 2012; El Ouardighi et al., 2016a,b).

The demand function, now \( D : \mathbb{R}^{2+} \times \mathbb{R} \to \mathbb{R}^+ \), is twice continuously differentiable. The demand \( D \) depends on the price \( p \), advertising \( a \), and temperature \( w \).

\[
D = D(p, a, w).
\]

Demand falls with price and rises with advertising and temperature; there are diminishing returns of advertising. Plus, it is more difficult to increase the demand by reducing the price when temperature and advertising are high compared to when they are low. There is a synergy phenomenon at the demand-side between advertising and temperature.\(^2\)

\[
D_p < 0, \ D_a > 0, \ D_w > 0, \ D_{aa} \leq 0, \ D_{pw} \leq 0, \ D_{pa} \leq 0, \ D_{aw} \geq 0. \tag{3}
\]

Demand functions satisfying these assumptions include \( D = k_0 p^{k_1} w^{k_2} a^{k_3} \), \( D = k_0 + k_1 p + k_2 w + \frac{1}{k_3} a^{k_3} \), and \( D = k_0 e^{k_1 p} e^{k_2 w} e^{k_3 a} \) with the parameters \( k_1 < 0, \ k_0, k_2 > 0 \) and \( 1 > k_3 > 0 \). Note that the demand is supposed concave in \( a \) above to ensure interior solutions for advertising.

The profit function, now \( \pi : \mathbb{R}^{2+} \times \mathbb{R} \to \mathbb{R}^+ \), is twice continuously differentiable. It writes

\[
\pi(p, a, w) = p D(p, a, w) - a.
\]

\(^2\)All these classical assumptions are verified with our dataset as reported in Appendix A.2.
Note the similar role of temperature and advertising that both increase demand and thus revenue. The difference is that advertising is expensive for the firm, whereas temperature is free. We look for interior solutions for the price and advertising, assuming that they exist. The necessary and sufficient first- and second-order conditions for \(p\) and \(a\) read

\[
\pi_p = 0 \implies D + pD_p = 0 \implies e_p = 1, \quad (4a)
\]
\[
\pi_a = 0 \implies pD_a - 1 = 0 \implies e_a = \frac{a}{pD}, \quad (4b)
\]
\[
\pi_{pp} \leq 0 \implies D_p + D_p + pD_{pp} \leq 0 \implies -2D_p - pD_{pp} \geq 0, \quad (4c)
\]
\[
\pi_{aa} \leq 0 \implies pD_{aa} \leq 0 \implies D_{aa} \leq 0, \quad (4d)
\]
\[
\pi_{pp} \pi_{aa} - \pi_{pa}^2 \geq 0 \implies (2D_p + pD_{pp})pD_{aa} - (D_a + pD_{pa})^2 \geq 0, \quad (4e)
\]

with the advertising elasticity of demand \(e_a = D_a \frac{a}{D}\). Let us explain the three second-order conditions (4c)-(4e) that ensure concavity of \(\pi\) with respect to \(p\) and \(a\). Condition (4c) corresponds to (2b) and has been discussed already. Condition (4d) verifies that advertising has diminishing effects on demand. Condition (4e) is technical, without direct interpretation.

The two first-order conditions (4a) and (4b) impose

\[
e_p = 1, \quad e_a = \frac{a}{pD},
\]

expressing that the price and advertising are such that the price elasticity of demand is unitary and the advertising elasticity of demand equals the share of advertising expenses over the total revenue.

Note that comparing (4a) and (4b) dictates the rule of thumb of advertising

\[
\frac{e_a}{e_p} = \frac{a}{pD},
\]

best known as the condition of Dorfman and Steiner (1954). This condition stipulates
that the ratio of demand elasticities with respect to advertising and price $e_a/e_p$ equalizes the advertising intensity, measured by the profit-maximizing amount of advertising relative to total revenues $\frac{a}{pD}$. Plus, a monopoly sets a price such that the price elasticity of demand is unitary, which is verified by (4a) indicating $e_p = 1$. That is, the condition of Dorfman and Steiner (1954) simplifies to $e_a = \frac{a}{pD}$, given by (4b). Note that this simplified condition of Dorfman and Steiner (1954) informs the relationships between the optimal values of price level and advertising expense. But, it remains silent about any causal implication.

The condition above ignores temperature interactions, and thus the impact of temperature on pricing and advertising. The role played by temperature is explicit by decomposing the first-order conditions (4a) and (4b). Such decomposition is achieved with the time differentiation of these conditions.

$$\begin{align*}
\dot{p} &= 0 \implies \dot{p}(-2D_p - pD_{pp}) + \dot{a}(-D_a - pD_{pa}) = \dot{w}(D_w + D_{pw}), \quad (5a) \\
\dot{a} &= 0 \implies \dot{a}(-D_a - pD_{ap}) = \dot{w}D_{aw}, \quad (5b)
\end{align*}$$

Recall $D_{ij} = D_{ji}$, $\forall i, j = p, a, w$. Define $H_2 = (2D_p + pD_{pp})D_{aa} - (D_a + pD_{pa})^2$. Note $H_2 \geq 0$ because of the Hessian condition (4e). If $H_2 = 0$, the dynamics of $p$ and $a$ hold unknown. If $H_2 > 0$, the dynamics of $p$ and $a$ are given by

**Proposition 2.** For a branded product, the impact of higher temperature on price and advertising is characterized by

$$\begin{align*}
\dot{p} H_2 &= \dot{w} \left[ -pD_{pa}(D_w + D_{pw}) + D_{aw}(D_a + pD_{pa}) \right], \\
\dot{a} H_2 &= \dot{w} \left[ D_{aw}(-2D_p - pD_{pp}) + (D_a + pD_{pa})(D_w + D_{pw}) \right], \\
\end{align*}$$

for all $t > 0$.

**Proof.** Solve (5a)-(5b) with the rule of Cramer. 

Proposition 2 informs that temperature has an ambiguous impact on both price and
advertising. The direct effect of advertising and temperature on demand $D_a$ and $D_w$ and the indirect effect of advertising and temperature $D_{aw}$ promote a positive impact. But, the indirect effects of price with advertising and temperature $D_{pa}$ and $D_{pw}$ exert an opposite influence. If the direct effects outweigh the indirect effects tied to price, then price and advertising increase with temperature. In this case, when temperature increases, the firm maximizes profit by charging and advertising more. Conversely, if the indirect effects linked to price overcome the direct effects, then price and advertising decrease with temperature. In this situation, after an increase in temperature, the firm is better off charging and advertising less. For a general demand function, the impact of temperature on price and advertising is ambiguous, depending on the relative strength of the direct and indirect effects on demand. Assuming away the indirect effects of price simplifies the analysis.

**Corollary 2.** For a branded product, if there is no indirect effect of price at the demand level ($D_{pw} = D_{pa} = 0$), the impact of higher temperature on price and advertising is characterized by

$$
\dot{p} \ H_2 = \dot{w} \ D_{aw} D_a,
$$

$$
\dot{a} \ H_2 = \dot{w} \left[ D_{aw} \left( -2D_p - pD_{pp} \right) + D_a D_w \right],
$$

for all $t > 0$.

**Proof.** Substitute $D_{pw} = D_{pa} = 0$ in Proposition 2.

If price is not linked to advertising and temperature at the demand level, then results follow intuition. Corollary 2 shows that price and advertising increase when temperature augments. That is, price and advertising are complement to temperature. If the demand function is additively separable in the price with $D = h(p) + l(a, w)$, then the indirect effects of price vanish ($D_{pw} = D_{pa} = 0$) and Corollary 2 applies. An instance of demand function additively separable in price is $D = k_0 w^{k_2} a^{k_3} - k_1 p$, where $k_0 w^{k_2} a^{k_3}$ measures
the market potential. Omitting all indirect effects of price simplifies further the analysis, interestingly offering stronger but less intuitive results.

**Corollary 3.** For a branded product, if there is no indirect effect at the demand level \( D_{pw} = D_{pa} = D_{aw} = 0 \), the impact of higher temperature on price and advertising is characterized by

\[
\begin{align*}
\dot{p} H_2 &= 0, \\
\dot{a} H_2 &= \dot{w} D_a D_w.
\end{align*}
\]

for all \( t > 0 \).

**Proof.** Substitute \( D_{pw} = D_{pa} = D_{aw} = 0 \) in Proposition 2. 

Omitting the indirect effects simplifies greatly the analysis. Following Corollary 3, price remains constant (contrasting with Corollary 1) and advertising increases with temperature. In other words, price is independent from temperature and advertising is complementary to temperature. Disregarding the indirect effects, it is intuitive that advertising increases with temperature, but it is more surprising that price is constant over time. If the demand function is additively separable with \( D = h(p) + l(w) + m(a) \), the indirect effects disappear \( (D_{pw} = D_{pa} = D_{aw} = 0) \) and Corollary 3 is verified. An example of additive separable demand function is \( D = k_0 + k_1 p + k_2 w + \frac{1}{k_3} a^{k_3} \) previously presented.

Proposition 2 and Corollaries 2 and 3 hold while the classical condition of Dorfman and Steiner (1954) is verified. The condition of Dorfman and Steiner (1954) shows the necessary linkage between optimal values of price and advertising, but it describes no causal relationship with temperature.

Propositions 1 and 2 show that when temperature increases, the adaptation of the marketing-mix, that is of price and advertising, is unknown, both for the non-branded (price decision) and branded (price and advertising decisions) products. Indeed, the direct effects of price and advertising in response to a temperature increase seem intuitive. But
the analysis reveals that indirect effects—between price on the one side and advertising and temperature on the other side—play out in an opposite way. The total effect of temperature on price and advertising depends thus on the relative weights of the direct and indirect effects, and the theoretical analysis allows no general conclusion. Indeed, indirect effects at the demand level imply an ambiguous impact of temperature on pricing and advertising (Propositions 1 and 2 for a general demand function), whereas the absence of indirect effects drives a clear impact of temperature (Corollaries 1, 2, and 3 for a more specific demand function).

Additive separability of the demand function possesses clear-cut implications: Corollary 2 indicates that the additive separability of price in the demand function is a sufficient (but not necessary) condition for price and advertising to increase with temperature; Corollary 3 points that the additive separability of the demand function with respect to all arguments is a sufficient (but not necessary) condition for price to be constant and advertising to augment with temperature. In practice, thus, concluding about the impact of temperature imposes to provide empirical evidence about the specific properties of the demand function in a given market. Still, the theoretical analysis highlights the need to integrate both direct and indirect effects at the demand level in the empirical analysis.

4 Estimation Strategy and Data

To estimate the theoretical model, we use the case of drinking yogurt in France. The market of drinking yogurt in France is historically dominated by one leading brand, namely Yop. We use data from 2008-2011, because at that time, the mass market was mainly divided into Yop and the retailer brands.3 No competitor had managed to enter the mass market with a brand on a national or regional scale. At that time, peripheral brands existed, but their geographical coverage and thus their sales were limited. For instance Michel & Augustin was a high-end positioning brand focusing on the Parisian market, but it had little effect on the mass market of drinking yogurt. In other words, the French market of drinking yogurt can be approximated as a local duopoly market, with

3This insight comes from a discussion with a former product manager of drinking yogurts.
the products of Yop and of retailers. The market of drinking yogurts thus appears as a
textbook case of a market with a branded product, for which both price and advertising
need to decided, and a non-branded product, for which only price is a decision.

Consider the following demand (volume of sales) function, obeying the theoretical
assumptions formalized in Equations (1) and (3)

\[ Sales = e^{k_0} Price^{k_1} Advertising^{k_2} Temperature^{k_3}, \]

with \( k_0, k_2, k_3 > 0 \) and \( k_1 < 0 \). This demand function lets all direct and indirect effects
(identified in the previous section) play a role at the demand level.

Taking the logarithms yields

\[ \ln(Sales) = k_0 + k_1 \ln(Price) + k_2 \ln(Advertising) + k_3 \ln(Temperature). \] (9)

To test Equation (9) we then need data on the demand, price, advertising, and temper-
ature. For each variable, we obtained weekly data for the period from October 2008 to
September 2011. Data on demand, price, and temperature are easily measurable. First,
data on the demand and prices are extracted from a French retail panellist representative
of generalist department stores. Because of privacy and confidentiality issues, we use
aggregated data at the brand-level. For each brand, we aggregated the data from the
7,000 retail stores with more than 400 square meters of sales area and a food department
(it does not include neither hard-discount stores, nor gas stations, convenience stores,
and specialist department stores). We use \( Sales \) (in volume) as a variable to capture the
demand. This variable represents the kilograms of drinking yogurt sold in the French
market. We define the \( Price \) variable as the average price per kilogram when the prod-
uct is not sold under a promotional offer. Finally, we define the \( Temperature \) variable,
which represents the outdoor average temperature in France on a weekly basis, in de-
grees Fahrenheit. These data are from the InfoClimat StatIC database covering 1,168
measuring points located all over France.

Data on advertising efforts by brand are not directly available. However, we construct
two advertising variables from the media-buying agency Havas from Mediametrie’s audience measurement reports covering 5,000 households (11,600 persons over the age of 4). Mediamat is the French measurement panel of TV audiences in France, extracted on a daily basis for each program broadcast by national TV stations (TNT, cable and satellite TV, ADSL). This panel is elaborated to be representative of socio-demographic characteristics of French households and of the TV offers. We define two advertising variables based on TV media exposure of targeted shoppers over 35 housewives with children (due to the fact that drinking yogurt is mostly consumed at home by children between the ages of 8 and 15, over-35 housewife makes purchase decision and is therefore the media target). The first is Advertising, which is defined as the Gross Rating Point (GRP), that is, the average number of contacts achieved by an advertising campaign out of 100 people in the target population. To handle the problem of missing data due to the logarithmic transformation of observations where Advertising equals zero, we added one before this transformation \( \ln(\text{Advertising} + 1) \). For the robustness tests, we use an alternative variable accounting for the lasting impact of GRP, namely \( \text{Stock of Advertising} \) at time \( t \) defined by \( \text{Stock of Advertising}_t = \text{Advertising}_t + 0.95 \times \text{Stock of Advertising}_{t-1} \). The stock of advertising accounts for the carrying over impact of the GRP, the current GRP weighting more than the past GRP.

In addition, to reduce potential bias generated by omitted variables, we add two supplementary control variables. The first is a dummy variable named Yop, this variable takes the value of one if yogurt brand is Yop, and zero otherwise. This variable is helpful to capture the power of brand (which influences the value of the theoretical model’s parameter \( k_0 \)), as well as unobserved elements of the advertising strategy (which could influence the value of the theoretical model’s parameter \( k_2 \)). Eventually, we define a control variable named Promotion, which represents the volume (in kilograms) of sales under promotional offers in the outlet through brochure promotions, promotional packs, price reductions, or loyalty programs.

\(^4\)This insight comes from a discussion with a former product manager of drinking yogurts.
<table>
<thead>
<tr>
<th>Table 2: Descriptive statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Sales volume</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Price</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Advertising</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Stock of Advertising</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Temperature</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Observations</td>
</tr>
</tbody>
</table>

Notes. * p < 0.05, ** p < 0.01. Sample means and standard deviations (in parentheses) are reported for each variable. Brands are classified into two groups: Yop and retailers’ brands. F-tests are the $f$-statistic under the hypothesis that the brand means are identical.

We proceed then to estimate the following model:

$$\ln(Sales_{it}) = \alpha + \beta_1 \ln(Price_{it}) + \beta_2 \ln(Advertising_{it}) + \beta_3 \ln(Temperature_{it}) + \beta_4 \ln(X_{it}) + \epsilon_{it},$$

(10)

where $i$ and $t$ index the brand (Yop and retailer) and the time, $X$ corresponds to the vector of control variables $Promotion_{it}$ and $Yop_i$, and $\epsilon_{it}$ measures the error term. Appendix A.2 links the empirical estimation of (10) to the theoretical modeling presented in the previous section.

Table 2 presents summary statistics for the variables that we use in the analysis. Column 1 reports the sample mean and the standard deviation for the entire sample. In the next two columns, we classify the brands into two groups according to the Yop variable. Columns 2 and 3 show that, the volume of sales, but also the price and advertising are higher for Yop than for the retailers brand. The last column reports the F-statistics under the hypothesis that the variables have the same mean for Yop and the retailers’ brand. The F-statistic shows that, excepting temperature, differences between Yop and retailers’ brand are significant for all the variables.
Table 3: Determinants of sales using GRP as a measure of advertising

<table>
<thead>
<tr>
<th>Dependent variable: Log of sales volume of drinking yogurts</th>
<th>OLS (1)</th>
<th>IV (2)</th>
<th>OLS (3)</th>
<th>OLS (4)</th>
<th>OLS (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>-0.786**</td>
<td>-0.764**</td>
<td>-0.759**</td>
<td>-12.637**</td>
<td>-0.568**</td>
</tr>
<tr>
<td></td>
<td>(0.216)</td>
<td>(0.274)</td>
<td>(0.210)</td>
<td>(2.512)</td>
<td>(0.203)</td>
</tr>
<tr>
<td>Advertising</td>
<td>0.021**</td>
<td>0.039**</td>
<td>0.012*</td>
<td>0.012*</td>
<td>-0.578**</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.012)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.086)</td>
</tr>
<tr>
<td>Temperature</td>
<td>0.686**</td>
<td>0.675**</td>
<td>0.533**</td>
<td>-1.650**</td>
<td>0.593**</td>
</tr>
<tr>
<td></td>
<td>(0.050)</td>
<td>(0.072)</td>
<td>(0.063)</td>
<td>(0.481)</td>
<td>(0.051)</td>
</tr>
<tr>
<td>Yop</td>
<td>0.815**</td>
<td>0.800**</td>
<td>-0.478</td>
<td>0.816**</td>
<td>0.778**</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.043)</td>
<td>(0.340)</td>
<td>(0.032)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>Temperature × Yop</td>
<td>0.326**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.085)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price</td>
<td></td>
<td>2.991**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.625)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Advertising</td>
<td></td>
<td></td>
<td></td>
<td>0.146**</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.021)</td>
<td></td>
</tr>
</tbody>
</table>

Observations | 310 | 306 | 310 | 310 | 310
R-squared | 0.924 | 0.922 | 0.928 | 0.930 | 0.932
RMSE | 0.143 | 0.144 | 0.140 | 0.137 | 0.136
Hansen J p-value | .0704

Note: * p < 0.05, ** p < 0.01. The constant is not reported. All variables are expressed in log form. The estimators are OLS ordinary least squares and IV instrumental variables. Heteroskedastic and auto-correlation robust standard errors are in parentheses. The Hansen J-test reports the p-values for the null hypothesis of the instruments validity. Each regression controls for promotion (brochure promotions, promotional packs, price reduction, or loyalty program).

5 Results

Table 3 reports the estimates of Equation (10) to determine the variables with influence on sales of drinking yogurts. Appendix A.1 verifies that the estimates are robust with an alternative measure of advertising. Table 3 presents both direct effects (Columns 1 & 2) and moderating effects (Columns 3, 4, and 5) of sales determinants.

Columns 1 & 2 of Table 3 present the direct effects of price, advertising, temperature, and brand on drinking yogurt sales (promotion is a control variable). The estimates verify that drinking yogurt is weather-sensitive because temperature fosters its sales. Further, sales also decrease with price and increase with advertising. All variables affecting sales of drinking yogurt are significant at the 1% level (Column 1). Following Column 1, we now quantify the direct effect of a variation in price, advertising, and temperature on sales.
Considering the price variable, we measure price elasticity of demand: a 10% increase of price reduces sales of drinking yogurt by 7.86%. The advertising variable possesses a significant impact on sales, but to a lesser extent: a 10% increase in advertising increases sales by 0.21%. Considering the temperature variable, an increase in temperature has a significant (Herrmann, 1998) and positive impact on sales: a 10% increase in temperature increases sales by 6.86%. Finally, the brand variable, it has a positive impact on sales: brand increases product sales by 81.5%. The findings led to show that sales are directly negatively affected by prices and positively affected by temperature, advertising, and brand.

Except for temperature which is an exogenous variable, there is potential endogeneity in the explanatory variables. We therefore use instrumental techniques (Column 2) to address this issue, and use 1-period and 2-period lags of explanatory variables as instruments. There is no significant variation in the coefficients, the various $R^2$ and the Root Mean Square Error are similar. The row for the Hansen $J$-test reports the $p$-values for the null hypothesis of the validity of the over-identifying restrictions. We do not reject the null hypothesis of instrument validity. Therefore we conclude that there is no endogeneity bias.

We now look to which extent temperature interferes with the effects of brand, advertising, and price on sales. Columns 3, 4 and 5 of Table 2 present the moderating effects of temperature on the relationship between sales and brand, price, and advertising. Column 3 presents the estimates to evaluate if the temperature influences Yop sales differently compared to the retailers’ brand. Results show that temperature significantly influences the brand effect. More precisely, the coefficient of the moderating variable suggests that brand advantage increases as temperature rises. Figure 1 illustrates better this result. The solid line represents the advantage of Yop brand over the retailers’ brand, and the dotted line represents the average temperature during the period of study. When temperature levels are in the average range, Yop sales increase 81.5% more than the retailers’, and this gap increases if temperature goes above the average. Hence, the advantage of

\[ \text{Appendix A.2 explains why the coefficients estimated in Column 1 benefit from an elasticity interpretation.} \]
the Yop brand over retailers’ increases as temperature rises.

Because demand is elastic to price, we then analyzed to which extent temperature affects price elasticity of demand. Column 4 presents the estimates. Price and brand continue to have a significant direct effect on sales at the 1% level, advertising at a the 5% level, and the moderating effect is positive and significant at the 1% level. This means that temperature moderates the effect of a price variation on demand. More precisely, the negative effects of increasing prices are lower as temperature increases. These findings are illustrated by Figure 2, which presents the marginal effects of price at different levels of temperature. The higher the temperature, the lower the price elasticity of demand. This result—variation of demand elasticity by season—is in line with Marion and Walker (1978). In other words, the higher the temperature (during the so-called “Yop season”), the higher a positive margin effect need to be so to offset a negative volume effect.

Finally, we wonder if the advertising impact on sales varies according to temperature. Column 5 presents the estimates. The coefficient of the interaction $Temperature \times Advertising$ suggests that the positive effects of advertising are reinforced as temperature increases. We compute the marginal effects of advertising, and we illustrate these effects in Figure 3. Figure 3 depicts that the higher the temperature, the greater is the advertising elasticity of demand. In addition, it can be noticed that the positive impact of advertising on sales starts from a defined temperature (50 degrees Fahrenheit). Figure 4 illustrates the density distribution of observations with and without advertising. When temperature is below 50 degrees Fahrenheit, there are few observations with advertising.

To test the robustness of our findings, we replicate the estimates using a stock of

\[ \text{Note that in the case of the moderating variable, we can consider either predictor to moderate the effect of the other. Hence, price could also moderate the effects of temperature. The effect of temperature on sales equals } -1.650 + 2.991 \ln(Price). \text{ It is then positive if } Price > 1.73. \text{ Because the average firm size of our sample is 2.21, the average effect of temperature is then positive.} \]

\[ \text{According to a former product manager of drinking yogurts, the “Yop season” corresponds to the months of higher temperature.} \]
the GRP as a measure of advertising; estimates are similar (see Table A1). Moreover, estimates for the specification with interaction between advertising and temperature suggest that the positive effects of advertising on sales starts at 36.6 degrees Fahrenheit. This could explain why there is advertising expense when temperature is lower than 38 degrees Fahrenheit (Figure 4).

6 Conclusion

In this study, we examine the effect of temperature on the marketing-mix (pricing and advertising) of weather sensitive products. In the first stage, we formulate an analytical model to theoretically examine the effect of temperature increase on price and advertising. The model is general enough to encompass the effect of seasonal factors that increase the demand. With a general demand function, no formal guarantee exists about complementarity or substitutability between temperature and the marketing elements. Consequently, specific investigations have to be conducted for each market. In the second stage, we provide an empirical investigation of the French drinking yogurt market. In this market, empirical evidence shows that price and advertising are complementary to temperature. That is, the retailer maximizes profit by charging and advertising more when temperature increases. Results also reveal that a rise in temperature increases more the sales of the branded product, which become even more expensive. Thus, the market power of the brand augments with temperature. This is in line with anecdotal evidence that high temperature helps trigger impulse buying of shoppers, which is typically more brand-oriented.

By pointing to the influence of temperature, and more broadly to weather and seasonality factors, this article offers a more comprehensive understanding of the marketing-mix for weather sensitive products. Product managers and retailers would gain from adapting their pricing and advertising policies to weather conditions. In effect, conducting a weather dependent marketing-mix, both for branded and non-branded products, enables wider profiting opportunities. More broadly, profit opportunities still unknown for
weather sensitive products call for future research on the optimal relationships between controllable variables (such as price and advertising) and uncontrollable variables (such as temperature and weather).
Figure 1: Effects of Yop brand on sales volume

Notes: The solid line represents the marginal effects computed from estimates reported in Column 3 of Table 3. The dashed line around the solid represents a 95% confidence interval. The vertical dotted line represents the average temperature during the period of study. Temperature values are in degrees Fahrenheit.
Figure 2: Effects of price increases on sales volume

Notes: The solid line represents the marginal effects computed from estimates reported in Column 4 of Table 3. The dashed line around the solid represents a 95% confidence interval. The vertical dotted line represents the average temperature during the period of study. Temperature values are in degrees Fahrenheit.
Figure 3: Effects of advertising increases on sales volume

Notes: The solid line represents the marginal effects computed from estimates reported in Column 5 of Table 3. The dash line around the solid represent a 95% confidence interval. The vertical dotted line represents the average temperature during the period of study. Temperature values are in degrees Fahrenheit.
Figure 4: Distribution of observations by temperature

Notes: *With advertising* corresponds to those observations in which $GRP > 0$, and *Without advertising* are those in which $GRP = 0$. Temperature values are in degrees Fahrenheit.
A Appendix

A.1 Robustness Check

Table A1: Determinants of sales using the stock of GRP as a measure of the carrying-over effect of advertising

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>IV</th>
<th>OLS</th>
<th>OLS</th>
<th>OLS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>Price</td>
<td>-0.823**</td>
<td>-0.609*</td>
<td>-0.763**</td>
<td>-11.976**</td>
<td>-0.701**</td>
</tr>
<tr>
<td></td>
<td>(0.214)</td>
<td>(0.240)</td>
<td>(0.212)</td>
<td>(2.450)</td>
<td>(0.207)</td>
</tr>
<tr>
<td>Stock of Advertising</td>
<td>0.054**</td>
<td>0.046**</td>
<td>0.028*</td>
<td>0.030*</td>
<td>-0.200**</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.012)</td>
<td>(0.013)</td>
<td>(0.012)</td>
<td>(0.056)</td>
</tr>
<tr>
<td>Temperature</td>
<td>0.649**</td>
<td>0.683**</td>
<td>0.534**</td>
<td>-1.537**</td>
<td>0.496**</td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
<td>(0.064)</td>
<td>(0.064)</td>
<td>(0.472)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>Yop</td>
<td>0.486**</td>
<td>0.530**</td>
<td>-0.513</td>
<td>0.629**</td>
<td>0.679**</td>
</tr>
<tr>
<td></td>
<td>(0.079)</td>
<td>(0.085)</td>
<td>(0.328)</td>
<td>(0.083)</td>
<td>(0.085)</td>
</tr>
<tr>
<td>Temperature × Yop</td>
<td></td>
<td></td>
<td></td>
<td>0.292**</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.090)</td>
<td></td>
</tr>
<tr>
<td>Temperature × Price</td>
<td></td>
<td></td>
<td></td>
<td>2.820**</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.615)</td>
<td></td>
</tr>
<tr>
<td>Temperature × Stock of Advertising</td>
<td></td>
<td></td>
<td>0.056**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.012)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observations: 310 306 310 310 310
R-squared: 0.925 0.926 0.928 0.931 0.930
RMSE: 0.142 0.140 0.140 0.137 0.137
Hansen J p-value: .908

Note: * p < 0.05, ** p < 0.01. The constant is not reported. All variables are expressed in log form. The estimators are OLS ordinary least squares and IV instrumental variables. Heteroskedastic and auto-correlation robust standard errors are in parentheses. The Hansen J-test reports the p-values for the null hypothesis of the instruments validity. Each regression controls for promotion (brochure promotions, promotional packs, price reduction, or loyalty program).
A.2 Demand Estimation

An instance of demand function copying with assumptions (3) (theoretical modeling in Section 3) is the Cobb-Douglas

\[ D = k_0 p^{k_1} a^{k_2} w^{k_3}, \]

with \( k_1 < 0, k_0, k_3 > 0 \) and \( 0 < k_2 < 1 \).

Accounting for various controls and the error term \( c \) (empirical estimation in Section 4) enriches the previous formulation to

\[ D = k_0 p^{k_1} a^{k_2} w^{k_3} c. \]

According to Column 1 of Table 3, the estimation of the demand function writes

\[ D = k_0 p^{-0.788} a^{0.021} w^{0.686} c. \]

It is easy to verify that the estimated demand function copes with assumptions (3). In effect, simple derivations yield

\[ D_p = -0.788 \frac{D}{p} < 0, \quad D_a = 0.021 \frac{D}{a} > 0, \quad D_w = 0.686 \frac{D}{w} > 0, \]

\[ D_{aa} = 0.021(0.021 - 1) \frac{D}{a^2} \leq 0, \quad D_{pw} = -0.788 \times 0.688 \frac{D}{pw} \leq 0, \]

\[ D_{pa} = -0.788 \times 0.021 \frac{D}{pa} \leq 0, \quad D_{aw} = 0.021 \times 0.686 \frac{D}{aw} \geq 0. \]

Also, with a Cobb-Douglas (or constant elasticity) demand function, the coefficient of each variable corresponds to the demand elasticity of this variable. The price, advertising,
and temperature elasticity of demand write

\[ e_p = -\frac{\partial D/D}{\partial p/p} = -\frac{\partial D/\partial p}{D/p} = 0.788, \]
\[ e_a = \frac{\partial D/D}{\partial a/a} = \frac{\partial D/\partial a}{D/a} = 0.021, \]
\[ e_w = \frac{\partial D/D}{\partial w/w} = \frac{\partial D/\partial w}{D/w} = 0.686, \]

offering a direct economic interpretation.

Elasticity interpretation is the following. When product price increases by 10%, then demand decreases by 7.88%; when advertising expense increases by 10%, then demand increases by 0.21%; when outdoor temperature increases by 10%, then demand increases by 6.86%.
References


